Has the Euro Area Transitioned to a Zero Interest Rate and Low Inflation Equilibrium? Empirical Evidence and Policy Options

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Abstract

Persistently low inflation rates and the liquidity trap in the euro area have caused concerns of a transition to a new equilibrium with the interest rate remaining near zero for a prolonged period and inflation remaining below target. Using a structural model, we estimate the probability of such an equilibrium transition in the euro area. The model includes a discount factor shock that moves the natural real rate of interest. Further, the representative agent's expectation formation follows a forecast averaging process. Based on past forecast performance, the agent assigns probabilities to forecasts which endpoints are either based on inflation returning to target and the nominal interest rate returning to positive values or inflation remaining below target and the interest rate remaining at zero. The combination of negative discount factor shocks and decreasing inflation expectations can induce a transition to the low inflation and zero interest rate equilibrium. Based on the probabilities that agents assign to the two equilibria, we find that the probability of an equilibrium transition has increased from 20% in 2008 to around 42% in 2020. Besides negative demand shocks, the secular decline in the natural real interest rate contributed to the partial disanchoring of inflation expectations. We simulate policies that may reduce the risk of transitioning to the low inflation equilibrium and that are promising for transitioning back to the targeted inflation equilibrium.

Keywords: Zero Lower Bound, Low Inflation, Equilibrium Transitions, Natural Interest Rate, Inflation Expectations
JEL-Codes: E31, E32, E42, E52

1 Introduction

Headline inflation has been below the ECB's target of close but below 2% for the last seven years except for a few months in 2017 and 2018. Core inflation has been below 2% in every single month since 2009. Headline inflation was even temporarily negative during some months in 2015 and 2016, and has turned negative again during the Corona crisis. At the same time the possibilities of the ECB to expand monetary policy, and thereby increase inflation and inflation expectations have been limited by the effective lower bound (ELB) on nominal interest rates. Consequently, inflation expectations are low. The Survey of Professional Forecasters (SPF) predicts 0.3% HICP inflation for 2020, 0.9% for 2021, and 1.3% for 2022, and the probability distribution of the forecasts shows an increasing skewness to the left. The combination of a decline in inflation alongside of an increasing risk of de-anchoring of inflation expectations, modest economic growth, and an overall weak and uncertain economic outlook concerns policy makers and academics alike. Since the ELB has been binding for several years now, concerns of the liquidity trap being not only a temporary phenomenon, but the euro area economy transitioning to a new equilibrium with interest rates remaining new zero for a prolonged period as well as inflation remaining below target—a *Japanification* of the euro area economy—are growing.

It is well known that macroeconomic models that explicitly account for the ELB display two equilibria (see, e.g., Benhabib et al., 2001a,b): (1) a targeted equilibrium with inflation fluctuating around target and the ELB being non-binding most of the time, and (2) a below-target equilibrium with the ELB frequently binding and inflation being below the inflation target. In this paper, we estimate whether the euro area has transitioned from the targeted equilibrium to the below-target equilibrium using a structural model. While standard solution techniques applying a linearization around a unique stable equilibrium rule out multiplicity in solution paths, we incorporate the unstable nature that dwells within the New Keynesian model to study how the euro area transitions between the two equilibria. Following Lansing (2019a), our model includes a discount factor shocks that affects the natural real rate of interest, and an expectation formation process of the representative agent that weighs forecasts based on the two different equilibria. The agents assign probabilities to forecast rules which are based on the targeted and the below-target equilibrium depending on past forecast performance.

We estimate a linear version of the model as to match the behavior of the euro area economy and derive a minimum state variable solution. We apply a reverse engineering solution to match the shock realizations with data from 1999Q1 to 2019Q4. Specifically, we employ time series for quarterly and annual inflation, the nominal interest rate, the output gap, and the natural real interest rate based on Holston et al. (2017). The assignment of weights to the forecasting rules associated with each equilibrium lets the economy endogenously fluctuate between equilibria. The endogenous weights on the two forecasting rules yield time-varying probabilities of being in the targeted or the below-target equilibrium. In light of the recent review of monetary policy frameworks at central banks, we additionally simulate policies that may reduce the risk of transitioning to the low inflation equilibrium and that are promising for transitioning back to the targeted inflation equilibrium.

Our analysis yields five key results. First, we find that the probability of being in the below-target equilibrium has increased over the course of the Great Financial Crisis (GFC) and its aftermath including the European sovereign debt crisis from 21% in 2007Q1 to 42% in 2019Q4. We show that this increase is not only driven by a secular decline in the natural real interest rate, but also by the occurence of negative demand shocks. Second, simulating the model shows that the forecast averaging solution, which fluctuates between the two equilibria is better suited than a unique-equilibrium solution in matching the moments of our data sample. Third, the reverse engineering solution demonstrates that the model-generated inflation expectations resemble the overall path of the SPF's inflation expectations. Fourth, our result is qualitatively robust to employing alternative time series for inflation. Finally, optimal monetary policy that aims at remaining in the targeted equilibrium is best described by a more hawkish central bank.

This paper relates to the literature addressing multiple equilibria in DSGE models. Originally, Benhabib et al. (2001a,b) have shown that New Keynesian models exhibit multiple equilibria if the ELB is accounted for even if the central bank adheres to the Taylor principle. Researchers have coped with the associated unstable dynamics by introducing sunspot shocks, that induce exogenous regime switches, see, e.g., Mertens and Ravn (2014) for an application including fiscal policy shocks as well as Schmitt-Grohé and Uribe (2017) for confidence shocks. Aruoba et al. (2018) solve a two-equilibria, non-linear model with sunspot shocks for the US and Japan, and average regime probabilities across model specifications. For a sample period from the 1980s to 2015, they find that while the US is likely in the targeted equilibrium at the end of sample, Japan has likely switched to the below-target equilibrium.

While introducing sunspot shocks provides a computationally elegant way to cope with multiplicity in solution paths, models switch exogenously between equilibria. Our methodological approach is closely related to Lansing (2019a), who also uses a minimum state variable solution and applies reverse engineering techniques, such that the model fluctuates endogenously between equilibria.[1] In doing so, the model solution does not yield a binary time series indicating which equilibrium the economy is in, but a time-varying weight assigned to each equilibrium, so that actual dynamics consists of a mix of both equilibria. A further advantage of this approach is that equilibrium transitions occur endogenously, so that an analysis of policy options avoiding transitioning to the below-target equilibrium or transitioning back to the targeted equilibrium are feasible. Lansing (2019a) finds that the US economy is on a path to the below-target equilibrium since the GFC, while possible exit strategies prescribe that the central bank reaction function should account for the natural real interest rate gap. Recently, Cuba-Borda and Singh (2020) distinguish between two possible driving forces associated with the Japanese liquidity trap: expectations-driven or secular stagnation. The authors, however, find inconclusive results. We contribute to the literature by studying equilibrium transitions in case of the euro area. This paper is highly preliminary work in progress.

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[1] For background information on related applications of reverse engineering solutions, see Gelain et al. (2018), Lansing and Markiewicz (2018), as well as Lansing (2019a,b).

2 Model

The two-equilibria New Keynesian model we employ consists of an IS curve, a Phillips curve, and a monetary policy rule in spirit of Taylor (1993). While the IS curve relates the current output gap to the expected future output gap and some measure of the real interest rate, the Phillips curve equates current inflation to discounted expected future inflation and some measure of the output gap. The monetary policy rule explicitly accounts for the ELB. Instead of deriving the full model, we provide the key model equations, refer the interest reader to Lansing (2019a), and devote more space to the model's solution strategy. The IS and Phillips curve are denoted as:

$$y_t = E_t y_{t+1} - \alpha[i_t - E_t \pi_{t+1} - r_t] + v_t \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t, \tag{2}$$

where y_t is the output gap, i_t is the nominal interest rate, π_t is the inflation rate, r_t is the short-run natural rate of interest, and E_t denotes the rational expectations operator. The parameters α describe the interest rate sensitivity in the Euler equation, β denotes the time discount factor, and κ captures the degree to which the output gap enters the Phillips curve, respectively. The disturbances v_t and u_t represent a demand and cost push shock, respectively, whereby the shocks are assumed to follow a standard, autoregressive process of the form:

$$v_t = \rho_v v_{t+1} + \epsilon_{v,t}, \qquad \epsilon_{v,t} \sim N[0, \sigma_v^2(1 - \rho_v^2)] \tag{3}$$

$$u_t = \rho_u u_{t-1} + \epsilon_{u,t}, \qquad \epsilon_{u,t} \sim N[0, \sigma_u^2(1 - \rho_u^2)]. \tag{4}$$

Distinct from many macro models, we assume the natural real rate of interest r_t^* not to be constant, but subject to short- and long-run shocks. This deviation is motivated by the estimated decline in the natural real interest rate in many advanced economies including the euro area (Laubach and Williams, 2003; Holston et al., 2017). The natural real interest rate follows the functional form of a random walk, where ϵ_t denote short-run and η_t long-run shocks, respectively:

$$r_t^* = r_{t-1}^* + \eta_t, \qquad \eta_t \sim N(0, \sigma_\eta^2), \tag{5}$$

while the real interest rate is derived from:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) r_t^* + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_\epsilon^2). \tag{6}$$

We restrict r_t to follow a stable path by imposing that $|\rho_r| < 1$. In line with the literature we treat r_t^* as unobservable, while agents gather information on r_t^* via a Kalman filter estimate, whose functional form is denoted in Equation 17 and whose derivation can be found in the appendix to Lansing (2019a).

The central bank is modeled to follow a monetary policy rule, that explicitly accounts for the ELB:

$$i_t^* = \rho i_{t-1}^* + (1-\rho)[E_t r_t^* + \pi^* + g_\pi(\bar{\pi}_t - \pi^*) + g_y(y_t - y^*) + g_r(r_t - E_t r_t^*)], \tag{7}$$

where $\rho \in [0, 1)$ captures the degree of interest rate smoothing. The parameters g_π, g_y and g_r represent elasticities to deviations in the inflation gap, of the output gap from target as well as the natural rate gap, respectively. π^* is the inflation target, while i_t^* denotes the desired nominal interest rate, which is truncated by the ELB in the following, computationally convenient form, disabling negative interest rates:

$$i_t = 0.5 i_t^* + 0.5\sqrt{(i_t^*)^2}. \tag{8}$$

Inflation does not enter the monetary policy rule directly, but we assume the broad inflation measure $\bar{\pi}_t$ to capture the ECB's inflation target variable. We model it as a weighted moving-average between current inflation and past inflation measure by setting ω appropriately:

$$\bar{\pi}_t = \omega \pi_t + (1-\omega)\bar{\pi}_{t-1}. \tag{9}$$

3 Data and Estimation

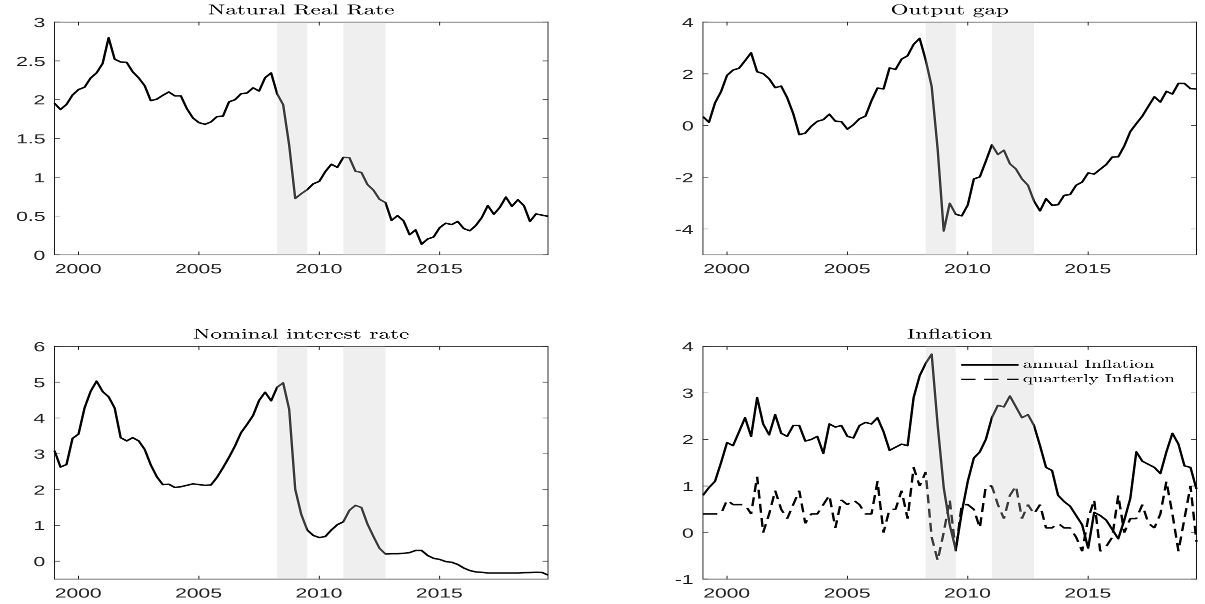
We estimate the structural parameters of the model described in Section 2 using Bayesian techniques to match the behavior of the EA economy. We employ five time series: the output gap, quarterly and annual inflation, nominal interest rate, and the natural real rate of interest. To circumvent the non-linearity implied by Equation 8, we use data only from before the ZLB becomes binding, i.e. from 1999Q1 to 2012Q2. We also simplify the model slightly as we keep the natural real interest rate constant and set it to the sample average of $\bar{r^*} = 1.8\%$. This leaves us with three shocks in the model, so that by exact identification three time series enter the estimation. Figure 1 plots the relevant data.

We employ the structural estimates from Holston et al. (2017) for the natural real rate of interest rate, which is plotted in the top left panel of Figure 1. While there is less movement in the series prior to the GFC, afterwards a downwards trend is clearly visible. Its minimum has been reached in 2014 with four consecutive quarters of negative real rates. The natural real interest rate increases slightly in the following, but oscillates between 0% and 1%.

The AMECO data base hosted by the European Commission provides the output gap at an annual frequency only. We interpolate to quarterly frequency using the interpolation method by Chow and Lin (1971) and quarterly series on real GDP. The GFC triggered a sharp drop in the output gap entering negative territory, from which the output gap recovered successively before it faces another contraction during the European sovereign debt crisis. Since then, the output gap increases.

For the central bank's policy rate, we apply the 3-month Euro Interbank Offered Rate (Euribor), which unlike the official ECB's interest rates condenses information on supply and demand of funds in the European interbanking market in a single time series. The Euribor is closely correlated with the ECB's rate of main refinancing operations and commonly used in the literature as a proxy for the nominal short-

Figure 1: Euro area time series



Note: Shaded areas show recession periods officially dated by the Euro Area Business Cycle Network. Sources: The Natural Real Rate is taken from the website of the Federal Reserve Bank of New York. The output gap stems from the AMECO data base hosted by the European Commission. Data on inflation are constructed from HICP inflation series taken from Eurostat. The nominal interest rate is the 3-month Euribor rate.

term interest rate (see e.g. Fagan et al. (2005)). As can be seen from the bottom left panel of Figure 1, the Euribor fell from 5% to 2% in the aftermath of the dotcom bubble, rising again to 5% until the onset of the GFC. After a small recovery around 2011/2012, the Euribor entered negative territory in the second quarter of 2015 and remains negative ever since.

Inflation is approximated using the Harmonised Index of Consumer Prices (HICP) for the euro area. The bottom right panel of Figure 1 displays the quarterly inflation rate as dashed line, which is computed as the log-difference quarter over quarter, and the annual inflation rate as solid line, which is computed as the change in prices from a year ago. Both inflation measures indicate a substantial decline following the GFC, while only the annual estimates seem to be affected by the European sovereign debt crisis. Even though inflation recovers gradually, the ECB's target rate of close but below 2% annually is only met in 2015Q1, while quarterly inflation turned negative in 2009Q2 as well as 2010Q1.

For the three time series that enter the model's estimation, we choose the set of three series that is associated with the highest initial likelihood derived by the Kalman filter, which are quarterly inflation, the output gap, and the nominal interest rate. For the priors, we follow the literature on estimating DSGE model for the EA (Smets and Wouters, 2003) and adopt initial guesses from empirical model (Eser et al., 2020). We run the Bayesian estimation using a diffuse multivariate Kalman filter, which uses $20,000$ MCMC replication to elicit proper posterior results, which comprised together with the priors in Table 1.

Table 1: Priors and posteriors of estimation

parameter	type	mean	st.dev.	mode	5%	95%
structural parameters						
α	G	0.05	0.50	0.05	0.05	0.05
β	G	0.99	0.50	0.87	0.79	0.94
κ	G	0.05	0.50	0.06	0.03	0.09
ρ	G	0.80	2.00	0.34	0.22	0.44
g_π	G	2.00	2.00	1.03	1.00	1.07
g_y	G	1.00	2.00	0.13	0.10	0.17
st.dev. of the innovations						
σ_u	IG	0.003	0.01	0.004	0.002	0.005
σ_v	IG	0.003	0.01	0.004	0.003	0.005
σ_ϵ	IG	0.003	0.01	0.048	0.038	0.057
persistence of the exogenous processes						
ρ_u	B	0.80	0.10	0.58	0.42	0.73
ρ_v	B	0.50	0.10	0.87	0.82	0.92

Notes: The prior distribution types are abbreviated as standardized beta (B), gamma (G), and inverse gamma (IG).

4 Solution Method

We introduce multiplicity in solution paths by constraining the central bank by the ELB. Consequently, we restrict the nominal interest rate to be positive in the targeted equilibrium, whereas we assume an interest rate of zero for the below-target equilibrium. Computing equilibria yields the following functional relationships, where the equilibria shown here must not be interpreted as steady states, since the natural interest rate is a state variable and subject to short- and long-run shocks as in Equations 6 and 5:

Targeted equilibrium ($i_t > 0 \; \forall t$)

$$\pi_t = \pi^*$$
$$y_t = y^* = \pi^*(1-\beta)/\kappa$$
$$i_t^* = i_t = r_t^* + \pi^*$$

Below-target equilibrium ($i_t = 0 \; \forall t$)

$$\pi_t = -r_t^*$$
$$y_t = -r_t^*(1-\beta)\kappa$$
$$i_t^* = (r_t^* + \pi^*)[1 - g_\pi - g_y(1-\beta)/\kappa].$$

Solving the model with standard techniques around a unique, non-stochastic steady state would per assumption ignore the multiple equilibria that dwell inside the basic New Keynesian model. We conserve the multiplicity in solution paths by employing the minimum state variable (MSV) approach together with the method of undetermined coefficients. The MSV approach is able to identify a unique solution that does not exhibit unstable dynamics (McCallum, 1983, 1999). The method of undetermined coefficients integrates out the expectations operator of the structural equations by guessing a linear functional form of the solution that only depends on current and past observations of state variables. As a result, the explicit decision rules express the endogenous variables as linear functions of the state variables.

The state variables for the targeted inflation equilibrium are $(r_t - E_t r_t^*)$, $\bar{\pi}_t$, $(i_t^* - E_t r_t^* - \pi^*)$ as well as the demand shock v_t, and the cost-push shock u_t. For the below-target equilibrium, the state variables boil down to $(r_t - E_t r_t^*)$ and the two shocks. Iterating the linear decision rules one period ahead yields the local forecasting rule for either equilibrium. In this model set-up, agents base their decisions on forecasts for the output gap and inflation. For the targeted inflation equilibrium, the forecast rules are

$$E_t^{targ} y_{t+1} = y^* + \mathcal{A}_{11}\rho_r(r_t - E_t r_t^*) + \mathcal{A}_{12}(\bar{\pi}_t - \pi^*) + \mathcal{A}_{13}(i_t^* - E_t r_t^* - \pi^*) + \mathcal{A}_{14}\rho_v v_t + \mathcal{A}_{15}\rho_u u_t \tag{10}$$

$$E_t^{targ} \pi_{t+1} = y^* + \mathcal{A}_{21}\rho_r(r_t - E_t r_t^*) + \mathcal{A}_{22}(\bar{\pi}_t - \pi^*) + \mathcal{A}_{23}(i_t^* - E_t r_t^* - \pi^*) + \mathcal{A}_{24}\rho_v v_t + \mathcal{A}_{25}\rho_u u_t, \tag{11}$$

where $\mathcal{A}$ denotes the decision rule coefficients of the MSV solution to the targeted inflation equilibrium. For the below-target equilibrium, the forecast rules are:

$$E_t^{defl} y_{t+1} = -E_t r_t^*(1-\beta)/\kappa + \mathcal{B}_{11}\rho_r(r_t - E_t r_t^*) + \mathcal{B}_{14}\rho_v v_t + \mathcal{B}_{15}\rho_u u_t \tag{12}$$

$$E_t^{defl} \pi_{t+1} = -E_t r_t^*(1-\beta)/\kappa + \mathcal{B}_{21}\rho_r(r_t - E_t r_t^*) + \mathcal{B}_{24}\rho_v v_t + \mathcal{B}_{25}\rho_u u_t, \tag{13}$$

where $\mathcal{B}$ denotes the decision rule coefficients of the MSV solution to the below-target equilibrium.

Our model fluctuates endogenously between the targeted and the below-target equilibrium. We generate this property by assuming that agents are aware of both forecasting rules and try to forecast the output gap and inflation optimally by fitting a weight μ_t to the targeted inflation forecasting rule, while $(1 - \mu_t)$ is assigned to the below-target one:

$$E_t y_{t+1} = \mu_t E_t^{targ} y_{t+1} + (1-\mu_t) E_t^{defl} y_{t+1} \tag{14}$$

$$E_t \pi_{t+1} = \mu_t E_t^{targ} \pi_{t+1} + (1-\mu_t) E_t^{defl} \pi_{t+1}, \tag{15}$$

where $(0 \leq \mu_t \leq 1)$ has to hold. Based on Lansing (2019a), the agents are interpreted as econometricians, who try to optimize the forecast performance. Agents incorporate a backward-looking window of T_w periods of data into their forecast and set μ_t as to minimize the root mean squared forecasting error (RMSFE):

$$\begin{aligned} RMSFE_{t-1} = \sum_{j=1}^{T_w} \Bigg\{ & \frac{1}{T_w} \left[y_{t-j} - \mu_t E_{t-j-1}^{targ} y_{t-j} - (1-\mu_t) E_{t-j-1}^{defl} y_{t-j} \right]^2 \\ & + \frac{1}{T_w} \left[\pi_{t-j} - \mu_t E_{t-j-1}^{targ} \pi_{t-j} - (1-\mu_t) E_{t-j-1}^{defl} \pi_{t-j} \right]^2 \Bigg\}^{0.5}. \end{aligned} \tag{16}$$

Given the forecasts for the output gap and inflation, the non-linear system of equations yields time series for the four endogenous variables y_t, π_t, i_t^* and i_t. We interpret the weight μ_t as an indirect, time-varying probability for the targeted inflation equilibrium and $(1 - \mu_t)$ as an implicit probability for the below-target equilibrium. An equilibrium transition is induced by a relative improvement in forecasting performance of a forecasting rule. Whether a forecasted data point is closer to the realized value, depends not only on the state variables, but also on the decision rule coefficients related to either equilibrium. The values for these decision rule coefficients stem from the model's calibration, which is introduced in the next section.

The remaining parameters are calibrated according to Table 2. The parameter ω is chosen optimally such that $\bar{\pi}_t \cong [\prod_{j=0}^{3}(1+\pi_{t-j})]^{0.25} - 1$, i.e. the weight on current inflation is optimally informative on the moving average representation of inflation. For the euro area, the value of ω is 0.5547.

Table 2: Baseline calibration

Parameter	Value	Description
π^*	2	ECB's average inflation target
ω	0.5547	$\bar{\pi}_t \cong$ 4-quarter inflation rate
ρ_r	0.875	Persistence parameter for natural real rate
λ	0.015	Kalman gain for $E_t r_t^*$
σ_ϵ	0.0018	Standard deviation of temporary shock to r_t
σ_η	0.0002	Standard deviation of permanent shock to r_t
T_w	8	Agents forecasting horizon

We estimate the observable, expected natural real interest rate $E_t r_t^*$ using the Kalman filter expression in Equation 17, whilst the parameters ρ_r and λ are chosen such that the time series of $E_t r_t^*$ matches the moments of the original series by Holston et al. (2017). The approach is adopted from Lansing (2019a), who solves for φ given the Kalman gain formula, and finally for σ_ϵ and σ_η such that the model-generated standard deviation of δr_t corresponds to the data equivalent.

$$E_t r_t^* = \lambda[(r_t - \rho_r r_(t-1))/(1-\rho_r)] + (1-\lambda)E_t r_t^* \tag{17}$$

$$\lambda = (-(1-\rho_r)^2\varphi + (1-\rho_r)\sqrt{(1-\rho_r^2\varphi^2 + 4\varphi))}/2. \tag{18}$$

Figure 2: Time series for the natural interest rate

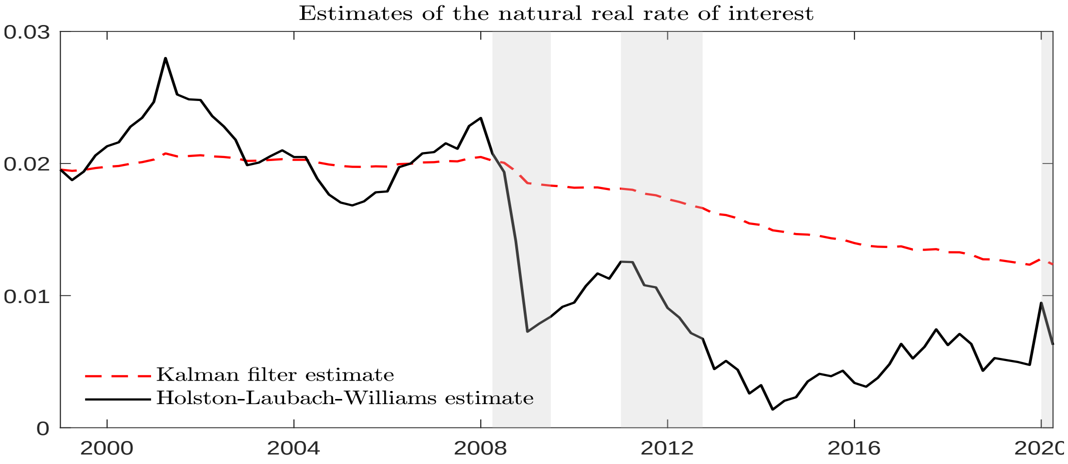


Figure 2 plots the original series from Holston et al. (2017) and the Kalman filter estimate for $E_t r_t^*$, which shows that the observable approximation via the Kalman filter closely resembles the structural estimate of the natural real rate of interest.

Based on the estimated and calibrated of the structural parameters, the decision rule coefficients in $\mathcal{A}$ for the targeted inflation equilibrium, take the following values:

$$\mathcal{A} = \begin{bmatrix} 0.39 & -0.04 & -0.03 & -0.11 & 6.95 \\ 0.10 & -0.01 & -0.00 & 1.97 & 1.68 \\ 0.07 & 0.30 & 0.34 & 0.74 & 1.23 \end{bmatrix} \tag{19}$$

For the below-target equilibrium, the decision rule coefficients in $\mathcal{B}$ are:

$$\mathcal{B} = \begin{bmatrix} 0.46 & 0.00 & 0.00 & 0.14 & 8.05 \\ 0.16 & 0.00 & 0.00 & 2.01 & 1.97 \\ 0.08 & 0.30 & 0.34 & 0.77 & 1.44 \end{bmatrix} \tag{20}$$

5 Results

5.1 Results from the simulation exercise

We simulate the model for 100,000 periods, compute moments of relevant variables and compare those to equivalents of our data sample for the euro area from 1999Q1 to 2019Q4. Due to the endogenous transitioning inherent in the model's solution strategy, the simulation-generated moments can be computed separately for either equilibrium as well as the forecast averaging solution. Table 3 lists the results.

The ECB's interest rate has been close to or at zero since 2012Q2, whereas the ELB has constrained monetary policy thereafter, which accounts for 23% of the sample period. Both the targeted as well as the forecast averaging solution undershoot with 0% and 16.90%, respectively, the data moments, while the below-target equilibrium hits the ELB close to four-fifths of its simulated periods. For 19 quarters has the ECB been affected by the ELB, which coincides with the average ELB-duration in our sample. The forecast averaging solution yields an estimate of 10%, for the targeted equilibrium 2%, and 25% for the below-target equilibrium solution.

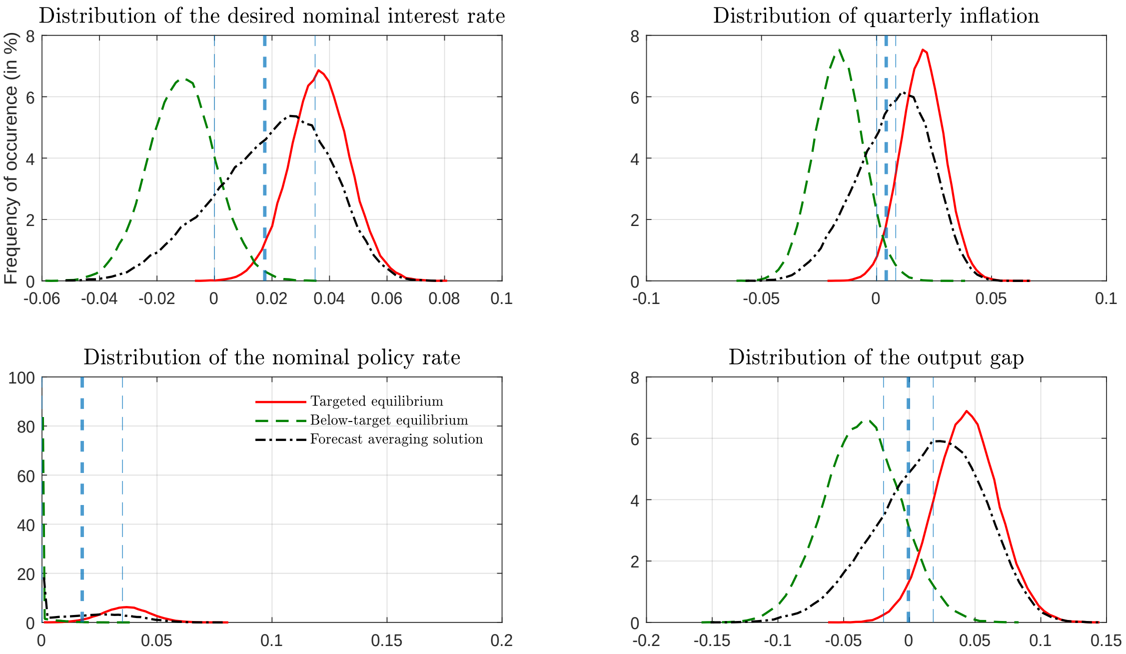
In terms of the output gap, the average value is overshot by the targeted equilibrium as well as the forecast averaging solution, whereas the standard deviation and the correlation are close the data equivalents. It holds true that the forecast averaging solution is best in matching the first- and second-order moments for the annual inflation measure and the desired nominal interest rate. In case of the nominal interest rate, the forecast averaging solution yields an average of 2.18%, which is closest to the 1.75% average in the data. The correlations of the observed variables are approximated fairly close.

Next to first- and second-order moments, we compare the distribution of the simulated variables with the data sample. Since our sample only comprises 84 observations, we refrain from plotting the whole distribution, but compare the simulated distributions to two-thirds of the sample's probability mass. Figure 3 plots the distribution of the desired nominal interest rate, quarterly inflation rate, the nominal policy rate as well as the output gap for the single-equilibrium and the forecast averaging solution. The bold dashed vertical lines indicate the mean, while the thin vertical lines delimit plus and minus one

Table 3: Moments of the simulation exercise

Statistics	Data	Targeted EQ	Below-target EQ	Forecast Averaging
Share of periods at ZLB	23%	0%	83.82%	16.90%
Average ZLB-duration	19	2.07	24.52	10.10
Maximum ZLB-duration	19	5	227	146
Output gap				
Mean y_t	-0.00%	4.34%	-3.58%	1.59%
Std y_t	0.02	0.02	0.03	0.04
Corr y_t, y_{t-1}	0.95	0.86	0.87	0.94
Annual inflation				
Mean $\pi_{4,t}$	1.70 %	2.00%	-1.65%	0.72%
Std $\pi_{4,t}$	0.01	0.01	0.01	0.01
Corr $\pi_{4,t}, \pi_{4,t-1}$	0.88	0.93	0.94	0.98
Desired nominal interest rate				
Mean i_t^*	1.75%	3.66%	-1.14%	1.98%
Std i_t^*	0.02	0.01	0.01	0.02
Corr i_t^*, i_{t-1}^*	0.98	0.94	0.94	0.98
Nominal interest rate				
Mean i_t	1.75%	3.66%	0.10%	2.18%
Std i_t	0.02	0.01	0.03	0.02
Corr i_t, i_{t-1}	0.98	0.94	0.88	0.98

Figure 3: Distributions of endogenous variables



Note: Outliers excluded from the bottom left panel.

standard deviation of the distribution.

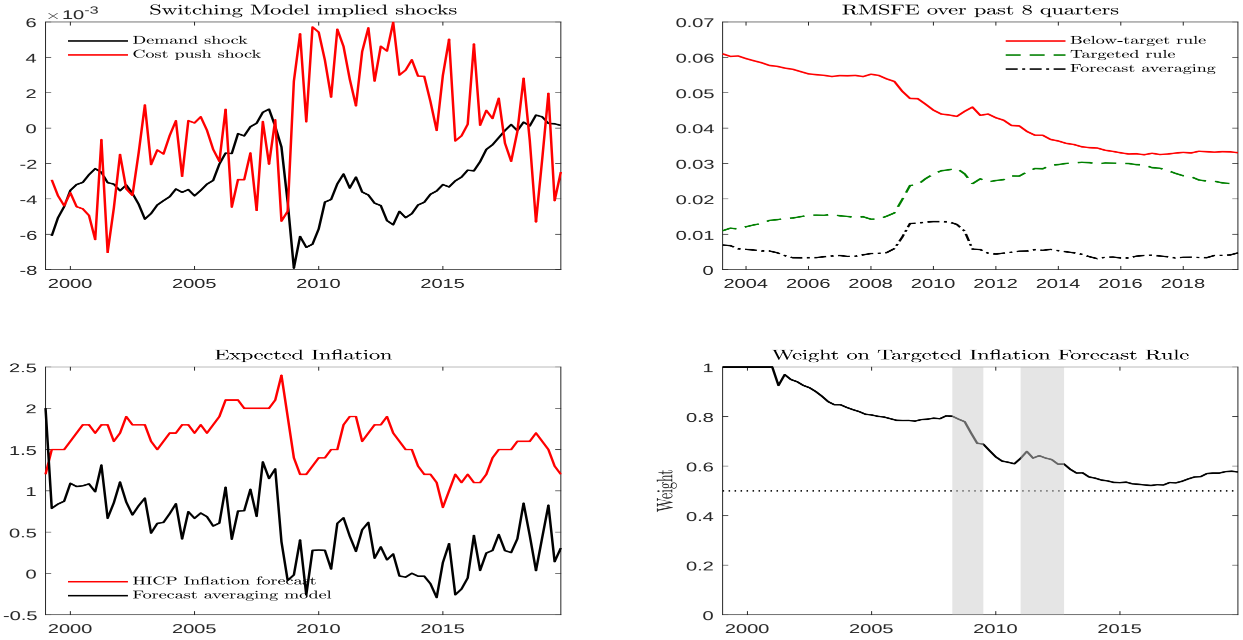
From the distributions of the desired nominal interest rate, the quarterly inflation, and the output gap, it is evident that the forecast averaging solution is suited best to match the euro area's economic behavior. The forecast averaging solution approximates the probability mass of the data counterparts best. The model generates normally distributed values for the desired nominal interest rate, but not for the nominal policy rate due to the ELB constraint. Instead, the distributions inherit two maxima, one of which is the ELB constraint, while the other centers at 3.5%. Only in case of the nominal policy rate does neither a single-equilibrium solution nor the forecast averaging solution approximate the data-related distribution closely.

5.2 Results from the reverse engineering solution

By means of a reverse engineering solution, we use the model's equilibrium transition behavior to elicit time-varying probabilities for the targeted equilibrium and the below-target equilibrium. We employ data on the natural real interest rate, inflation, the output gap and the nominal interest rate into the forecasting rules for either equilibrium. Agents optimally weigh their forecast weight assigned to the targeted equilibrium forecasting rule. The associated shock series solve the model equations. Additionally, the model solution generates time series for inflation and output gap expectations. Figure 4 plots the central results.

The top left panel shows the cost-push and demand shock series that solve the model in the reverse

Figure 4: Result of the reverse engineering solution



Note: The inflation expectations data is taken from the ECBs Survey of Professional Forecasters, specifically the one-year ahead percentage change in the all items Harmonised Index of Consumer Prices.

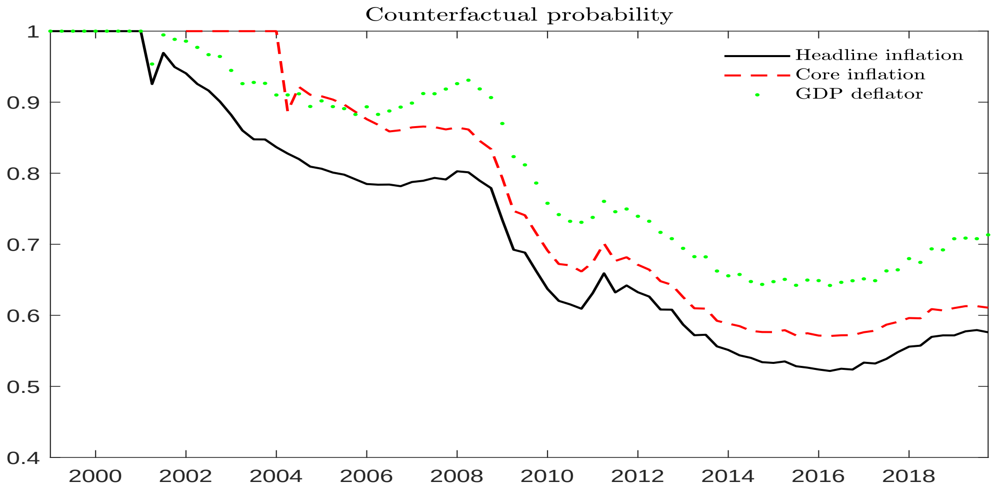
engineering solution. While the demand shock manifests predominantly negatively, the cost-push shock is much more volatile. Especially in the first seven years of our sample, the reverse engineering solution implies large negative cost-push shocks. Thereafter, the cost-push shock shifts from highly positive to rather negative realizations in the aftermath of the GFC. Until the sample's end, the cost-push shock realizes negatively.

The top right panel plots the RMSFE for the forecasting rules. The RMFSE is computed separately for hypothetical model solutions that apply only a single forecasting rule. Evidently, the below-target forecasting rule performs worst along most of the sample. While the targeted equilibrium forecasting rule performs relatively better, the forecast averaging solution is associated with the lowest error measure throughout the whole sample.

The bottom left panel compares the model-generated inflation expectations with exogenous data on inflation expectations taken from the ECB's SPF. The model-generated expectations are based on an optimal weight assigned to the targeted equilibrium forecasting rule relative to the below-target rule as in Equation 11.

We annualize the model-generated quarterly forecasts and compare those to the one-year ahead percentage change in the all items HICP. The panel highlights two aspects: on the one hand, the reverse engineering solution undershoots the inflation expectations in the euro area systematically. On the other hand, however, the forecast averaging solution is well-suited to replicate the overall trend of expectation dynamics. It follows the path of the SPF's inflation expectations closely, which supports the suitability of the equilibrium transition model to capture actual economic behavior in the euro area. The solution

Figure 5: Weight of alternative inflation series



Note: In the baseline specification, HICP headline inflation is used. For Alternative 1, we use GDP deflator series, while Alternative 2 employs HICP core inflation.

replicates the sharp drop in inflation expectations following the GFC as well as the moderate contraction around 2012/2013. Strikingly, both inflation expectation measures fluctuate below the ECB's inflation target of close but below 2% for most of the sample.

The bottom right panel plots the weight assigned to the targeted equilibrium forecasting rule, which implies a probability for the associated equilibrium. For the initial 9 quarters, μ_t is restricted to 1, before the weight is assigned endogenously by the agent's optimal forecasts of inflation and the output gap. While the probability of the targeted inflation equilibrium decreases slightly in the early 2000s, we find a large contraction starting around the onset of the GFC. From then on, the probability decreases successively, before it settles around 55% in 2017. In the course of 2018 it increases again with an implied probability of close to 58% by the end of our sample.

6 Robustness checks

To challenge the robustness of our results, we conduct reverse engineering solutions using alternative data specifications. Since the basic New Keynesian model is centered around inflation dynamics, we employ two alternative measures for euro area inflation. As a first alternative to our baseline specification, we use GDP deflator series, while we use HICP core inflation as a second alternative. Euro are core inflation is only available since 2002Q1, which is why the implied probability is plotted from then onward. The associated weights on the targeted equilibrium forecasting rule are plotted in Figure 5.

Qualitatively, our finding holds true that the probability of being in the targeted equilibrium has decreases substantially in the last two decades. In case of GDP deflator series, the contraction starts subdued and is less pronounced compared to the baseline specification. For the core inflation series, the probability lies in between the other two. Thus, this robustness exercise highlights that using alternative inflation series confirms the result qualitatively, whereas the quantitative extent depends on the exact inflation measure.

7 Possible countermeasures to the equilibrium transition

How can monetary policy be conducted to prevent the economy from drifting into the below-target equilibrium? We scrutinize this question by a counterfactual simulation of the sample period. We take the shocks from the reverse engineering solution as given, i.e. we conserve the derived time series for u_t, v_t, η_t, and ϵ_t. By letting these shocks enter the simulation and re-calibrating the monetary policy rule, we derive how the endogenous variables would have evolved under alternative policies. To do so, we substantially increase the reaction to the inflation gap, output gap, and the natural real rate gap, respectively, whilst keeping the policy smoothing parameter ρ constant. Table 4 lists the baseline and counterfactual calibration.

Table 4: Counterfactual calibration

Parameter	Baseline	Counterfactual	Description
g_y	0.13	1.00	Output gap elasticity
g_π	1.03	1.50	Inflation gap elasticity
g_r	0.00	1.80	Natural real rate gap elasticity
ρ	0.34	0.34	Smoothing parameter

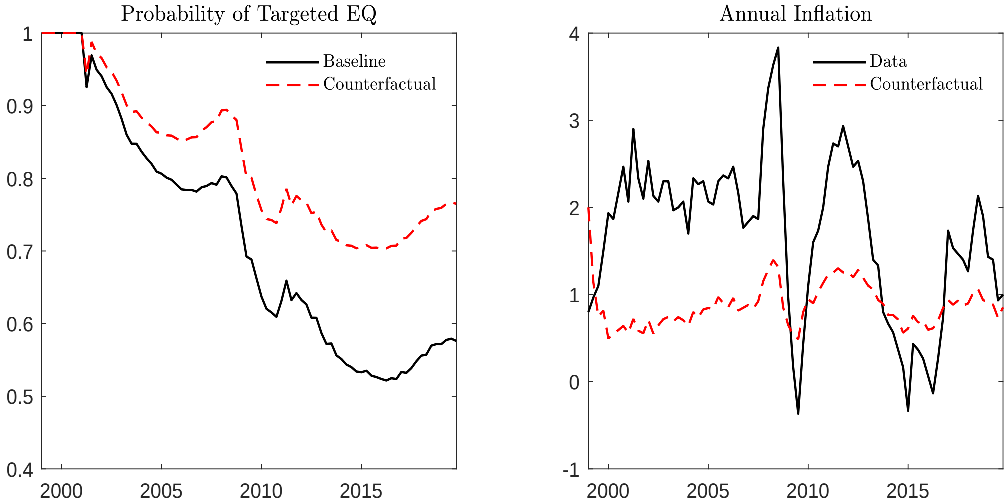
The results from the counterfactual simulation are plotted in Figure 6, which contrasts the weight on the targeted EQ forecasting rule and the associated annual inflation series in the data with their counterfactual counterparts. In the model set-up with two equilibria, the central bank would be well-advised to follow an additional objective next to the inflation target: monetary policy should maximize the probability of being in the targeted equilibrium. The original inflation objective is severely impeded by a possible transition to the below-target equilibrium, where monetary policy is constrained by the ZLB. For that reason, we plot both the probability of the targeted equilibrium and how annual inflation would have evolved under an alternative policy regime.

A more hawkish central bank would achieve a higher probability of being in the targeted equilibrium in the sample period. The counterfactual probability of being in the targeted equilibrium would lie well above the baseline series, while the overall decline would still exist but be less pronounced. For the annual inflation measure, we observe a less volatile series that would still undershoot the ECB's inflation target. So, a more hawkish central bank contributes to preventing a equilibrium transition, but would still struggle to achieve the inflation target. More research on policy options is needed.

8 Conclusion

In this paper, we addressed whether the euro area has transitioned to a new equilibrium with inflation below target and the nominal interest rate constrained by the ELB by means of a reverse engineering

Figure 6: Counterfactual simulation



solution in a two-equilibria New Keynesian model. Our findings indicate that the relative probability of the below-target equilibrium has increased substantially throughout the sample period from 1999Q1 to 2019Q4. By the end of the sample, the probability of being in the below-target equilibrium amounts to close to 42%.

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