# Expectations and term premia in ESM bond yields*

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**Abstract**

The European Financial Stability Facility (EFSF) was set up as a temporary solution to the Greek sovereign debt crisis in June 2010, and in October 2012 it evolved into the European Stability Mechanism (ESM), an institution offering a permanent solution and a new capital based structure to bond holders. The EFSF and ESM have so far disbursed €254.5 billion to five countries in the form of loans financed by bond issuance. The EFSF continues to rollover financing of existing debt. The ESM remains an active issuer and a lender of last resort for Euro Area countries that lose market access and for those in need of Pandemic Crisis Support. This paper offers the first study of the term structure of EFSF bond yields and a decomposition into expected interest rates and risk premia, based on a state of the art no-arbitrage term structure model. A joint model of the EFSF curve and the swap curve allows to further identify the liquidity and credit components of EFSF yields, and it offers a pricing model to EFSF investors and beneficiary member states.

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# 1 Introduction

The European Financial Stability Facility (EFSF) was set up as a temporary solution to the Greek sovereign debt crisis in June 2010, and in October 2012 it evolved into the European Stability Mechanism (ESM), an institution offering a permanent solution and a new capital based structure to bond holders. The ESM remains an active issuer and a lender of last resort for euro area countries that may be unable to tap the primary markets. The EFSF and ESM have so far disbursed €254.5 billion to five countries in the form of fixed income securities. The bond stock of debt amounts at €150 billion (EFSF) and €85 billion (ESM) with average annual flow of issuance of about €10 billion each year. These sizes make the EFSF / ESM the biggest supranational issuer in the EMU, yet demand is much greater than supply, so liquidity is guaranteed at some benchmark maturities.

Fixed income markets need liquid benchmark yield curves to price new bonds issued by small issuers due to lack of available market data. In practice, the issue of limited liquidity affects market quotes, and therefore the ability of investors to buy-sell their holdings at a reasonable price. Some maturities can be more affected than others, particularly the very short or the very long ones, depending on investors demand. All along comes the issue of short time series for new issuers that economists need for interest rate projections.

In recent years there has been a misalignment between liquid bond yields such as German and French yields and interest rate swaps that is also a very liquid market in the EMU area. Particularly, bonds prices have been affected by central bank's asset purchases and regulatory requirements that have lowered their yields, anything else equal. This is most noted for AAA yields after 2009 (banking crisis) and 2016 (Central Clearing Counterparty) as demand for High Quality Liquid Assets has increased.[1] On the other hand, swaps have been less affected, as they do not offer the same protection due to the fact they are not cash based.

These considerations led us to choose the EURIBOR swap curve as a benchmark, as it reflects more clearly dynamics of the natural rate of interest. The EURO swap market has become one of the largest and most liquid markets in the world (Remolona and Wooldridge (2003)). The hedging and positioning activity increased substantially in Euro denominated interest rate swaps since (Dalla Fontana et al. (2019)).

This paper contribution is threefold. Firstly, this paper is the first study fitting a no-arbitrage model to the EFSF yield curve, and to offer a full decomposition of the observed historical EFSF yields into the expectation and term premia component.[2] We find that the

[1] See Ranaldo et al. (2020).

[2] The model could be applied to ESM bond yields, as these can be and are priced in reference to the swap

term premia on the EFSF curve has increased sharply during the 2011-2012 crisis, and that it is also on the rise in the more recent months.

Secondly, when the term structure model is estimated jointly with the swap and spot curves, we assume that the latter carries an additional premium stemming from an additional pricing factor. A regression analysis shows that such additional premium and pricing factor are related to liquidity and credit conditions. In a comparative analysis based on Germany and France government bond yields we find that the liquidity/credit factor is larger in those spot curves more heavily affected by liquidity conditions such as funding liquidity cost, bid/offer spreads, and asset purchases by ECB.

Thirdly, this paper introduces methodological contribution consisting in a novel approach to price the yield curves of small issuers and / or illiquid markets. This is achieved by including in the model a benchmark liquid curve, and specifying an additional factor which is ex-ante attributed to liquidity/credit.

The paper is organized as follows. Section 2 describes the no arbitrage model used in the paper, Section 3 describes the data, Section 4 illustrates the decomposition of EFSF yields in expectations and term premia, Section 5 discusses the liquidity/credit pricing factor. Section 6 concludes. Further technical details are in the Appendix.

## 2 Model

Since the seminal work of Vasicek (1977) a large part of research has focused on Gaussian Affine Term Structure Models (GATSM). Prominent contributions in this tradition include Duffie and Kan (1996), Dai and Singleton (2000), Duffee (2002), and Ang and Piazzesi (2003).

Let $y_t$ denote a vector of yields on a set of zero-coupon bonds of maturity $\tau = 1, ..., N$. In the Duffie and Kan (1996) canonical term structure model, the yields are driven by an $n$-dimensional vector of unobservable risk factors $S_t$:

$$y_t = A_S^{\mathbb{Q}} + B_S^{\mathbb{Q}} S_t + \Sigma_y \varepsilon_t^y, \tag{1}$$

$$\Delta S_t = K_{0S}^{\mathbb{P}} + K_{1S}^{\mathbb{P}} S_{t-1} + \Sigma_S \varepsilon_t^{\mathbb{P}}, \tag{2}$$

where $A_S^Q$ and $B_S^Q$ are $N \times 1$ and $N \times n$ coefficient matrices, $K_{0S}^{\mathbb{P}}$ is a $n \times 1$ vector, $K_{1S}^{\mathbb{P}}$ is a $n \times n$ matrix, and $\Sigma_y$ and $\Sigma_S$ are lower triangular Cholesky factor matrices. The disturbances $\varepsilon_t^y$, $\varepsilon_t^{\mathbb{P}}$ are *i.i.d.* $N(0, I)$ vector processes and are mutually independent.

curve. In this study we focus on EFSF only due to the larger available sample size.

Equations (10)-(11) constitute a factor model in which the yields depend linearly on the factors $S_t$ trough the intercept vector $A_S^Q$ and the factor loadings $B_S^Q$. These equations do not make explicit the role of the no arbitrage assumption. Such assumption further implies that $A_S^Q$ and $B_S^Q$ are (highly) nonlinear functions of some deep parameters $\Theta_S^{\mathbb{Q}} = \{K_{0S}^Q, K_{1S}^Q, \Sigma_S, \rho_{0S}, \rho_{1S}\}$, i.e. $A_S^{\mathbb{Q}} = A(\Theta_S^Q)$ and $B_S^{\mathbb{Q}} = B(\Theta_S^Q)$. Specifically, the elements in any generic row $\tau$ of $A_S^{\mathbb{Q}}$ and $B_S^{\mathbb{Q}}$ must obey a set of (highly) nonlinear restrictions ensuring that there are no arbitrage opportunities:

$$A_S^{\mathbb{Q}}(\tau) = -A_\tau/\tau, \; A_{\tau+1} = A_\tau + K_{0S}^{\mathbb{Q}\prime} B_\tau + 0.5 B_\tau' \Sigma_S \Sigma_S' B_\tau - \rho_{0S}, \tag{3}$$

$$B_S^{\mathbb{Q}}(\tau) = -B_\tau/\tau, \; B_{\tau+1} = B_\tau + K_{1S}^{\mathbb{Q}\prime} B_\tau - \rho_{1S}, \tag{4}$$

with initial conditions $A_0 = B_0 = 0$. The deep parameters $\Theta_S^{\mathbb{Q}}$ describe the evolution of the state variables under the so-called equivalent martingale measure:

$$\Delta S_t = K_{0S}^{\mathbb{Q}} + K_{1S}^{\mathbb{Q}} S_{t-1} + \Sigma_S \varepsilon_t^{\mathbb{Q}}, \tag{5}$$

as well as the dynamics of the instantaneous risk free rate $r_t$:

$$r_t = \rho_{0S} + \rho_{1S} S_t, \tag{6}$$

where $K_{0S}^{\mathbb{Q}}$ is a $n \times 1$ vector, $K_{1S}^{\mathbb{Q}}$ is a $n \times n$ matrix, $\rho_{0S}$ a scalar, $\rho_{1S}$ a $n \times 1$ vector, and $\varepsilon_t^{\mathbb{Q}}$ is an *i.i.d.* $N(0, I)$ vector process. With no loss of generality, we use the normalization $\rho_{0S} = 0$ and $\rho_{1S}$ =1 a $n \times 1$ vector of ones.

Here $\mathbb{Q}$ and $\mathbb{P}$ denote the risk neutral and physical measures of probability. Under the $\mathbb{P}$ measure agents' risk aversion implies that prices need to be predictable to some extent, producing the expected returns necessary to compensate investors for bearing risks. Under this measure the states follow the dynamics described by (2). Under the $\mathbb{Q}$ probability measure, prices are a martingale, which resembles a hypothetical situation in which investors are risk neutral. Under this measure, the states behave according to (5). The existence of the equivalent martingale measure $\mathbb{Q}$ is a necessary and sufficient condition of the absence of arbitrage. Conversion from the $\mathbb{P}$ to the $\mathbb{Q}$ measure can be achieved using a variable transformation described by a Radon-Nikodym derivative that, together with the risk free rate (6), forms the pricing kernel.[3]

It is important to distinguish the assumption of absence of arbitrage and the additional specification restrictions inherent in a GATSM. In particular, there are other assumptions

[3] In particular, under the $\mathbb{Q}$ measure the price of an asset $V_t$ that does not pay any dividends at time $t+1$ satisfies $V_t = E_t^{\mathbb{Q}}[\exp(-r_t)V_{t+1}]$, where $r_t$ is the short term rate. Under the $\mathbb{P}$ measure the price is $V_t = E_t^{\mathbb{P}}[(\xi_{t+1}/\xi_t)\exp(-r_t)V_{t+1}]$, where $\xi_{t+1}$ is the Radon-Nikodym derivative. The term $(\xi_{t+1}/\xi_t)\exp(-r_t)$ is referred to as the stochastic discount factor (or pricing kernel).

which are not required to guarantee the absence of arbitrage, but are needed in order to estimate the model or to compute quantities of interest. For example, the use of a VAR(1) for the law of motion of the factors under the $\mathbb{P}$ measure.[4] Similarly, no arbitrage only requires the existence of a pricing kernel, but it is silent about the form of such kernel. A log-normal form is typically chosen, which provides tractability.[5]

These additional assumptions can improve efficiency, but can lead to misspecification.

## 2.1 A Joint model of swap and spot rates

In the empirical application, the $N$-dimensional $y_t$ vector of yields is partitioned into $N^{swap}$ swap rates $y_t^{swap}$ and $N^{spot}$ spot rates $y_t^{spot}$:

$$y_t = \begin{bmatrix} y_t^{swap} \\ y_t^{spot} \end{bmatrix}. \tag{7}$$

We set $n = 4$, which means there are a total of four factors driving $y_t$. However, we assume that while the short term spot rate responds to all four factors, the short term swap rate only depends on the first three factors. This is implemented by augmenting the model with an equation for the instantaneous swap rate:

$$r_t^{swap} = \delta_{0S} + \delta_{1S} S_t, \tag{8}$$

where $\delta_{1S}$ is a 4-dimensional vector with the last element equal to 0. This implies that:

$$r_t^{spot} - r_t^{swap} = S_t^{(4)}, \tag{9}$$

i.e. the difference between the spot and swap rate is given by the fourth factor $S_t^{(4)}$.[6] It also implies that the entire swap curve will not depend on the fourth factor. As we shall

[4] Duffee (2011b) shows that it is entirely possible for the factors to follow richer dynamics in the physical measure than in the risk neutral measure and that this translates to the presence of hidden factors which -while not useful in explaining the cross-section of yields- can help in explaining their dynamics. Similarly, Joslin, Priebsch, and Singleton (2012) show that a VAR representation (under the physical measure) including measures of real economic activity and inflation captures better the dynamics of the term structure. In this paper, we illustrate the proposed approach using the simpler framework offered by yields-only models, but our approach can be naturally extended to models allowing for macroeconomic factors.

[5] In particular it is assumed that $\ln(\xi_{t+1}/\xi_t) = -0.5\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_t^{\mathbb{P}}$, which implies the stochastic discount factor is log-normal with conditional mean $-r_t$ $-0.5\Lambda_t'\Lambda_t$ and conditional variance $\Lambda_t'\Lambda_t$. Further assuming a linear price of risk $\Lambda_t = \lambda_0 + \lambda_1 S_t$, the relation between the coefficients of the factor dynamics under the two measures is: $K_{jS}^{\mathbb{Q}} = K_{jS}^{\mathbb{P}} - \Sigma_S\lambda_j;\ \ j = 0, 1$. Once can also add a constraint on the variabillity of prices of risk, for example costraining their Sharpe Ratios as in Duffee (2010):$\sqrt{\Lambda_t'\Sigma_S^{-1}\Sigma_S^{-1}\Lambda_t} < c$.

[6] To be precise, it is $r_t^{spot} - r_t^{swap} = \delta_{1S} S_t^{(4)}$, but recall we are using a normalization in which the autoregressive coefficients on the short term rates are one, hence $\delta_{1S} = 1$.

see, the fourth factor prices the specific risks associated with the spot rates, which in our application are EFSF bonds.

## 2.2 Estimation

Traditional no arbitrage term structure models entail a high level of nonlinearity - evident in the restrictions (3) and (4) - that makes the estimation extremely difficult and often unreliable (Duffee (2011a,b), Duffee and Stanton (2012), Hamilton and Wu (2012)). Some recent literature has successfully addressed this issue. Hamilton and Wu (2012) propose a strategy to estimate such models using a series of transformations and OLS estimation. Christensen, Diebold and Rudebusch (2011) proposed a no-arbitrage term structure model based on the Nelson and Siegel (1987) exponential framework. In this paper, we use the representation proposed by Joslin, Singleton and Zhu (2011), which is equivalent to the canonical representation of Duffie and Kan (1996), but parametrized in such a way that estimation is considerably simplified.

Joslin, Singleton and Zhu (2011) (JSZ) derive the following equivalent representation for equations (1) and (2):

$$y_t = A_P^{\mathbb{Q}} + B_P^{\mathbb{Q}} P_t + \Sigma_y \varepsilon_t^y, \tag{10}$$

$$\Delta P_t = K_{0P}^{\mathbb{P}} + K_{1P}^{\mathbb{P}} P_{t-1} + \Sigma_P \varepsilon_t^{\mathbb{P}}. \tag{11}$$

In (10) and (11), the factors $P_t = W\tilde{y}_t$ are portfolios composed of $N$ yields priced without error $\tilde{y}_t = A_S^{\mathbb{Q}} + B_S^{\mathbb{Q}} S_t$, and $\Sigma_P$ is the Cholesky factor of their conditional variance. Details of the transformation leading from (1)-(2) to (10)-(11) can be found in the appendix. The advantage of the JSZ rotation stems from the fact that the least-squares projection of the observable factors $P_t^o = Wy_t$ onto their lagged values will nearly recover the maximum likelihood estimates of $K_{0P}^{\mathbb{P}}$ and $K_{1P}^{\mathbb{P}}$ to the extent that $P_t^o \approx P_t$. Moreover, the intercepts $A_P^{\mathbb{Q}} = A(\Theta_P^Q)$ and the loadings $B_P^{\mathbb{Q}} = B(\Theta_P^Q)$ depend on a smaller set of deep parameters $\Theta_P^{\mathbb{Q}} = \{k_\infty^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_P\}$, where $\lambda^{\mathbb{Q}}$ are the (ordered) eigenvalues of $K_{1S}^{\mathbb{Q}}$ and $k_\infty^{\mathbb{Q}}$ is the first element of $K_{0S}^{\mathbb{Q}}$ (the remaining elements of this vector being zero).

Clearly, the model at hand is a linear Gaussian state space system. Equations (10) and (11) (or, in the equivalent canonical representation, equations (1) and (2)) are respectively the transition and measurement equation. Equations (5) and (6) are implicitly embedded in the model trough the restrictions (3) and (4) on the factor loadings: it is these restrictions that impose the absence of arbitrage. Estimation can be performed via maximum likelihood.

# 3 Data

All data is at monthly frequency, with a monthly data point equal to the average of the daily data points in any given month. The swap curve $y_t^{swap}$ is based on the 6-month Euribor swap rates, ranging from April 2000 to April 2020. The EFSF zero coupon equivalent rates are provided by the ESM and are obtained using Svesson's method, and range between April 2012 to April 2020. The spot curves $y_t^{spot}$ of Germany and France are sourced from Bloomberg and are available from April 2000 to April 2020.

We select the most liquid maturities and include a swap rate with very short maturity as a proxy for the risk free rate. For the EFSF the maturity choice is limited as it is a supranational smaller market, so maturities start at 1-year. Analysis of the fitting showed that a minimum 6 maturities is needed per curve, and that the swap, Germany and France can be estimated with a 3 month rate, but this is not the case for the EFSF curve. The swap, Germany and France curves can be estimated up to 20-year with a small fitting error, but that is not the case for EFSF, as the long-rate affects the yield decomposition assumption of the no-arbitrage model (relationship between short rate and long rate). Based on these considerations, we chose maturities 1, 2, 3, 5, 7, 10 -years for the spot curves (EFSF, Germany, France) and 3-month and 1, 2, 3, 4, 5, 7, 10 -years for the swap curve.

Since the EFSF was established only in 2012, yields for the period 2000-2012 are note available. While estimation using a shorter sample is possible, the time series dimension of the EFSF yields is too limited to provide reliable results, and for this reason, we backcast the EFSF data to April 2000, using an average of the zero-coupon equivalent yields for the countries entering the EFSF capital, with weights given by the respective proportions of capital assigned to each country. Specifically, The EFSF data set between April 2000 and March 2012 has been reconstructed using a 'synthetic' curve based on the ESM capital keys by shareholder:[7]

$$y_{t=2000/4:2012/3}^{EFSF} = \frac{\Sigma_{i=1}^{5} w_i y_t^c}{\Sigma_{i=1}^{5} w_i}$$

We only used the first 5 shareholders in order of share's size (Germany, France, Italy, Spain, The Netherlands) with the following weights $w_i$: France=20.24; Italy =17.79; Germany=26.96; Spain= 11.8; Netherlands=5.7, which represents more than 80% of the total capital. The data for the curves Netherlands, Italy and Spain are from Bloomberg. The rationale behind this choice is that the ESM and EFSF curves are very similar[8] and that

[7] See http://esm.europa.eu

[8] ESM yields only trade a couple of basis points below EFSF yields. The yield difference is mainly due to the capital structure for the ESM and a guarantee structure for the EFSF. In this study, we assume the two structures offer the same investor's protection.

EFSF rates tend to co-move with the core EMU countries.[9]

Figure 1 shows the historical curves by issuers, and Figure 2 shows the spreads between the swap and the spot rates (this figure also includes data on Italian bonds, for reference). Our synthetic instrument shows a reasonable behavior before 2012. Between 2000 and 2009 the 10-year swap spread (Figure 2 and Table 4-timeline of events) was narrow like the ones of Germany, France and Italy. The tight swap spreads for most countries are in line with a period of positive economic growth for the euro area, when inflation was also close or on target at 2%. After 2009 the financial and sovereign debt crisis have marked a regime change, especially for Italy, as we can see from a significant widening of the Italian swap spread and a rise on the Italian 5-year CDS Between 2009 and 2012 the EFSF swap spread widens at the margins due to the Italian effect. From 2012, we have real EFSF observations. In 2012-13 a wider swap spread is also justified by the initial illiquidity of EFSF bond market.

# 4 Decomposing the EFSF curve

## 4.1 Model implied yields and in sample fit

Estimation of the model provides the model-implied yields:

$$\tilde{y}_t = A_S^{\mathbb{Q}} + B_S^{\mathbb{Q}} S_t, \tag{12}$$

which depend on parameters belonging to the risk neutral measure $\mathbb{Q}$. These are model-based estimates of the actual yields, and are illustrated in Figure 3.

The difference between actual and model based yields:

$$y_t(\tau) - \tilde{y}_t(\tau) = \hat{\varepsilon}_t(\tau), \tag{13}$$

is the fitting error. A straightforward measure of in-sample fit of the model is simply given by the root mean squared errors:

$$RMSE(\tau) = \sqrt{\frac{1}{T} \Sigma_{t=1}^{T} \hat{\varepsilon}_t^2(\tau)},$$

which are reported in Table 1. The small fitting errors suggest that while EFSF rates are potentially problematic because of reduced liquidity at the shortest and longest maturities, the swap curve can be exploited as a (liquid) benchmark for pricing.

[9] The correlation between EFSF, Germany, and France is between 95.6% and 99.9%, depending on maturity.

### 4.2 Instantaneous risk-free rate

The no arbitrage model retrieves the instantaneous risk-free rates via equation (6), which we report in Figure 4. The short rate observations are aligned until 2007 and within the ECB 'corridor' (difference between deposit facility and marginal lending facility), which confirms our conjecture that swaps are the best proxy of the natural rate of interest, as they stay within the ECB corridor. In 2008, the swap rate rises due to the banking crisis. After 2008, swap spread tighten again, all market rates fall after 2015, but only swap rate stays within the ECB corridor. The historical behavior suggests that short maturity swap rates have been closer to the ECB monetary policy stance than bond rates and they are less sensitive to regime changes.

We believe that the reason for this difference between swap and bonds is mainly due to the key characteristic of the bond that is 'a bond is a cash' instrument, so you can buy and sell to make cash immediately. Swaps do not require cash to enter. Yet, an investor can take a directional position on future interest rates using swaps, same as he'd do with bonds. An investor can buy and sell bonds. The purchase may require financing like for a bank and it may come at a cost. If the investor holds cash, like asset managers, they may need to hold cash until the right opportunity to buy comes. In a negative interest rate environment this costs too. So, either way, independently of the type of investor, liquidity bears a cost and this cost is reflected in the bond yield, but not in the swap rate.

Other factors may affect the bond yield and not the swap. Bonds can be used as 'collateral' that can be used by investors in the repo market or by banks to borrow money from the ECB. This lowers financing cost, hence the cost of liquidity that is priced in the bond yield. Swap and bonds have different credit rating, one is reflective of banking risk and the other of sovereign risk. This may also affect the swap spread. We argue that the swap spread is minimally affected by credit factors, as swaps are mostly collateralized on highly rated bonds or cash. Eventually, it is the cost of holding collateral or cash to hedge the credit risk in a swap position to affect the spread. Once again, it is the liquidity cost to be priced in the swap spread and it is cash related from collateralisation. Finally, bonds can be used as HQLAs (high quality liquid collateral) to meet capital requirements by regulators. Hence, this is another characteristic that makes bonds and swaps no substitutes to each other.

### 4.3 Expectations

The model implied yields can be further decomposed into an expectation component and a term premium. To compute the expectation component, we first compute the conditional expectations of the factors:

$$E_t[S_{t+\tau}] = \bar{S} + e^{K_{1S}\tau} \times (S_t - \bar{S}), \tag{14}$$

where $\bar{S} = -(K_{1S}^{\mathbb{P}})^{-1} K_{0S}^{\mathbb{P}}$.[10] Then using (6) it is possible to derive the conditional expectations of the short term rate:

$$E_t[r_{t+\tau}] = \rho_{0S} + \rho_{1S} E_t[S_{t+\tau}]. \tag{15}$$

Finally, application of the expectations hypothesis gives the Expectations Hyphothesis (EH) consistent swap yields:

$$y_t^*(\tau) = \frac{1}{\tau} \sum_{j=1}^{\tau} E_t[r_{t+\tau}]. \tag{16}$$

Note that this expectation is computed under the physical measure $\mathbb{P}$.[11] Importantly, in the case of the swap curve the expectation is based only on 3 factors, i.e. $\rho_{0S} = 0,\ \rho_{1S} = [1\ 1\ 1\ 0]$. Instead, the expected spot curve is based on 4 factors , i.e. $\rho_{0S} = 0,\ \rho_{1S} = [1\ 1\ 1\ 1]$. Specifically, we have that:

$$y_t^* = y_t^{*swap} = \frac{1}{\tau} \sum_{j=1}^{\tau} E_t S_{1t+\tau} + \frac{1}{\tau} \sum_{j=1}^{\tau} E_t S_{2t+\tau} + \frac{1}{\tau} \sum_{j=1}^{\tau} E_t S_{3t+\tau} \tag{17}$$

and

$$y_t^{*spot} = y_t^* + \frac{1}{\tau} \sum_{j=1}^{\tau} E_t S_{4t+\tau}. \tag{18}$$

In what follows, we take the stand that $y_t^* = y_t^{*swap}$ represents a measure of the EH-consistent yields, while the term $\frac{1}{\tau} \sum_{j=1}^{\tau} E_t S_{4t+\tau}$ represents an additional expectation component that arises on the spot rates.[12]

## 4.4 Term premia

The term premia are the difference between the model-implied yields $\tilde{y}_t(\tau)$ and the EH-consistent yields:[13]

$$TTP_t^{spot}(\tau) = \tilde{y}_t^{spot}(\tau) - y_t^*(\tau), \tag{19}$$

[10] Note that $e^{K_{1S}^{\mathbb{P}}\tau} \approx (K_{1S}^{\mathbb{P}} + I)^\tau$ since $\ln(K_{1S}^{\mathbb{P}} + I)^\tau = \tau \ln(I + K_{1S}^{\mathbb{P}}) \approx \tau K_{1S}^{\mathbb{P}}$.

[11] The same expectation under the $\mathbb{Q}$ measure would almost coincide with $\tilde{y}_t(\tau)$, the (small) difference consisting in a Jensen's inequality term. This happens because under the $\mathbb{Q}$ measure, the EH holds and there are no term premia.

[12] Refer also to corporate bond pricing literature, where defaultable bonds are priced by discounting future cash flows using a default - and liquid - adjusted short rate. Contributions in this tradition include Duffie-Sinleton (1999), Longstaff et al (2005), Driessen (2005).

[13] See equation (4) in Dai and Singleton (2002).

The premia depend on parameters belonging to both the $\mathbb{Q}$ and the $\mathbb{P}$ measures. Note that in expression (19) we use the EH-consistent swap expected yields, not the expected spot yields, as a reference point.

Solving for the model implied yields $\tilde{y}_t^{spot}(\tau)$ and recalling from (13) that they differ from the observed yields by a measurement error, we have:

$$y_t^{spot}(\tau) = y_t^* + TTP_t^{spot}(\tau) + \hat{\varepsilon}_t^{spot}(\tau). \tag{20}$$

The expression above makes clear that $y_t^{spot}(\tau)$ can be decomposed in the EH-consistent expected swap rates $y_t^*$ and a term premium. The term premia for the EFSF curve, for all maturities, are shown in Figure 5. Figure 6 shows the term premia for all the curves, at the 10-year maturity. A decomposition of yields into expectations and term premia can be found in Figure 7 and Figure 8.

The term premia change over time and are of course increasing with maturity. Among all issuers considered, the lowest $TTP_t^{spot}$ premium is the one of Germany as one would expect from safe heaven. Both term premia and expectations decrease over time in all curves, with expectations leveling out at negative yield level after 2015.

# 5 Pricing factors

The estimated $S_t$ factors are shown in Figure 9. As is clear in the figure, these are in line with the usual interpretation of level, slope, and curvature factors. The last panel in the figure shows the fourth factor $S_t^{(4)}$, which coincides with the difference between the instantaneous risk-free spot and swap rates, as described in equation (9). This factor is particularly relevant as its cumulated forecasts represents the difference between the EH-consistent curves:

$$\frac{1}{\tau}\sum_{j=1}^{\tau} E_t S_{4t+\tau} = y_t^* - y_t^{*spot} \tag{21}$$

which is illustrated in figure (10). The figure shows the factor $S_{4t}$ (in bold) together with the spreads $y_t^* - y_t^{*spot}$ for different maturities.

## 5.1 Determinants of the fourth factor

The factor $S_{4t}$ determines a wedge between the EH-consistent rates of the spot and the swap curve. A time series plot of the fourth factor for the EFSF, Germany, and France yield curve is displayed in Figure (11).

In this Section we show that this factor can be interpreted as a liquidity/credit factor. Specifically, we estimate the following regression model:

$$S_{4t} = c + \phi_1 S_{4t-1} + \phi_2 \Delta CDS_t + \phi_3 \Delta XL_t + \phi_4 VSTOXX_t + \phi_5 D_t + \varepsilon_t,$$

where $c$ is an intercept, $\Delta CDS_t$ is the change in credit default swaps rates, $\Delta XL_t$ is the change in a measure of excess liquidity,[14] $VSTOXX_t$ is the Euro STOXX 50 area volatility index by DB and Goldman Price, $D_t$ is a set of dummy variables picking up extreme events in the dates August 2007 and November 2011, and $\varepsilon_t$ is a white noise disturbance. We estimate the model using OLS and correcting the standard error for heteroschedasticity via the White (1980) robust estimator. Results are displayed in Table 2.

For the sample starting in June 2011, time series of bid-ask spreads are also available, allowing to run the regression:

$$S_{4t} = c + \phi_1 S_{4t-1} + \phi_2 \Delta CDS_t + \phi_3 \Delta XL_t + \phi_4 VSTOXX_t + \phi_5 D_t + \phi_6 BA_t + \varepsilon_t,$$

where $BA_t$ is the bid-ask spread. Bid-asks are calculated by ESM using MTS high frequency data. They are defined as the average of best-bid ask spreads relative to the best mid price at the same time across minutes within a particular day, provided that the spreads are below threshold spread. EFSF data not used due data issues. We estimate the model using OLS and correcting the standard error for heteroschedasticity via the White (1980) robust estimator. Results are displayed in Table 3.

These regression results show that the $S_{4t}$ factor captures relative liquidity conditions of swap/bond markets. After 2011, bid offer spreads are more important determinants than broader liquidity measures controlled by ECB monetary policy. Germany's $S_{4t}$ factor moves with the risk sentiment as it is a safe heaven asset, while the EFSF $S_{4t}$ factor is linked to France's credit default risk.

## 6 Conclusions

In this paper, we fitted a no-arbitrage affine term structure model to the EFSF yield curve, and offered an historical decomposition of the observed historical EFSF yields into the expectation and term premia component. We have also fitted France and Germany government yield curves, which showed the robustness of the model to both liquid and non-liquid markets.

[14] Specifically, $XL_t$ is the ECB eurozone excess liquidity defined as deposits at the deposit facility net of the recourse to the marginal lending facility.

We found that the term premia on the EFSF curve has increased sharply during the 2011-2012 crisis, and that it is also on the rise in the more recent months. Both term premia and expectations decrease over time in all curves, with expectations leveling out at negative yield level after 2015.

We also found that the relatively more illiquid curves – and specifically the one of supranational bonds – do show a larger liquidity/credit factor. Regression analysis confirmed starkly that the additional factor is related to liquidity and credit conditions.

Finally, the paper introduced a novel approach to price the yield curves of small issuers and / or illiquid markets. This is achieved by including in the model a benchmark liquid curve, and specifying an additional factor which is ex-ante attributed to liquidity/credit.

## Appendix A: derivation of the JSZ representation

To make this paper self-contained, we derive here the JSZ representation of the GATSM. A more rigorous and detailed description can be found in JSZ. The evolution of $n$ risk factors (a $n$-dimensional state vector) is given by:

$$\Delta S_t = K_{0S}^{\mathbb{P}} + K_{1S}^{\mathbb{P}} S_{t-1} + \Sigma_S \varepsilon_t^{\mathbb{P}} \tag{22}$$

$$\Delta S_t = K_{0S}^{\mathbb{Q}} + K_{1S}^{\mathbb{Q}} S_{t-1} + \Sigma_S \varepsilon_t^{\mathbb{Q}} \tag{23}$$

$$r_t = \rho_{0S} + \rho_{1S} S_t, \tag{24}$$

where $\mathbb{Q}$ and $\mathbb{P}$ denote the risk neutral and physical measures, $r_t$ is the short term rate, $\Sigma_S$ is the Cholesky factor of the conditional variance of the states and the errors are i.i.d. Gaussian random variables. The model-implied yield on a zero-coupon bond of maturity $\tau$ is an affine function of the state $S_t$ (Duffie and Kan (1996)):

$$\widetilde{y}_t(\tau) = A_\tau(\Theta_S^{\mathbb{Q}}) + B_\tau(\Theta_S^{\mathbb{Q}}) S_t \tag{25}$$

where $\Theta_S^{\mathbb{Q}} = \{K_{0S}^{\mathbb{Q}}, K_{1S}^{\mathbb{Q}}, \Sigma_S, \rho_{0S}, \rho_{1S}\}$ and the functions $A_\tau(\Theta_S^{\mathbb{Q}})$ and $B_\tau(\Theta_S^{\mathbb{Q}})$ are computed recursively and satisfy a set of Riccati equations:

$$\begin{aligned} A_{\tau+1} &= A_\tau + K_{0S}^{\mathbb{Q}\prime} B_\tau + 0.5 B_\tau' \Sigma_S \Sigma_S' B_\tau - \rho_{0S} \\ B_{\tau+1} &= B_\tau + K_{1S}^{\mathbb{Q}\prime} B_\tau - \rho_{1S} \end{aligned}$$

with initial conditions $A_0 = B_0 = 0$. Here the use of the symbol $\tilde{}$ highlights that the yields $\widetilde{y}_t$ are assumed to be perfectly priced by the model, i.e. (25) does not contain any measurement error.

A preliminary result (Joslin, 2007) is that (22), (23), and (24) can be re-parametrized as follows:

$$\begin{aligned} K_{0S}^{\mathbb{Q}} &= (k_{\infty}^{\mathbb{Q}}, 0, ..., 0), && (26)\\ K_{1S}^{\mathbb{Q}} &= J(\lambda^{\mathbb{Q}}) \text{ (real Jordan form)}, && (27)\\ \Sigma_S &= \text{lower triangular},\\ \rho_{0S} &= 0,\\ \rho_{1S} &= \mathbf{1} \text{ (vector of ones)}. \end{aligned}$$

The $\lambda^{\mathbb{Q}}$ are the (ordered) eigenvalues of $K_{1S}^{\mathbb{Q}}$. Note that in this case knowledge of $k_{\infty}^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_S$ will be sufficient to compute the loadings so we can write $A(\Theta_S^{\mathbb{Q}}) = A(k_{\infty}^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_S)$ and $B(\Theta_S^{\mathbb{Q}}) = B(\lambda^{\mathbb{Q}})$.

Now consider $n$ linear combinations of $N$ yields (that is, portfolios), and label them $P_t = W\widetilde{y}_t$. JSZ show that i) the state vector $S_t$ which is in general unobservable can be replaced by the observable portfolios $P_t$ by means of an invariant transformation, and ii) the $Q$distribution of the observable portfolios $P_t$ is entirely characterized by $\Theta_P^{\mathbb{Q}} = \{k_{\infty}^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_P\}$ where $\Sigma_P$ is the Cholesky factor of the conditional variance of $P_t$.[15]

To derive the JSZ rotation we start from getting a measurement equation in terms of the states $P_t$. Rewrite the measurement equation (25) by stacking by columns the equations for different yields:

$$\underset{N\times 1}{\widetilde{y}_t} = \underset{N\times 1}{A(\Theta_S^{\mathbb{Q}})} + \underset{N\times n}{B(\Theta_S^{\mathbb{Q}})}\underset{n\times 1}{S_t} \tag{28}$$

with $\widetilde{y}_t = [\widetilde{y}_t(\tau_1), ..., \widetilde{y}_t(\tau_N)]'$, $A(\Theta_S^{\mathbb{Q}}) = [A_{\tau_1}, ..., A_{\tau_N}]'$, and $B(\Theta_S^{\mathbb{Q}}) = [B'_{\tau_1}, ..., B'_{\tau_N}]'$. By premultiplying (28) by $W$ the measurement equation can be stated as:

$$P_t = A_W + B'_W S_t, \tag{29}$$

where

$$A_W = WA(\Theta_S^{\mathbb{Q}}) \tag{30}$$

and

$$B'_W = WB(\Theta_S^{\mathbb{Q}}). \tag{31}$$

From (29) we can get an expression for $S_t$:

$$S_t = B_W'^{-1}(P_t - A_W), \tag{32}$$

[15] The parameter $k^{\infty}$ under Q-stationarity (and if the multiplicity of the first eigenvalue $\lambda_1^Q$ is $m_1 = 1$) is related to the risk neutral long run mean of the short rate as follows: $k_{\infty}^Q = -\lambda_1^Q r_{\infty}^Q$. As a result, it is possible to define equivalently $\Theta_P^Q = r_{\infty}^Q, \lambda^Q, \Sigma_P$.

and substituting (32) into the measurement equation (28) gives:

$$\widetilde{y}_t = A_p + B_p P_t \tag{33}$$

with:

$$A_p = (I - B(\Theta_S^{\mathbb{Q}})B_W'^{-1}W)A(\Theta_S^{\mathbb{Q}}), \tag{34}$$
$$B_p = B(\Theta_S^{\mathbb{Q}})B_W'^{-1}, \tag{35}$$

while using (29) to compute the conditional variance of $P_t$ gives:

$$\Sigma_P \Sigma_P' = B_W' \Sigma_S \Sigma_S' B_W. \tag{36}$$

Note that since $B(\Theta_S^{\mathbb{Q}}) = B(\lambda^{\mathbb{Q}})$ and $B_W = WB(\Theta_S^{\mathbb{Q}})$, the matrix $\Sigma_S$ can be derived under knowledge of $\lambda^{\mathbb{Q}}$ and $\Sigma_P$, and in turn knowledge of $k_\infty^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_S$ yields the coefficients in $A(\Theta_S^{\mathbb{Q}}) = A(k_\infty^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_S)$. It follows that knowledge of $\Theta_P^{\mathbb{Q}} = k_\infty^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_P$ allows one to compute $A_p$ and $B_p$. Turning to the equations (22), (23), and (24), applying (29) to both sides and then substituting $S_{t-1}$ using (32) we obtain the JSZ canonical form corresponding to the measurement equation (33):

$$\Delta P_t = K_{0P}^{\mathbb{P}} + K_{1P}^{\mathbb{P}} P_{t-1} + \Sigma_P \varepsilon_t^{\mathbb{P}} \tag{37}$$

$$\Delta P_t = K_{0P}^{\mathbb{Q}} + K_{1P}^{\mathbb{Q}} P_{t-1} + \Sigma_P \varepsilon_t^{\mathbb{Q}} \tag{38}$$

$$r_t = \rho_{0P} + \rho_{1P} P_t. \tag{39}$$

The relation between the two representations is given by:

$$K_{1P}^{\mathbb{Q}} = B_W' K_{1S}^{\mathbb{Q}} B_W'^{-1}, \tag{40}$$
$$K_{0P}^{\mathbb{Q}} = B_W' K_{0S}^{\mathbb{Q}} - K_{1P}^{\mathbb{Q}} A_W, \tag{41}$$
$$\rho_{1P} = B_W^{-1} i, \tag{42}$$
$$\rho_{0P} = -A_W \rho_{1P}, \tag{43}$$
$$K_{1P}^{\mathbb{P}} = B_W' K_{1S}^{\mathbb{P}} B_W'^{-1}, \tag{44}$$
$$K_{0P}^{\mathbb{P}} = B_W' K_{0S}^{\mathbb{P}} - K_{1P}^{\mathbb{P}} A_W, \tag{45}$$

where $K_{1S}^{\mathbb{Q}} = J(\lambda^{\mathbb{Q}})$ and $K_{0S}^{\mathbb{Q}} = k_\infty^{\mathbb{Q}} e_{m_1}$ with $e_{m_1}$ a vector of zeros except for the entry $m_1$ which is one ($m_1$ being the multiplicity of the first eigenvalue $\lambda_1^Q$).

Now assume the portfolios (and therefore the yields) are measured with error. In this case we define the observed yields as $y_t = \widetilde{y}_t + \Sigma_y \varepsilon_t^y$. The definition of the portfolios stays the same: $P_t = W\widetilde{y}_t$ as before, but now these differ from the observed portfolios $P_t^o = Wy_t$ so one needs to filter out the unobserved states $P_t$. The state space system is:

$$\Delta P_t = K_{0P}^{\mathbb{P}} + K_{1P}^{\mathbb{P}} P_{t-1} + \Sigma_P \varepsilon_t^{\mathbb{P}} \tag{46}$$

$$y_t = A_p + B_p P_t + \Sigma_y \varepsilon_t^y. \tag{47}$$

The objects $K_{0P}^{\mathbb{P}}$ and $K_{1P}^{\mathbb{P}}$ can be concentrated out in a preliminary OLS step, $A_p$ and $B_p$ are parameterized by the vector of coefficients $\Theta_P^{\mathbb{Q}} = \{\lambda^{\mathbb{Q}}, k_\infty^{\mathbb{Q}}, \Sigma_P\}$ via (34), (35) and (36). Hence, the model is fully parameterized by:

$$\theta = (\lambda^{\mathbb{Q}}, k_\infty^{\mathbb{Q}}, \Sigma_P, \Sigma_y), \tag{48}$$

Considering that reasonable initial conditions for $\Sigma_P$ and $\Sigma_y$ are readily available, the size of the parameter set to be estimated is dramatically reduced.

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## Tables and figures

**Table 1: RMSEs for different maturities**

| **maturity (years)** | **RMSE (basis points)** |
|---|---|
| 0.25 | 26.898 |
| 1 | 7.183 |
| 2 | 0.946 |
| 3 | 0.822 |
| 4 | 5.393 |
| 5 | 0.783 |
| 7 | 1.42e-05 |
| 10 | 3.507 |
| 1 | 17.866 |
| 2 | 7.800 |
| 3 | 2.36e-06 |
| 5 | 11.270 |
| 7 | 18.078 |
| 10 | 24.756 |
| average | 8.565 |

**Table 2: determinants of the fourth factor.**

| $S_{4t} = c + \phi_1 S_{4t-1} + \phi_2 \Delta CDS_t + \phi_3 \Delta XL_t + \phi_4 VSTOXX_t + \phi_5 D_t + \varepsilon_t$ | | | | |
|---|---|---|---|---|
| | **EFSF** | **EFSF** | **France** | **Germany** |
| $c$ | -9.96E-05 | -8.82E-05 | -0.000213 | -0.000472*** |
| $S_{4t-1}$ | 0.892*** | 0.896** | 0.920** | 0.962*** |
| $\Delta CDS_t\ (FR)$ | 1.851*** | - | -1.014 | - |
| $\Delta CDS_t\ (DE)$ | - | 2.468** | - | -3.232*** |
| $\Delta XL_t$ | -0.016** | -0.015** | -0.0172*** | -0.0112* |
| $VSTOXX_t$ | -0.276 | -0.306 | -0.599 | 1.382** |
| $D_t$ (2007 : 8) | -0.0011*** | -0.001*** | -0.0022*** | -0.0019*** |
| $D_t$ (2011 : 11) | 0.0083*** | 0.008*** | 0.0068*** | -0.0011*** |
| $R^2$ (adj) | 0.940 | 0.939 | 0.903 | 0.953 |

Note: here $c$ is an intercept, $\Delta CDS_t$ is the change in credit default swaps rates, $\Delta XL_t$ is the change in a measure of excess liquidity, $VSTOXX_t$ is the Euro STOXX 50 area volatility index by DB and Goldman Price, $D_t$ are dummy variables picking up extreme events in the dates August 2007 and November 2011. Sample size: 204.

**Table 3: determinants of the fourth factor, sample starting 2011:6**

$S_{4t} = c + \phi_1 S_{4t-1} + \phi_2 \Delta CDS_t + \phi_3 \Delta XL_t + \phi_4 VSTOXX_t + \phi_5 D_t + \phi_6 BA_t + \varepsilon_t$

| | **EFSF** | **EFSF** | **France** | **Germany** |
|---|---|---|---|---|
| $c$ | 0.0004 | 0.0002 | -0.0001 | -0.0006*** |
| $S_{4t-1}$ | 0.892*** | 0.865*** | 0.695*** | 0.886*** |
| $\Delta CDS_t$ $(FR)$ | 1.847** | - | -0.745 | - |
| $\Delta CDS_t$ $(DE)$ | - | 2.852 | - | -3.475*** |
| $\Delta XL_t$ | -0.010 | -0.010 | -0.00037 | 0.0073* |
| $VSTOXX_t$ | -2.554 | -2.958* | -2.596* | 1.607* |
| $D_t$ $(2011:11)$ | 0.0087*** | 0.0077*** | 0.0074*** | 0.0003 |
| $BA_t$ $(FR)$ | -8.37E-06 | - | -0.000104** | - |
| $BA_t$ $(DE)$ | - | 1.43E-05 | - | -2.13E-05*** |
| $R^2$ | 0.953 | 0.955 | 0.774 | 0.950 |

Note: here $c$ is an intercept, $\Delta CDS_t$ is the change in credit default swaps rates, $\Delta XL_t$ is the change in a measure of excess liquidity, $VSTOXX_t$ is the Euro STOXX 50 area volatility index by DB and Goldman Price, $D_t$ a dummy variable picking up extreme event in November 2011, and $BA_t$ is the bid-ask spread. Sample size: 106.

**Table 4: timeline of events**

| | |
|---|---|
| Sep-01 | U.S. terrorist attacks |
| Jan-04 | Committee of European Banking Supervisors |
| Mar-08 | Bear Stearns |
| Sep-08 | Lehman Brothers |
| Apr-10 | Greece requests financial assistance |
| May-10 | EFSF program approved for Greece |
| May-10 | Start of the SMP |
| Nov-10 | Ireland Financial assistance |
| May-11 | Portugal Financial assistance |
| Oct-11 | Dexia resolution |
| Dec-11 | 3Y- LTROS announced |
| Feb-12 | 2nd EFSF programme for Greece |
| Mar-12 | ECB stops buying Greece in repo |
| Apr-12 | Bankia resolution |
| Jul-12 | Whatever it takes |
| Aug-12 | OMT |
| Oct-12 | ESM operating |
| Dec-12 | ESM assistance to Spain |
| Apr-13 | Cyprus Financial assistance |
| Jul-13 | Soft forward Guidance |
| Dec-13 | Spain and Ireland: End of financial assistance |
| Jun-14 | GovC decision TLTROS |
| Sep-14 | GovC decision ABS, covered bonds |
| Jun-14 | Portugal: End of financial assistance |
| Jan-15 | GovC decision, EAPP |
| Jun-15 | Greece fails to repay IMF loan |
| Aug-15 | ESM Board of Governors approves ESM programme for Greece |
| Dec-15 | Extension of QE |
| Jan-16 | EMIR regulation on CCPs |
| Mar-16 | Increase pace of APP |
| Mar-16 | Cyprus: End of financial assistance |
| Dec-16 | decrease pace of APP |
| Jan-17 | short term measures Greece |
| Jun-18 | Medium term measures of Greece approved by Europroup (efsf) |
| Sep-18 | ECB resumes Cyprus asset purchases |
| Aug-18 | Greece: end of financial assistance |
| Mar-20 | ECB announces more QE for pandemic |

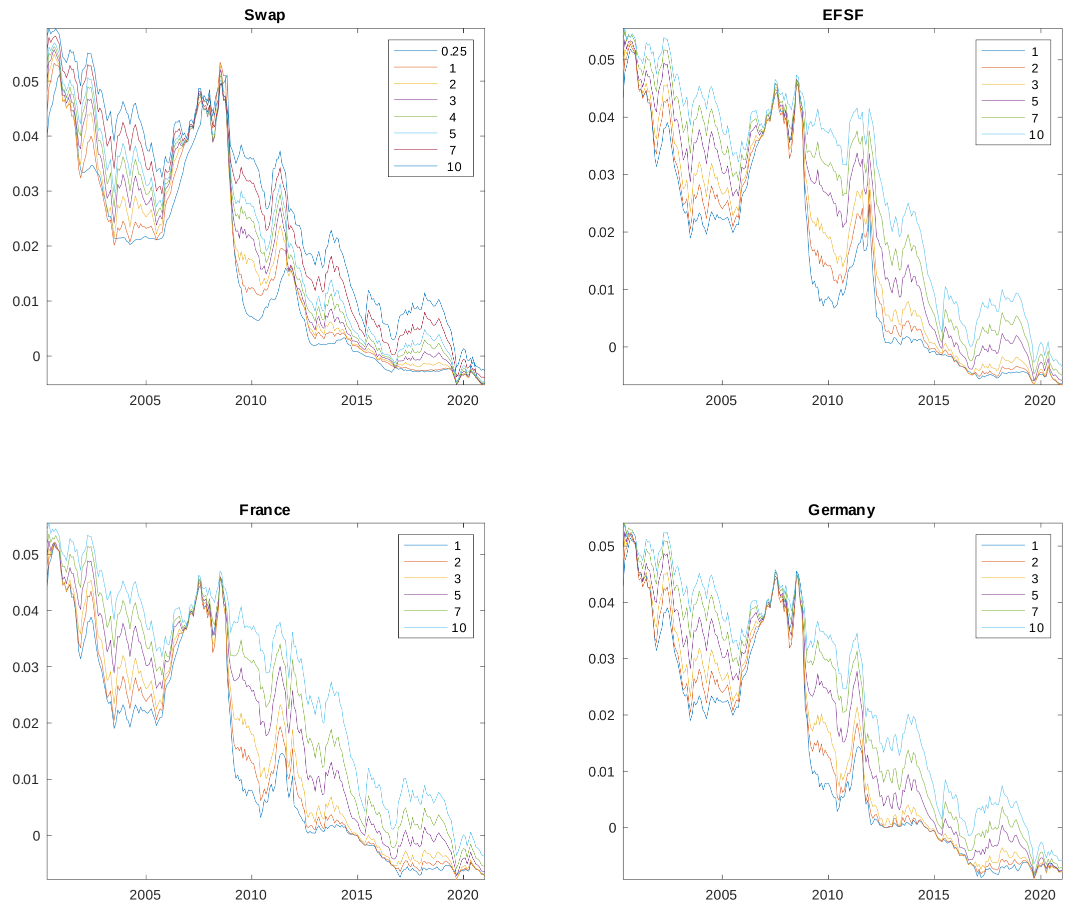


Figure 1: Data. Swap curve and spot curves for EFSF, France, and Germany. Units are expressed in levels, so for example $0.06 = 6\%$.

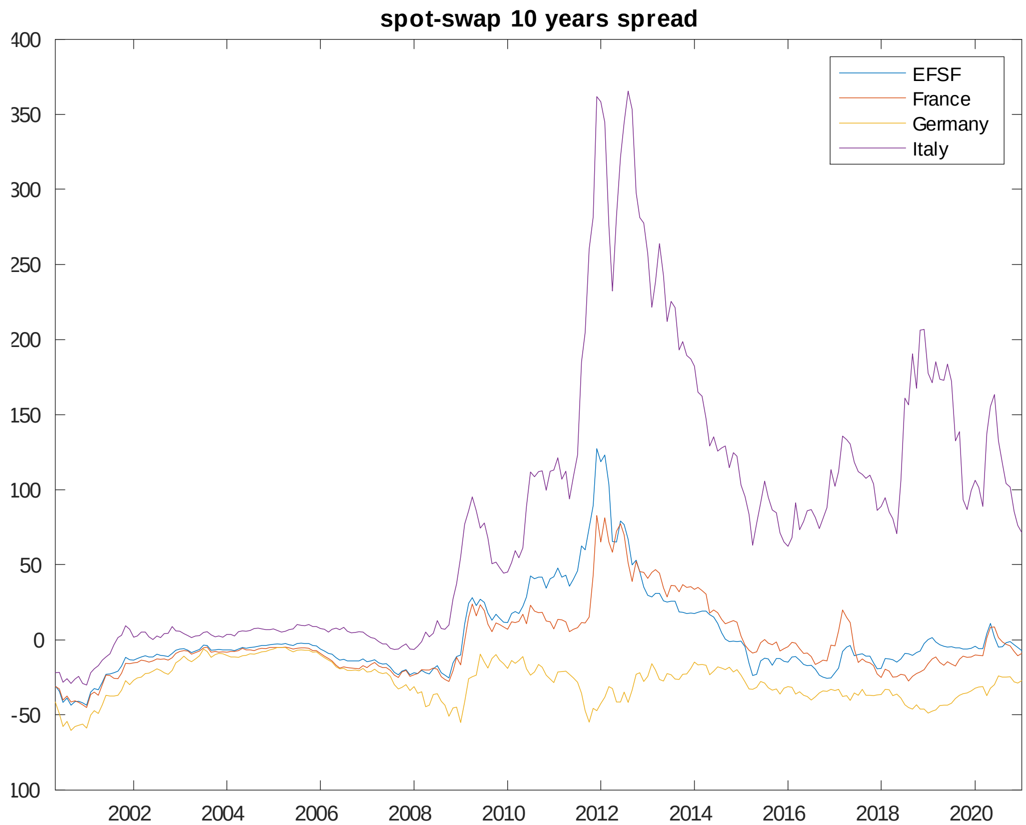


Figure 2: Spreads between swap and spot rates. Units are basis points.

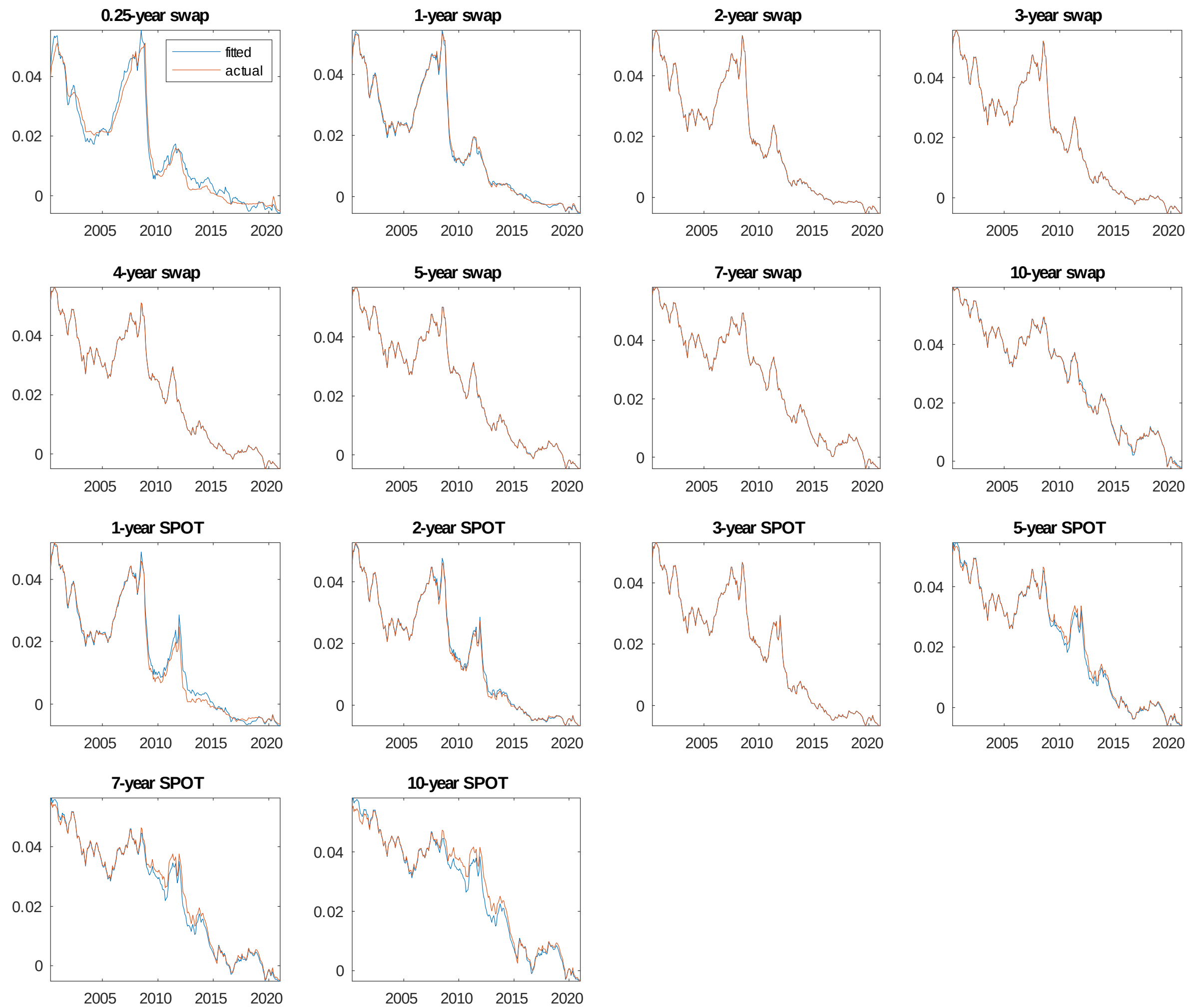


Figure 3: Actual and fitted yields, swap-EFSF curve. Units are expressed in levels, so for example 0.06 = 6%.

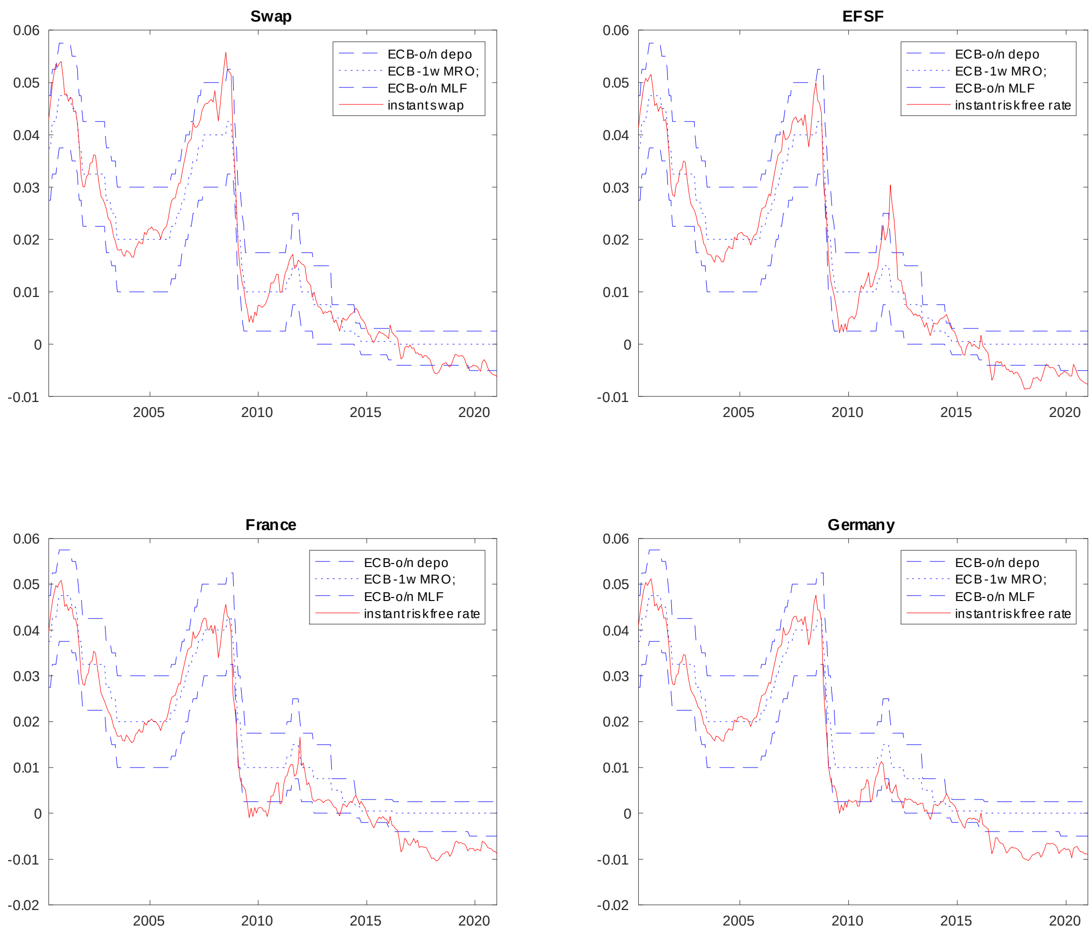


Figure 4: Instantaneous risk-free rates for alternative curves. Units are expressed in levels, so for example $0.06 = 6\%$.

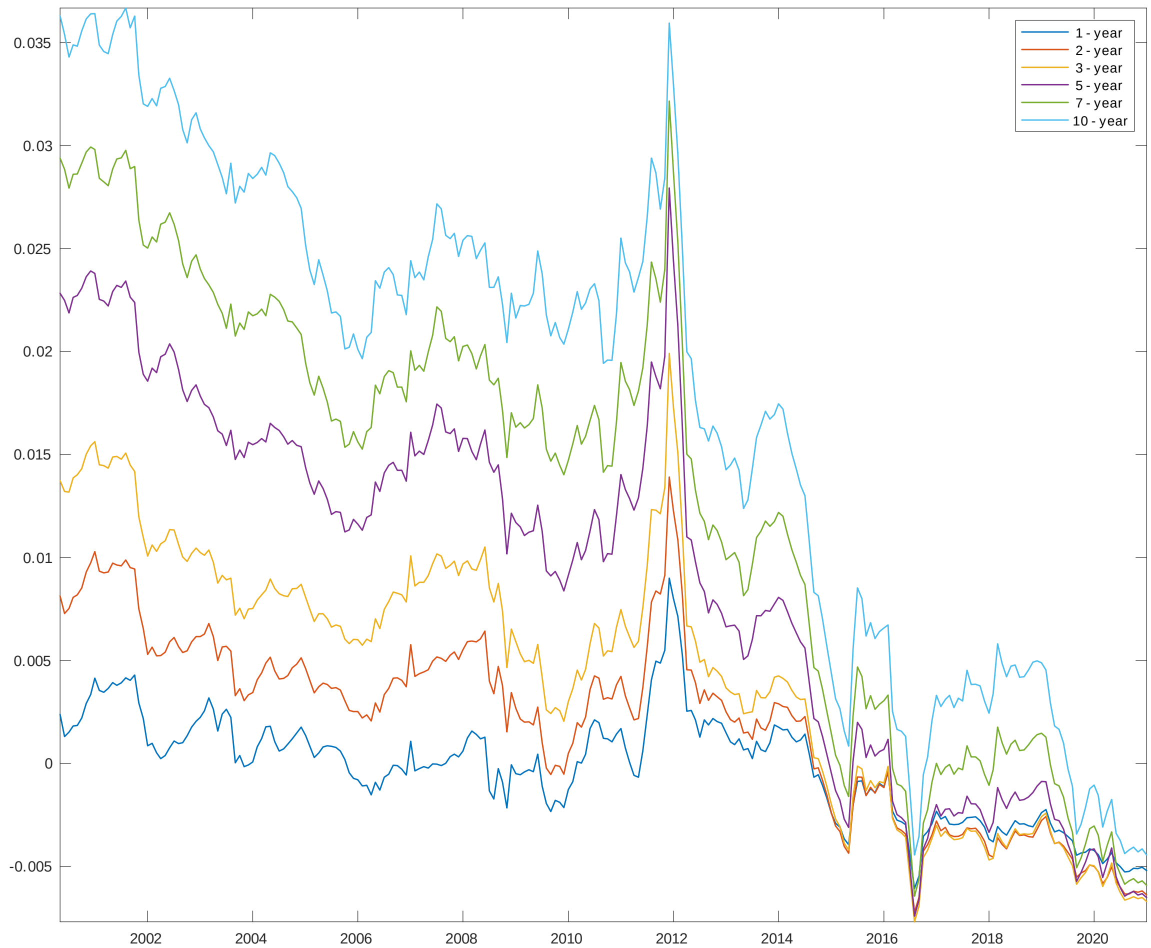


Figure 5: Term premia - EFSF curve. The term premium is the one shown in (19). Units are expressed in levels, so for example 0.03 = 3%.

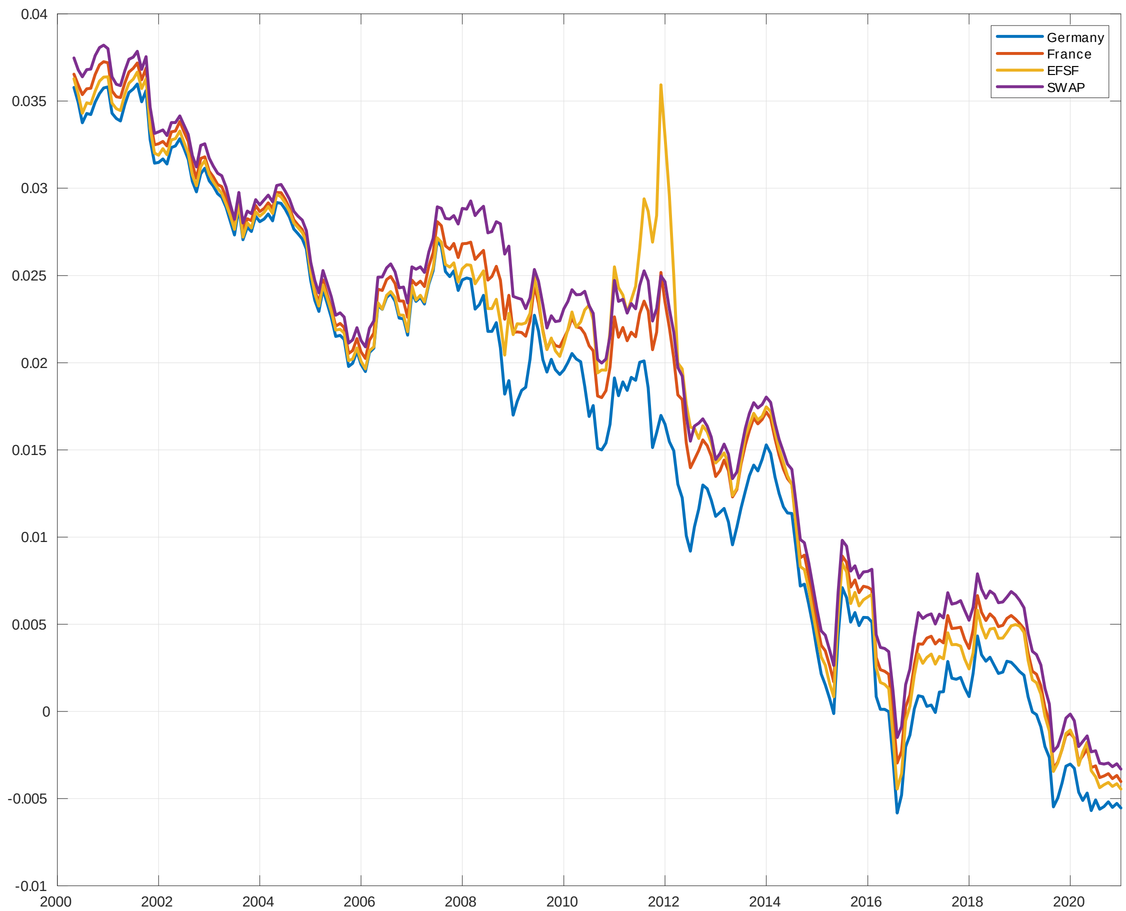


Figure 6: Term premia on the 10-year yield. The term premium is the one shown in (19). Units are expressed in levels, so for example 0.03 = 3%.

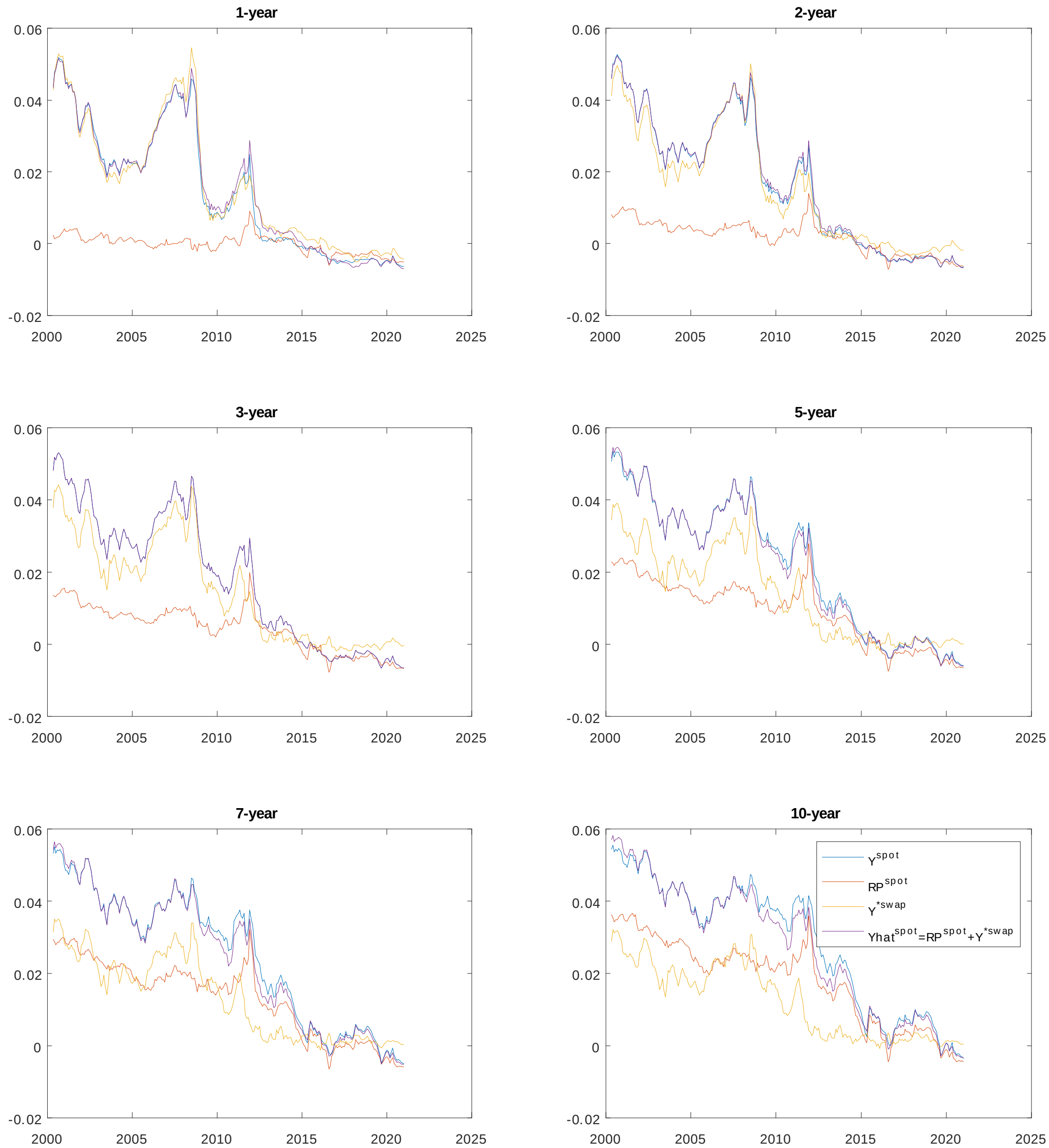


Figure 7: Decomposition of EFSF yields. The figure illustrates the decomposition shown in (20). The term premium is the one shown in (19). Units are expressed in levels, so for example $0.06 = 6\%$.

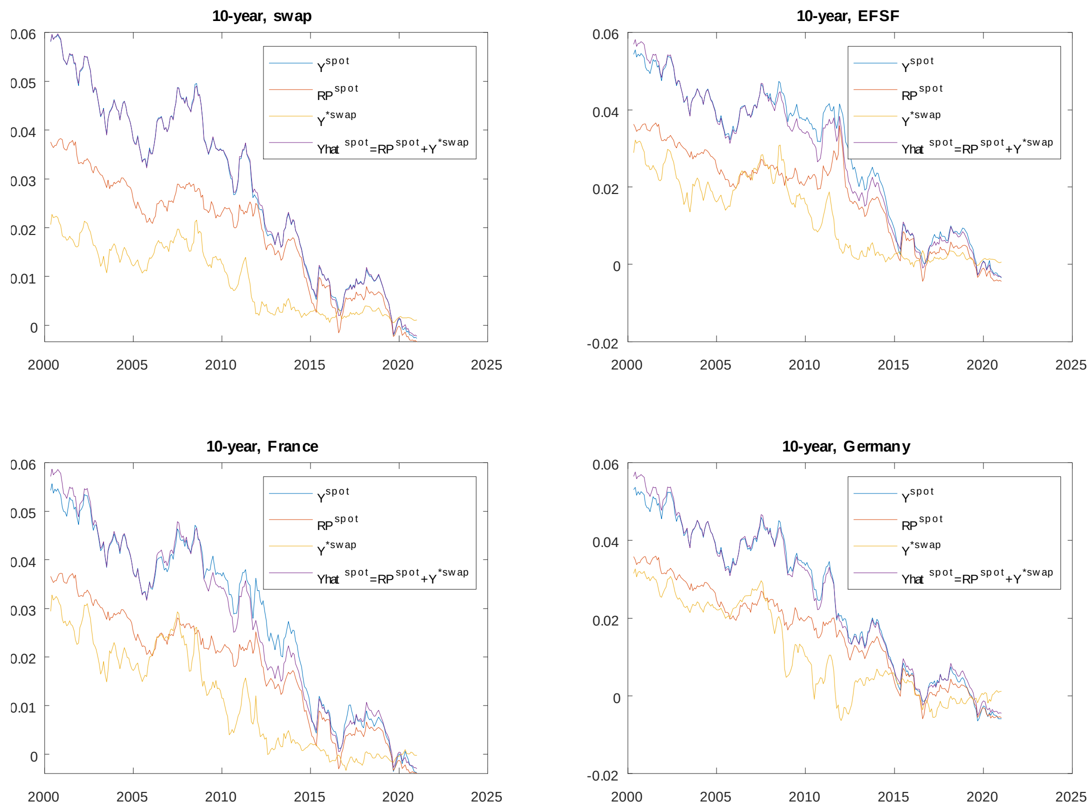


Figure 8: Decompositions for different curves. The figure illustrates the decomposition shown in (20). The term premium is the one shown in (19). Units are expressed in levels, so for example $0.06 = 6\%$.


Figure 9: Estimated $S_t$ latent factors, swap-EFSF curve. Units are expressed in levels, so for example $0.06 = 6\%$.

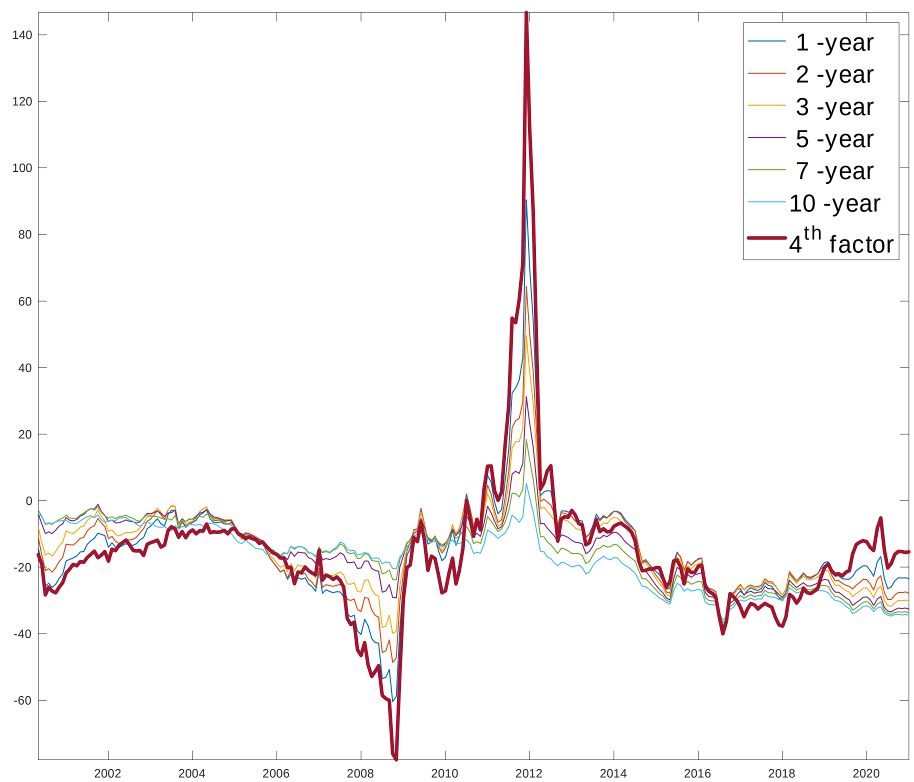


Figure 10: Spreads between EFSF and swap term premia. The bold line is the fourth pricing factor. Units are in basis points.

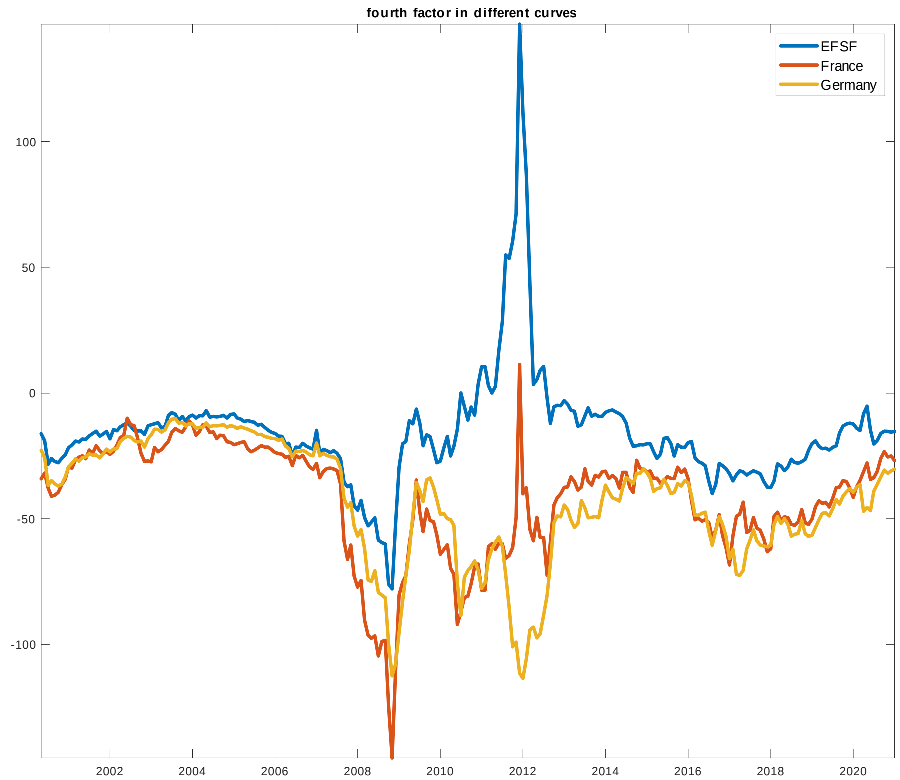


Figure 11: Fourth factor for EFSF, France, and Germany. Units are in basis points.