Optimal information structures in bilateral trade: maximizing expected gains from trade

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Abstract

With the goal of maximizing expected gains from trade, this paper analyzes the optimal information structure (and mechanism) in a bilateral trade setting. The difference in gains from trade in the optimal information structure and first best constitutes the minimal loss due to asymmetric information. With binary underlying types it is shown that more than 95% of first best can be achieved while the optimal mechanism without information design may achieve less than 90% of first best. For more general type distributions, the optimal information structure is a monotone partition of the type space and the optimal mechanism is deterministic. Necessary conditions for the optimal information structure are derived and a closed form solution is given for the binary type case.

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1. Introduction

Information asymmetries can lead to inefficiencies in economic transactions, see, for example, Akerlof (1970); Mirrlees (1971); Baron and Myerson (1982). Myerson and Satterthwaite (1983) established this result in what is arguably the most basic economic setting: bilateral trade. In their model, a buyer holds *private* information about his valuation and a seller about his costs. Myerson and Satterthwaite establish an inefficiency result but also derive the mechanism maximizing expected gains from trade (EGT) in their setting. This paper extends

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their analysis by considering not only the EGT maximizing mechanism but also the EGT maximizing information structure. More precisely, imagine that buyer and seller do not know their own valuation and costs perfectly but only have a private noisy signal, i.e. an estimate, of these variables. This paper derives the information structure, i.e. a mapping from true valuation and costs to signals, that maximizes EGT. EGT under this optimal information structure are consequently the maximal EGT that are attainable (by any information structure and mechanism) under the assumption that players will eventually hold the information they receive *privately*. Hence, the difference in EGT attained by the solution of this paper and first best constitutes the EGT loss that can be attributed to information asymmetries. Any additional EGT loss has to be blamed on suboptimal institutions, i.e. either a suboptimal mechanism or a suboptimal information structure.

The goal of the paper is therefore to derive the optimal information structure in order to establish the EGT loss due to information asymmetry in a bilateral trade setting. Of course, properties of the EGT maximizing information structure (and mechanism) are also of independent interest.

There are also literal interpretations of information design. For example, conventions and institutions like the legal framework for contracting affect the information of players. Many goods are indeed transacted at a time at which value and costs are not entirely clear. Tickets for flights are, for instance, bought and sold long before the actual travel implying that the true (fuel) costs of the flight are not perfectly known at the time of contracting. Similarly, tickets for music festivals are sold at a time at which the final line-up is still subject to changes which implies that neither costs nor valuation are perfectly known at the time of contracting. In other examples, buyers know only a set of key characteristics of the product but do not know all features when buying and more generally a seller's opportunity costs which depend on potential future buyers' valuations are uncertain. In this sense, the EGT optimal information structure gives an indication about the optimal point of time for contracting or the ideal set of known attributes.

Limiting the information of a player has several effects. Consider, for example, a buyer whose valuation is either high or low and suppose the information structure is such that he does not get any information about which of the two valuations has realized. This makes it impossible to establish whether his valuation is above or below the costs of the seller and therefore less information directly harms efficiency. On the other hand, giving a player less information also reduces his information rent. The latter effect relaxes the budget balance constraint and will therefore increase EGT. Section 1.1 illustrates this tradeoff using the canonical example with uniformly distributed costs and valuations. Section 1.2 summarizes the related literature and section 2 introduces the model formally. Section 3 derives the optimal mechanism for a given finite information structure. The main results of the paper are derived in section 4 on optimal information structures. As another prominent example, binary signal/type distributions are discussed in section 5. Section 6 concludes. Proofs and derivations that are standard in the literature are relegated to the appendix.

1.1. Example: Uniform type distribution

The canonical example in the bilateral trade literature assumes that the buyer's valuation (v) and the seller's cost (c) are uniformly distributed on $[0, 1]$. First best, i.e. trade if and only if valuation is above costs, then leads to EGT of $1/6 = 0.1\bar{6}$. Myerson and Satterthwaite (1983) showed that the second best mechanism implements trade if and only if $v - c \geq 1/4$ leading to EGT of $9/64 = 0.140625$. Without information design, only 84.375% of first best EGT can be achieved due to asymmetric information. In the uniform distribution setting, the double auction has an equilibrium that achieves this second best EGT (Chatterjee and Samuelson, 1983).

Now consider the following information structure: The buyer receives a high (low) signal if his valuation is above (below) 1/3. The buyer's expected valuation upon receiving the high (low) signal is therefore 2/3 (1/6). Simlarly, the seller receives a low (high) signal if his cost is below (above) 2/3 leading to expected costs of 1/3 (5/6) in case of low (high) signal. Consider the mechanism that induces trade at price 1/2 if and only if the buyer receives the high and the seller receives the low signal. Otherwise, no trade takes place and no transfers are made. Clearly, this mechanism is incentive compatible and satisfies interim participation constraints. Expected gains from trade are $(2/3 - 1/3) * (2/3)^2 = 4/27 \approx 0.148$ or 88.9% of first. This shows that a coarsening of the information structure can increase expected gains from trade. The reason is that less information reduces information rents which are at the heart of Myerson and Satterthwaite's inefficiency result.

Table 1 presents results of a numerical analysis in which – using the results of my paper – the optimal information structure with n buyer and n seller signals was derived.[1] For $n \geq 5$, the optimization algorithm used less than n types and

[1]The numerical analysis first creates a grid of possible information structures in the mono-

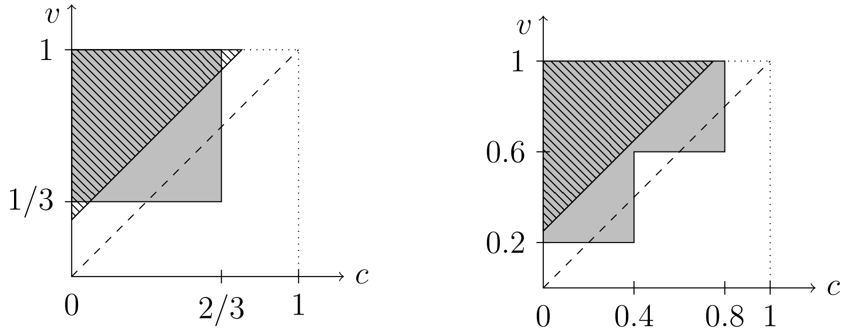

Figure 1: Trading types in the optimal mechanism with full information (hatched region) and optimal two/three element information structure (grey shaded region)

EGT did not increase further. Information design closes almost the whole gap to first best in this example as 97.55% of first best EGT can be achieved in the optimal information structure. Figure 1 illustrates the results by comparing which types trade under full information, i.e. the Myerson-Satterthwaite setup, with the optimal two and three element information structures.

n	EGT	EGT/EGT^{fb}	buyer signals	seller signals
1	0	0	0.5	0.5
2	0.148	.889	0.166, 0.666	0.333, 0.833
3	0.16	.967	0.1, 0.4, 0.8	0.2, 0.6, 0.9
4	0.163	.975	0.0503, 0.248, 0.547, 0.849	0.102, 0.354, 0.677, 0.925
5	0.163	.976	0.053, 0.232, 0.536, 0.841, 0.985	0.086, 0.339, 0.666, 0.892, 0.979

Table 1: EGT in optimal information structures for uniform type distribution on $[0, 1]$ (numerical analysis, rounded to third digit)

Some comparisons to the existing literature seem useful to put these results into perspective. First, Gottardi and Mezzetti (2019) optimally design the information structure under the assumption that a fixed price mechanism is used. They show that the binary information structure and mechanism mentioned above are also optimal in this class. Note that the optimal mechanisms in signal structures with more than two signals per player in table 1 are not fixed price mechanisms.

Second, Leininger et al. (1989) derived a class of step function equilibira in the double auction for the case without information design (i.e. the buyer/seller knows

tone partitional form, which are optimal by proposition 1, with n buyer and seller signals and maximizes EGT by brute force on this grid. The optimal information structure from this brute force method is used as a starting value for an optimization algorithm. The algorithm used is "MMA (Method of Moving Asymptotes)" as implemented in the NLopt package, see Johnson (2019) and Svanberg (2002). The code is available on the website of the author (`https://schottmueller.github.io/`). The usual disclaimer for numerical work applies, i.e. the solution is not exact and information partitions leading to even higher EGT are in principle possible.

his valuation/cost). As, for example, in the 2-step equilbirum only two bids are in the support of the bidding strategy, this might at first sight appear similar to the binary information structure above. However, the optimal binary information structure described above provides a nice example to demonstrate the power of information design. If a buyer with valuation 0.4 knew this valuation perfectly, he would be unwilling to trade at price 1/2. With information design, he is willing to trade because he only knows that his valuation is above 1/3. The coarser information structure implies less binding incentive compatibility constraints. As a result, the 2-step equilibrium of the double auction in Leininger et al. (1989) cannot achieve the same EGT that the binary information structure achieves. In fact, the 2-step equilibrium has every buyer with valuation above (below) 1/2 bidding 1/2 (0) and every seller with costs below (above) 1/2 bidding 1/2 (1).[2] Consequently, expected gains from trade are only $1/8 = 0.125$ or 75% of first best while information design with a binary information structure achieved 88.9%.

1.2. Literature

The setting is similar to Myerson and Satterthwaite (1983) who derive the mechanism maximizing expected gains from trade in a bilateral trade setting in which (i) trade is voluntary, (ii) the budget has to be balanced and (iii) the buyer (seller) privately knows his valuation (costs). Keeping (i) and (ii) this paper changes (iii) by deriving the information structure that maximizes expected gains from trade. Note that valuations and costs are independently distributed in Myerson and Satterthwaite (1983). Following the arguments of Cremer and McLean (1988), correlated types would allow to extract this private information at no cost and therefore allow for full efficiency. (In fact, telling each player both valuation and cost is one correlated information structure that eliminates private information altogether, see appendix B.) Information design could therefore achieve first best if the signal structures of buyer and seller were correlated. Hence, this paper extends the assumption in Myerson and Satterthwaite (1983) that types are independent by requiring that also signals of buyer and seller have to be independent.

The bilateral trade problem was extensively studied in the eighties. Chatterjee and Samuelson (1983) derived an equilibriumin strictly increasing strategies in the double auction. Myerson and Satterthwaite (1983) showed that this equilibrium achieves the maximally possible expected gains from trade in the standard example with uniformly distributed types. Leininger et al. (1989) use the same uniform

[2] Leininger et al. (1989) show that there is a continuum of 2-step equilibria in the double auction. I concentrate here on the one maximizing expected gains from trade.

example but derived two families of equilibria in the double auction. These achieve expected gains from trade between zero and the second best level. In particular, the equilibira in step-functions appear on first sight related to the monotone partition information structures derived in this paper. The crucial difference between those is that in a coarse information structure all types pooled on the same signal have the same information and therefore there is only one – "average" – incentive compatibility constraint for all those types. In the step-function equilibrium, all types make the same bid but have different valuations (respectively costs). Consequently, there is one incentive compatibility constraint for each type. Hence, only lower gains from trade can be realized in the step-function equilibrium than in an information structure that pools all those types that have the same bid in the step-function equilibrium on the same signal. Satterthwaite and Williams (1989) analyzed differentiable equilibira in a generalized double auction in which ganis from trade are not split equally in case of trade. Cramton et al. (1987) showed that the inefficiency result of Myerson and Satterthwaite (1983) does not hold if property rights are more evenly distributed, i.e. if there is not a seller owning the asset initially but if there are partners who both own initially a share of the item.[3]

This paper is related to a recent literature on information design as surveyed in Bergemann and Morris (2019). Within this literature the following papers consider bilateral trade settings. Closely related is Gottardi and Mezzetti (2019) who study how a mediator can help to maximize expected gains from trade. They develop a "shuttle diplomacy" protocol in which a mediator goes back and forth between the parties and provides more information to the initially uninformed parties with each visit. Effectively, this protocol establishes a particular information structure through which buyer and seller learn their valuation and cost. Gottardi and Mezzetti show that first best efficiency can be achieved using their shuttle diplomacy protocol. This appears to contradict the results of this paper but is easily explained by the fact that Gottardi and Mezzetti's protocol correlates the information of buyer and seller. As mentioned above, first best can be achieved with correlated information structures and therefore I extend the independence assumption of Myerson and Satterthwaite (1983) by assuming independent signals.[4] Less related is Roesler and Szentes (2017) who determine the consumer

[3]Another string of the literature that achieved an efficiency result considered markets with several buyers and sellers. Wilson (1985) showed that the double auction is efficient if the number of buyers and sellers is large. Rustichini et al. (1994) derived rates of convergence to efficiency as the number of market participants grows while Kojima and Yamashita (2017) propose a mechanism that is asymptotically efficient in a framework with interdependent valuations.

[4]Gottardi and Mezzetti (2019) also derive the information structure maximizing expected

surplus maximizing information structure if a monopolist seller makes a take-it-or-leave-it offer. The main differences to this paper are (i) the objective (consumer surplus vs. expected gains from trade), (ii) the seller has private information in this paper while he does not in Roesler and Szentes (2017) and (iii) the use of the optimal mechanism instead of a take-it-or-leave-it offer.[5] Technically, closest is Bergemann and Pesendorfer (2007) which derives the independent information structures that maximize revenue in independent private value auctions. As in this paper, the optimal information structure turns out to be a finite monotone partition. Apart from the objective (revenue vs. gains from trade) and the setting (auction vs. bilateral trade), the main difference is the presence of budget balance as an additional constraint in my setup. More precisely, the mechanism design problem cannot be written as an unconstrained maximization over virtual valuations as even when formulated in terms of virtual valuations/costs the problem is still subject to the budget balance constraint. While some proofs are similar in style to proofs in Bergemann and Pesendorfer (2007), the presence of this constraint complicates matters significantly. A technical contribution of this paper to the literature on information design is indeed the addition of this budget balance constraint in a setting with two-sided asymmetric information. Finally, Lang (2016, ch. 4) shows by means of an example that gains from trade can be higher if players have coarse information in a bilateral trade setting. However, he does not analyze information structures maximizing expected gains from trade.

2. Model

A single indivisible object may be traded between a buyer and a seller. The buyer's valuation for the object is distributed according to the cumulative distribution function (cdf) H_B with support on a bounded subset of $\mathbb{R}_+$. The buyer maximizes a linear utility function, i.e. he maximizes expected valuation minus expected payments. The seller's (opportunity) costs for making the object are distributed according to cdf H_S (on a bounded subset of $\mathbb{R}_+$) and the seller maximizes expected payments minus expected costs. EGT equal valuation minus costs if trade takes

gains from trade if a fixed price mechanism is used. In contrast, my paper uses the optimal mechanism which typically is not a fixed price mechanism.

[5]Condorelli and Szentes (2020) analyze a hold-up problem that can be viewed as a buyer choosing his distribution of valuations and privately learning his valuation after which a seller who only knows the distribution sets a profit maximizing price. In contrast to Roesler and Szentes (2017), the buyer is not restricted to choosing an information structure on a given "true type distribution" but can in fact choose the "true type distribution" itself. This difference to my paper is in addition to those already stated in the text above for Roesler and Szentes (2017).

place and zero otherwise.[6]

To make the problem interesting, I assume that the supports of H_B and H_S are overlapping and that for each type there are strictly positive gains from trade with some types of the other player:

Assumption 1 (Overlapping support)**.** $\min supp(H_S) < \min supp(H_B) < \max supp(H_S) < \max supp(H_B)$

A signal structure for the buyer $F: supp(H_B) \to \Delta(\Sigma_v)$ maps each valuation to a probability distribution over a set of signals $\Sigma_v \subset \mathbb{R}$. As the buyer cares only about his expected valuation it is without loss of generality to identify a signal with the expected valuation it induces. Hence, a signal v is understood to imply that the buyer has expected valuation v when receiving this signal. With this convention (and a slight abuse of notation), a signal structure can be described by a probability distribution over a set of expected valuations. A signal structure F is then feasible if and only if H_B is a mean preserving spread of F. The same applies to the seller: A feasible signal structure for the seller can be described by a distribution G over costs such that H_S is a mean preserving spread of G. A signal structure is then described by a feasible F and a feasible G. Note that the two distributions F and G are required to be independent as otherwise first best could be achieved easily by essentially eliminating the information asymmetry between buyer and seller, see appendix B.

Without loss of generality only incentive compatible direct revelation mechanisms are considered. A direct revelation mechanism ("mechanism" in the following) assigns to each pair of signals (v, c) a probability of trade $y(v, c) \in [0, 1]$ and a transfer $t_B(v, c) \in \mathbb{R}$ the buyer pays as well as a transfer $t_S(v, c) \in \mathbb{R}$ the seller receives. Incentive compatibility means that truthfully revealing his signal must yield a higher expected utility for a player than announcing another signal given that the other player announces his signal truthfully, i.e.

$$\int_{\mathbb{R}} vy(v,c) - t_B(v,c)\, dG(c) \geq \int_{\mathbb{R}} vy(v',c) - t_B(v',c)\, dG(c) \quad \text{for all } v' \in supp(F) \tag{ICB}$$

$$\int_{\mathbb{R}} t_S(v,c) - cy(v,c)\, dF(v) \geq \int_{\mathbb{R}} t_S(v,c') - cy(v,c')\, dF(v) \quad \text{for all } c' \in supp(G). \tag{ICS}$$

[6]EGT equal expected welfare if (i) budget surplus does not affect welfare and (ii) welfare denotes the sum of buyer and seller payoff.

Participation is voluntary at the interim stage: in order to be feasible a mechanism must not only be incentive compatible but also yield an expected utility of at least zero conditional on any signal. Denoting the buyer's (seller's) interim utility by U (Π), this can be written as

$$U(v) = \int_{\mathbb{R}} vy(v,c) - t_B(v,c)\, dG(c) \geq 0 \qquad \text{for all } v \in supp(F) \tag{PCB}$$

$$\Pi(c) = \int_{\mathbb{R}} t_S(v,c) - cy(v,c)\, dF(v) \geq 0 \qquad \text{for all } c \in supp(G). \tag{PCS}$$

Ex post budget balance requires $t_B(v,c) = t_S(v,c)$. However, it is well known that in the quasilinear utility model it is without loss of generality to use the weaker ex ante budget balance constraint in inequality form instead[7] (see appendix A for a standard proof)

$$\int_{\mathbb{R}} \int_{\mathbb{R}} t_B(v,c) - t_S(v,c)\, dF(v)\, dG(c) \geq 0. \tag{1}$$

Note that due to the convention that signals are the corresponding expected values, EGT equal $y(v,c)(v-c)$ if trade takes place between a buyer with signal v and a seller with signal c. The objective of this paper is to find the feasible information structure (F and G) and feasible mechanism (y, t_B and t_S) that maximize expected gains from trade subject to 1.

In the following I will refer to an element of the support of H_B or H_S as *type* and to an element of the support of F or G as *signal.* I will call the buyer's (seller's) signal structure *fully informative* if $F = H_B$ ($G = H_S$) and *noisy* otherwise.

3. Optimal mechanism for finite signal distributions

This section presents the optimal mechanism for a given finite information structure. Unsurprisingly, the derivation is similar to Myerson and Satterthwaite's analysis for a continuum of signals and therefore relegated to appendix A.

To fix notation for the finite signal case, let the buyer have signal $v_i \in \{v_1, \ldots, v_n\}$ with probability ω_i and the seller have signal $c_j \in \{c_1, \ldots, c_m\}$ with

[7] Strictly speaking this is not a "budget balance" but a "no budget deficit" constraint. In order not to overload the presentation with too much terminology, I stick to the customary "budget balance" terminology.

probability γ_j. Lower indices are assumed to denote lower signals. EGT equals

$$\sum_{i=1}^{n}\sum_{j=1}^{m} y(v_i, c_j)(v_i - c_j)\omega_i\gamma_j \tag{2}$$

where $y(v_i, c_j)$ is the probability of trade for v_i and c_j. Combining participation constraints, incentive compatibility constraints and 1 (see appendix A), one can derive the following implementability condition where $W_i = \sum_{k=1}^{i}\omega_k$ and $\Gamma_j = \sum_{k=1}^{j}\gamma_k$:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} y(v_i, c_j)\omega_i\gamma_j \left[v_i - (v_{i+1} - v_i)\frac{1 - W_i}{\omega_i} - c_j - (c_j - c_{j-1})\frac{\Gamma_{j-1}}{\gamma_j}\right] \geq 0. \tag{C}$$

The maximization problem of this paper can now be restated as maximizing EGT subject to (C) and standard monotonicty constraints on Y_S and $Y_B$that are necessary for incentive compatibility (see lemma 10 in appendix A). Neglecting these monotonicity constraints, the mechanism design problem becomes maximizing (2) subject to (C). Hence, the optimal decision rule y must maximize the Lagrangian

$$\mathcal{L}(y) = \sum_{i=1}^{n}\sum_{j=1}^{m} y(v_i, c_j)\omega_i\gamma_j \left[(1+\lambda)v_i - \lambda(v_{i+1} - v_i)\frac{1 - W_i}{\omega_i}\right.$$
$$\left. -(1+\lambda)c_j - \lambda(c_j - c_{j-1})\frac{\Gamma_{j-1}}{\gamma_j}\right] \tag{3}$$

where $\lambda \geq 0$ is the Lagrange parameter of (C). As the Lagrangian is linear in y, the optimal decision rule is

$$y^*(v_i, c_j) \begin{cases} = 1 & \text{if } v_i - (v_{i+1} - v_i)\frac{\lambda}{1+\lambda}\frac{1-W_i}{\omega_i} > c_j + (c_j - c_{j-1})\frac{\lambda}{1+\lambda}\frac{\Gamma_{j-1}}{\gamma_j} \\ \in [0,1] & \text{if } v_i - (v_{i+1} - v_i)\frac{\lambda}{1+\lambda}\frac{1-W_i}{\omega_i} = c_j + (c_j - c_{j-1})\frac{\lambda}{1+\lambda}\frac{\Gamma_{j-1}}{\gamma_j} \\ = 0 & \text{else.} \end{cases} \tag{4}$$

This leaves us with two questions: First, is it possible that (C) is non-binding? Second, will y^* satisfy the neglected monotonicity conditions? Since the signal distribution will be chosen in order to maximize EGT, it is unclear whether the usual monotone hazard rate conditions apply to W and Γ. In the following, it will be shown that it is typically not optimal to choose the information structure such that (C) is slack (in this case information is too coarse) or such that the monotonicity constraint is binding (in this case information is too fine).

4. Optimal information structure

For most of this section I take the number of signals as given. That is, it is assumed that the support of F contains no more than n signals and the support of G contains no more than m signals.[8] This restriction is useful for several reasons. First, it simplifies notation and expostion. Second, two results will be shown at a later point in the paper that emphasize the relevance of finite information structures. More precisely, finite information structures turn out to be optimal if the true type distributions H_B and H_S have finite support. Even if this not the case, it will be shown that finite information structures achieve EGT levels arbitrarily close to maximal EGT. In the following, n and m will be assumed to be at least two. The justification for this is the following lemma which establishes that pooling all types on one signal is never optimal.

Lemma 1. *The support of the signal distribution in the EGT maximizing information structure contains at least two elements for each player.*

Given the restriction to no more than n (m) buyer (seller) signals, I will show three main properties of the optimal information structure and mechanism: decision monotonicity, monotone partition structure and deterministic mechanism. Section 4.2 will then add a fourth property that holds if the type space is not too coarse: (C) binds.

Decision monotonicity refers to the monotonicity conditions for incentive compatibility given in the previous section: Y_S has to be decreasing and Y_B increasing. Note that $\lambda = 0$ in (4) would imply that y^* is the first best rule and clearly this leads to monotone Y_S and Y_B (though not necessarily strictly monotone). In order to verify that neglecting the monotonicity constraints for Y_B and Y_S in the derivation of (4) was immaterial provided that the signal structure is optimal, it is therefore sufficient to concentrate on the case $\lambda \neq 0$; that is, the case where (C) binds. Suppose now the buyer's monotonicity constraint was binding, that is $Y_B(v_i) = Y_B(v_{i+1})$ for some $i \in \{1, \dots, n-1\}$. The proof of the following lemma shows that "merging the two signals into one" would not affect the objective but strictly relax the binding constraint (C) – a contradicition. The intuition is that

[8] For notational convenience, I will then state and prove the results assuming that there are n (m) buyer (seller) signals and all these signals have strictly positive probability, i.e. $\omega_i > 0$ and $\gamma_j > 0$ for all i and j. If it is optimal to use only $n^* < n$ ($m^* < m$) signals though n (m) signals are allowed, the results obviously still hold as the solution is equivalent to the solution with n^* (m^*) in place of n (m).

merging the signals leads to coarser information and therefore to lower information rents. However, there is no downside in terms of EGT as $Y_B(v_i) = Y_B(v_{i+1})$ implies that the additional information present in the original information structure was not used to determine the efficient allocation. The lemma is stated and proven for a finite number of signals. However, this is for notational convenience only and the result holds generally as "merging signals" for which the monotonicity constraint binds generally relaxes (C) without affecting EGT.

Lemma 2. *If (C) binds in the EGT maximizing information structure with at most n buyer signals and m seller signals, then Y_S is strictly decreasing and Y_B is strictly increasing in the optimal mechanism.*
If (C) does not bind in the EGT maximizing information structure with at most n buyer signals and m seller signals, then either Y_S is strictly decreasing and Y_B is strictly increasing in the optimal mechanism or there exists $\tilde{n} \leq n$ and $\tilde{m} \leq m$ such that (i) $\tilde{n} + \tilde{m} < n + m$ and (ii) EGT with $\tilde{n}$ ($\tilde{m}$) buyer (seller) signals are no less than with n (m) buyer (seller) signals.

Lemma 2 establishes that Y_B and Y_S are strictly monotone if (C) binds. If (C) does not bind, then there exists a coarser information structure that achieves the same EGT and in which Y_B and Y_S are strictly monotone. In fact, the proof shows that this coarser information structure can be obtained by simply "merging signals", i.e. by assigning all signals v_i that have the same $Y_B(v_i)$ to the same new signal $\tilde{v}$.

The previous lemma established that the optimal decision rule is indeed characterized by (4) and neglecting the monotonicity constraints in its derivation is immaterial as Y_B and Y_S will be strictly monotone (or there exists another – coarser – information structure that is also optimal and in which Y_B and Y_S are strictly monotone). It is now worthwhile to return to (4). This optimality condition can be stated in terms of virtual valuations. That is, a buyer with signal v_i trades with a seller of signal c_j if his virtual valuaton exceeds the one of the seller. The virtual valuations are defined as

$$\begin{aligned} VV_B(v_i) &= v_i - (v_{i+1} - v_i)\frac{\lambda}{1+\lambda}\frac{1-W_i}{w_i} \\ VV_S(v_j) &= c_j + (c_j - c_{j-1})\frac{\lambda}{1+\lambda}\frac{\Gamma_{j-1}}{\gamma_j}. \end{aligned}$$

Strict monotonicity of Y_B implies that higher buyer signals must lead to higher virtual valuations. It also implies that between the virtual valuation of any two buyer signals there has to be the virtual valuation of a seller signal. The reason is that otherwise the two buyer signals would have the same probability of trade, i.e. the monotonicity constraint holds with equality and it would be better to merge the signals. Hence, seller and buyer signals will alternate in terms of virtual valuations.

The first main result establishes that the optimal information structure is a monotone partition of the type space. That is, each signal v_i corresponds to an interval of types.[9]

Proposition 1. *The optimal information structure with (at most) n buyer and m seller signals is a monotone partition (up to a measure zero set).*

The main idea behind the proof of proposition 1 is that an information structure that is not a monotone partition allows for both "mixing" and "demixing". Mixing refers here to the process of making an information structure coarser by moving two signals closer together. That is, if F assigns probabilities ω_i to v_i and ω_{i+1} to v_{i+1}, there is always a feasible information structure that uses the same probabilities but uses signals v'_i and v'_{i+1} a bit closer together. This can be achieved by sending the types that receive signal v_i (v_{i+1}) under F with some small probability the signal v'_{i+1} (v'_i) instead. If F is not a monotone partition, the opposite is possible: There is a feasible information structure that differs from F only by moving the signals v_i and v_{i+1} apart from one another. The proof shows that EGT can always be improved by one of the two operations if both, mixing and demixing, are possible. It follows that the optimal information structure has to be a monotone partition where further demixing is impossible.

Proposition 1 implies that finite type distributions lead to finite EGT maximizing information structures: A monotone partition of a type distribution with k elements in its support could lead to a signal structure with at most $2k - 1$ elements. For future reference, this is stated as a separate corollary.

Corollary 1. *Let the number of elements in the support of H_B (H_S) be finite and denote it by k. Then there exists an optimal signal structure for the buyer (seller) that contains at most $2k - 1$ elements in its support.*

Corollary 1 is important because arbitrary distributions of types H_S and H_B can be approximated arbitrarily closely by a probability distribution with finite

[9] If H_B is discrete, a monotone partition can assign a given type that has positive probability mass with some probability to v_i and some probability to v_{i+1}. It is then easier to think of a partition of $[0, 1]$ where each signal v_i corresponds to an interval $(a_i, b_i] \subset [0, 1]$ such that (i) signal v_i has probability mass $b_i - a_i$ in F and (ii) the types in $H_B^{-1}((a_i, b_i])$ receive signal v_i where H_B^{-1} is the generalized inverse of H_B. The optimal information structure with no more than n elements can therefore be completely described by $n - 1$ cutoffs.

support. For these distributions, the optimal signal structure is finite by corollary 1 and a monotone partition of the type space by proposition 1. Consequently, the optimal information structure can in general be approximated arbitrarily closely by a finite monotone partition of the type space. This property provides some justification for the focus on optimal finite information structures in the preceding lemmas and proposition. The following lemma formalizes the just mentioned approximation idea.

Lemma 3. *Take any information structure (F, G) and denote EGT in this information structure (using the optimal mechanism) by W_{FG}. Then for any $\varepsilon > 0$ there exists an information structure (F_n, G_n) with finite support such that EGT under (F_n, G_n) (using the optimal mechanism) are at least $W_{FG} - \varepsilon$.*

The previous results established properties of the optimal information structure. The following result describes the optimal mechanism given the optimal information structure. In particular, it establishes that the optimal mechanism is deterministic, i.e. that $y(v_i, c_j) \in \{0, 1\}$ for all $i \in \{1, \ldots, n\}$ and $j \in \{1, \ldots, m\}$. Note that the optimal mechanism for generic discrete information structures is not deterministic because generically the (C) does not hold with equality in deterministic mechanisms. In fact, the only reason for a stochastic mechanism is to relax the binding incentive compatibility constraint for some signal and thereby (C). The following result demonstrates that information design – in particular mixing the signal whose incentive compatibility constraint has to be relaxed with a worse signal – is a more efficient way of achieving the goal of relaxing these constraints.

Proposition 2. *Assume that a fully informative signal structure does not achieve first best EGT. Given the optimal information structure with at most n (m) buyer (seller) signals, $y(v_i, c_j) \in \{0, 1\}$ for all $i = 1, \ldots, n$ and $j = 1, \ldots, m$ in the optimal mechanism if (C) binds.*
If (C) does not bind, then there exists at least one optimal information structure such that $y(v_i, c_j) \in \{0, 1\}$ for all $i = 1, \ldots, n$ and $j = 1, \ldots, m$ in the optimal mechanism.

Proposition 2 establishes that it is without loss of generality to focus on deterministic mechanisms and consequently only such mechanisms will be considered in the remainder of the paper.

4.1. Necessary conditions for optimal information structure

This subsection derives a set of first order conditions that are satisfied by the optimal information structure with n buyer and m seller signals. To simplify notation suppose that H_B and H_S have densities h_B and h_S that are strictly positive on a bounded and convex support. Monotonicity of the optimal information structure implies then that the optimal information structure can be represented by cutoff values $(k_0, k_1, \dots, k_n)$ for the buyer where k_0 (k_n) is the infimum (supremum) of the support of H_B and $\omega_i = H_B(k_i) - H_B(k_{i-1})$ and $v_i = \int_{k_{i-1}}^{k_i} v \, dH_B(v)/\omega_i$. Similarly for the seller the information structure can be represented by a set of cutoff values $(g_0, g_1, \dots, g_m)$ such that g_0 (g_m) is the infimum (supremum) of the support of H_S and $\gamma_j = H_S(g_j) - H_S(g_{j-1})$ and $c_j = \int_{g_{j-1}}^{g_j} c \, dH_S(c)/\gamma_j$.

Note that the optimal cutoffs have to maximize the Lagrangian (3). That is, both $\partial\mathcal{L}/\partial k_i = 0$ and $\partial\mathcal{L}/\partial g_i = 0$ at the optimum. This gives a set of necessary conditions for the cutoffs $k_1, \dots, k_{n-1}$ and $g_1, \dots, g_{m-1}$. Namely,

$$\frac{\partial \mathcal{L}}{\partial k_i}/h_B(k_i) = Y_B(v_i)\left[(1+\lambda)k_i - \lambda\left(\frac{v_{i+1}-k_i}{\omega_{i+1}} - \frac{k_i - v_i}{\omega_i}\right)(1-W_i) + \lambda(v_{i+1}-v_i)\right]$$
$$+Y_B(v_{i+1})\left[-(1+\lambda)k_i + \lambda\frac{v_{i+1}-k_i}{\omega_{i+1}}(1-W_{i+1})\right] + Y_B(v_{i-1})\left[-\lambda\frac{k_i - v_i}{\omega_i}(1-W_{i-1})\right]$$
$$+\sum_{j=1}^{m}\left[y(v_i,c_j) - y(v_{i+1},c_j)\right]\left[-(1+\lambda)c_j\gamma_j - \lambda(c_j - c_{j-1}\Gamma_{j-1}\right] \stackrel{!}{=} 0 \quad (5)$$

$$\frac{\partial \mathcal{L}}{\partial g_j}/h_S(g_j) = Y_S(c_j)\left(-(1+\lambda)g_j - \lambda\frac{g_j - c_j}{\gamma_j}\Gamma_{j-1}\right) + Y_S(c_{j+2})\left[\lambda\frac{c_{j+1}-g_j}{\gamma_{j+1}}\Gamma_{j+1}\right]$$
$$+ Y_S(c_{j+1})\left[-(1+\lambda)g_j - \lambda\left(\frac{c_{j+1}-g_j}{\gamma_{j+1}} - \frac{g_j - c_j}{\gamma_j}\right)\Gamma_j - \lambda(c_{j+1}-c_j)\right]$$
$$+\sum_{i=1}^{n}\left[y(v_i,c_j) - y(v_i,c_{j+1})\right]\left[(1+\lambda)\omega_i v_i - \lambda(v_{i+1}-v_i)(1-W_i)\right] \stackrel{!}{=} 0. \quad (6)$$

One can substitute for w_i, v_i, W_i, c_i, γ_i, Γ_i the values above in order to express the conditions in terms of k_i and g_i. Nevertheless these first order conditions may, at first sight, not look useful as they still contain λ and Y_B as well as Y_S. However, note that (C) gives another equation which allows to solve for λ. While this still leaves Y_B and Y_S as unknowns, the previous results – in particular lemma 2 and proposition 2 – allow to limit the set of possible Y_B and Y_S to very few candidate solutions: As Y_B is strictly increasing, signal v_{i+1} will trade with more seller signals than signal v_i. As Y_S is strictly decreasing, v_{i+1} will in fact trade with one more seller signal than v_i. By proposition 2, this implies that $Y(v_{i+1}) = Y(v_i) + \gamma_j$ where c_j is the one additional signal with which v_{i+1} trades. Following this logic, it is clear that only very few options have to be considered. For the buyer, the main question is whether signal v_1 never trades or trades with seller signal c_1. Depending on this $Y_B(v_1) = 0$ or $Y_B(v_1) = \gamma_1$. In the first case, $Y_B(v_2) = \gamma_1$ while in the second case $Y_B(v_2) = \gamma_1 + \gamma_2$. Proceeding inductively, Y_B can be fully constructed in each of the two cases. Note that this procedure also constructs Y_S. Hence, one is left with two options for Y_B and Y_S and the system of necessary first order conditions can be solved for both cases. Comparing maximal EGT in all solutions of the set of first order conditions yields the optimal information structure with n (m) buyer (seller) signals.

As an example, consider the optimal information structure if H_B and H_S are the uniform distribution on $[0, 1]$ and $n = m = 2$. This implies that there is only one interior cutoff k (g) for the buyer (seller) and $v_1 = k/2$, $v_2 = (1 + k)/2$, $c_1 = g/2$, $c_2 = (1 + g)/2$, $\omega_1 = k = 1 - \omega_2$ and $\gamma_1 = g = 1 - \gamma_2$. Furthermore, $y(v_1, c_2) = 0$ as otherwise both players effectively have only one signal (and trade always). There are two options for $y(v_1, c_1)$ which can be either 1 or 0. The case $y(v_1, c_1) = 1$ is analyzed first. Strict monotonicity implies that $y(v_2, c_1) = 1$ and the assumption that there are two distinct buyer signals (and Y_B is strictly monotone) implies then $y(v_2, c_2) = 1$. Consequently, $Y_B(v_1) = g$, $Y_B(v_2) = 1$, $Y_S(c_1) = 1$ and $Y_S(c_2) = 1 - k$. Plugging these values in yields, after canceling terms,

$$\lambda g - (1 + \lambda)k(1 - g) + \frac{1 + \lambda}{2}(1 - g^2) = 0$$

for (5) and

$$-(1 + \lambda)gk - (1 - k)\lambda + \frac{1 + \lambda}{2}k^2 = 0$$

for (6). Constraint (C) can be written as

$$2gk - 2g + k - k^2 + gk^2 - kg^2 \geq 0 \qquad \text{with "=" if } \lambda \neq 0.$$

Consequently, one obtains three equations in the three variables λ, g and k. The only feasible solution for $\lambda \neq 0$, in the sense of $k, g \in (0, 1)$, of this system of equations is $k = 0.618034$ and $g = 0.381966$ which leads to EGT equal to 0.1459.[10] For $\lambda = 0$, one obtains $g = 1/3$ and $k = 2/3$. While this information structure satisfies (C), it only yields EGT of $1/9 < 0.1459$ and is therefore not

[10]More precisely, the solution is $\lambda = \frac{1}{5}\left(2\sqrt{5} - 5\right)$ and $g = \frac{1}{4}\left(6 - 2\sqrt{5}\right)$ and $k = \frac{1}{4}\left(2\sqrt{5} - 2\right)$.

optimal.

The second case is analyzed similarly and corresponds to $y(v_1, c_1) = 0$ which clearly implies $y(v_1, c_2) = 0$. By the strict monotonicity of Y_S and proposition 2, this implies $Y(v_2, c_1) = 1$ and $y(v_2, c_2) = 0$. Note that this mechanism can be implemented with a fixed price mechanism and consequently (C) will not bind. Equations (5) and (6) become (after canceling terms)

$$k = g/2 \qquad\qquad g = (1 + k)/2$$

which has the unique solution $g = 2/3$ and $k = 1/3$. EGT in this information structure equal $4/27 = 0.\overline{148} > 0.1459$ and therefore the optimal information structure with $n = m = 2$ equals $v_1 = 1/6$, $v_2 = 2/3$, $\omega_1 = 1/3$, $\omega_2 = 2/3$, $c_1 = 1/3$, $c_2 = 5/6$, $\gamma_1 = 2/3$, $\gamma_2 = 1/3$.

4.2. Binding constraint (C)

It remains to clarify whether constraint (C) typically binds in the optimal information structure. As already pointed out in Myerson and Satterthwaite (1983), first best may be achievable (with full information) if the type space is coarse and in these cases (C) is not binding. I present an example for this case and then argue that this is an artifact of the coarseness of the type space below.

Example 1: *Let seller and buyer types be binary. More precisely, let H_B (H_S) assign probability $1/2$ to each element in $\{2, 4\}$ (respectively $\{1, 3\}$). Consider the fully informative signal structure and note that trade if and only if valuation is above cost is implementable with the following transfers: $t(2, 1) = 2$, $t(4, 3) = 3$ and $t(2, 3) = 0$. To create a budget surplus let the buyer pay price 3 and the seller receive only payment 2 in case the types are $(4, 1)$. It is straightforward to see that incentive compatibility and participation constraints are satisfied.*

The main problem considered in this section was the problem of finding the optimal signal structure with no more than n (m) signals for the buyer (seller). As a next step, I want to change the example above slightly in order to show that in the optimal information structure of this problem the balanced budget constraint may not bind even if the type space is a continuum, the signal structure is not fully informative and first best cannot be achieved.

Example 2: *Let H_S and H_B be uniform distributions on $[0, 1]$. The optimal information structure for $n = m = 2$ was derived in section 4.1. As trade takes place only between v_2 and $c_1 < v_2$, it is clear that (C) does not bind.*

The main result of this subsection states that whenever a situation as in example 2 occurs, increasing the number of signals will result in strictly higher EGT.

As example 1 shows, this result cannot hold if the type space is coarse and therefore the following lemma is derived under the assumption that H_B and H_S have densities and their supports are identical intervals.

Lemma 4. *Let the support of H_B and H_S be identical intervals and let H_B and H_S be continuous. If (C) does not hold with equality under the optimal signal structure with at most n (m) buyer (seller) signals, then strictly higher EGT is obtained under the optimal signal structure with at most $n+1$ ($m+1$) buyer (seller) signals.*

The idea behind lemma 4 is simple: if (C) is slack, it is possible to introduce another cutoff close to the boundary of the support for one of the two players. This allows to either enable additional efficient trades or avoid inefficient trades. Because (C) was initially slack, it will remain slack if the newly created signal is close enough to the support boundary (and therefore has very low probability). Lemma 4, therefore, illustrates that the information structure is typically too coarse if (C) does not bind.

4.3. Discussion of assumption 1

I want to briefly discuss the role of assumption 1 which appears to be stronger than the overlapping support assumption in Myerson and Satterthwaite (1983). The main purpose of this assumption – apart from ruling out uninteresting cases – is to prove lemma 1 that states that the optimal information structure contains at least two signals for both players. Without assumption 1 in its strict form, this result does not necessarily apply as the following example illustrates: suppose H_S (H_B) is a uniform distribution on the binary set $\{1, 3\}$ ($\{1, 4\}$). Clearly, first best can be achieved with a signal structure that pools both seller types on one signal and is perfectly revealing for the buyer in combination with a fixed price mechanism at price $p \in [2, 4]$.

While the result of lemma 1 – at least two signals per player – simplifies notation, neither this result nor assumption 1 per se is necessary to derive lemma 2 or proposition 1. That is, the main results of the paper continue to hold if assumption 1 is violated. This is not too surprising as the proofs work on signal structures while the assumption concerns the original type distributions that do not play a role in the proofs.

Consequently, the analysis of the uniform example in section 1.1 is still in line with the results obtained of the paper although this example violates assumption 1.[11]

[11] It should be noted that the proof of lemma 1 goes through with minimal adaptations to

5. Optimal binary signal structure

One important implication of corollary 1 for the binary case is that the optimal information structure has at most three elements in its support if the true type distribution is binary. In fact, it will be shown below that the support of the optimal signal distribution will be binary if the type distribution is binary. As binary type distributions do not only provide more structure but are also often used in the (applied) literature (Kamenica and Gentzkow, 2011; Taneva, 2019), it makes sense to investigate the binary case in more detail.

Before exploiting the binary structure of the type distribution, two results are stated that make only use of the restriction $n = m = 2$. In other words, lemmas 6 and 5 also hold if only the signal distribution is restricted to be binary while the type distribution may not be binary.

The first result is that the optimal mechanism enforces trade if and only if the expected value is above expected cost. The second result states that trade takes place between players with "good signals" but not between players with "bad signals". For simplicity, the signals concerning costs (value) are denoted in this section by c_l (v_l) and c_h (v_h) with $c_h > c_l$ ($v_h > v_l$).

Lemma 5. *Consider the optimal mechanism under the optimal information structure for $n = m = 2$. Then the optimal mechanism enforces trade if and only if the buyer signal exceeds the seller signal.*

Lemma 6. *Consider the optimal mechanism under the optimal information structure for $n = m = 2$. Then $y(v_l, c_h) = 0$ and $y(v_h, c_l) = 1$.*

Intuitively, lemma 6 has to be true as $y(v_l, c_h) > 0$ would imply that $v_l \geq c_h$ and therefore trade irrespective of the signal at price $(v_l + c_h)/2$ would be optimal. This is outcome equivalent to pooling all types and cannot be optimal as information design could be used to rule out some inefficient trades; see lemma 1. Furthermore, $y(v_h, c_l) = 1$ has to hold as otherwise maximal EGT would be zero which is impossible given assumption 1.

The "only if" part of lemma 5 holds by (4). To illustrate the "if" part of lemma 5 consider signals c_h and v_l. Following the argument of the previous paragraph, $y(v_l, c_h) = 0$ and therefore $c_h \geq v_l$ as otherwise a fixed price mechanism and pooling all types would improve EGT.

also cover the case in which $supp(H_B) = supp(H_S)$ and this support is an interval; see the supplementary material for details.

The method of proof used here, i.e. arguing through fixed price mechanisms, is admittedly specific to the binary signal case and naturally leads – in conjunction with the optimality of deterministic mechanisms – to the question whether fixed price mechanims are EGT maximizing in the optimal binary information structure.[12] As the following subsection illustrates, this is not the case and the restriction to fixed price mechanisms is with loss of generality even for binary signal distributions.

5.1. Binary type distribution

This section considers the case where the true type distribution is binary. For this case, corollary 1 can be slightly strengthened and extended.

Corollary 2. *Let the true buyer valuation (seller cost) distribution have binary support. Then the optimal signal structure for the buyer (seller) has binary support and at least one element of the support is also an element of the support of the true valuation (cost) distribution.*

By corollary 2, the optimal signal distribution is binary if types are binary and one of the valuation signals as well as one of the cost signals must be fully informative. To simplify notation in this restricted setup, denote the true cost and valuation types by $\underline{c} < \bar{c}$ and $\underline{v} < \bar{v}$. Assumption 1 can then be written as $\underline{c} < \underline{v} < \bar{c} < \bar{v}$.

Lemma 7. *Consider the optimal mechanism under the optimal information structure for binary type support. Then $y(v_l, c_l) = 1 = y(v_h, c_h)$ and both $v_h = \bar{v}$ and $c_l = \underline{c}$.*

Lemma 7 leaves only the option that trade occurs unless both signals are "bad". One consequence of this is that the optimal mechanism in the optimal information structure is not a fixed price mechanism. The inequality $c_h > v_l$ holds as otherwise pooling all types would increase EGT. As signals c_h and v_l trade if and only if the other player has the "good signal" no fixed price mechanism can be optimal as no fixed price can make both c_h and v_l indifferent between trading and not trading.

Lemma 7 does not entirely describe the information structure as it does not indicate with which probability $\bar{v}$ ($\underline{c}$) types receive the v_l (c_h) signal. However, it turns out that (C) has to be satisfied with equality unless a fully informative signal structure yields a budget surplus.

[12] The converse of this is certainly true: Restricting oneself to fixed price mechanisms, the only relevant information is whether the type is above or below the fixed price and therefore binary signals are optimal; see Gottardi and Mezzetti (2019).

Lemma 8. *Consider the optimal mechanism under the optimal information structure for binary type support and assume that fully revealing signals violate (C). Constraint (C) will then bind in the optimal information structure.*

Hence, the search for the optimal information structure is equivalent to a maximization problem over two variables with one constraint or – as the constraint can be solved explicitly for one of the variables – an optimization problem over one variable without constraint, see appendix D. It is even possible to show that the objective in the latter problem is convex and therefore the solution is a corner solution. This means that one of the two players will have a fully informative signal while the other's signal has just enough noise to ensure that (C) holds. The optimal information structure is therefore one of the following two[13]

1. *buyer revealing*: $v_h = \bar{v}$, $v_l = \underline{v}$, $c_l = \underline{c}$ and $c_h = \frac{\gamma - \gamma_l^{BB}(\bar{\omega})}{1-\gamma_l^{BB}(\bar{\omega})}\underline{c} + \frac{1-\underline{\gamma}}{1-\gamma_l^{BB}(\bar{\omega})}\bar{c}$ while $\gamma_l = \gamma_l^{BB}(\bar{\omega})$ and $\omega_h = \bar{\omega}$
2. *seller revealing*: $v_h = \bar{v}$, $v_l = \frac{\bar{\omega} - \omega_h^{BB}(\underline{\gamma})}{1-\omega_h^{BB}(\underline{\gamma})}\bar{v} + \frac{1-\bar{\omega}}{1-\omega_h^{BB}(\underline{\gamma})}\underline{v}$, $c_l = \underline{c}$, $c_h = \bar{c}$ while $\gamma_l = \underline{\gamma}$ and $\omega_h = \omega_h^{BB}(\underline{\gamma})$.

It is straightforward to compute EGT in each of the two solution candidates above and the candidate achieving highest EGT is the optimal information structure. This comparison leads to the following result that completely describes the optimal information structure and mechanism in case of binary types.

Proposition 3. *Let the support of H_S and H_B be binary. Then the optimal information structure is* buyer revealing *if and only if*

$$\frac{(1-\underline{\gamma})(\bar{v}-\bar{c})}{(1-\bar{\omega})(\underline{v}-\underline{c})}\Bigg[\bar{\omega} - \frac{1}{2}\left(1 + \frac{\underline{\gamma}(\bar{v}-\underline{c})}{\bar{c}-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar{v}}\right) + \sqrt{\frac{1}{4}\left(1+\frac{\underline{\gamma}(\bar{v}-\underline{c})}{\bar{c}-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar{v}}\right)^2 - \frac{\underline{\gamma}\bar{\omega}(\bar{v}-\underline{v})+\underline{\gamma}(\underline{v}-\underline{c})}{\bar{c}-\underline{\gamma}\underline{c}-(1-\underline{\gamma})\bar{v}}}\Bigg]$$
$$\geq \underline{\gamma} - \frac{1}{2}\left(1+\frac{\bar{\omega}(\bar{v}-\underline{c})}{\bar{\omega}\bar{v}-\underline{v}+(1-\bar{\omega})\underline{c}}\right) + \sqrt{\frac{1}{4}\left(1+\frac{\bar{\omega}(\bar{v}-\underline{c})}{\bar{\omega}\bar{v}-\underline{v}+(1-\bar{\omega})\underline{c}}\right)^2 - \frac{\bar{\omega}(\bar{v}-\bar{c})+\bar{\omega}\underline{\gamma}(\bar{c}-\underline{c})}{\bar{\omega}\bar{v}-\underline{v}+(1-\bar{\omega})\underline{c}}}$$

and seller revealing *if the reverse inequality holds.*

[13]The function $\gamma_l^{BB}(\omega_h)$, which is defined in appendix D, gives the γ_l necessary to satisfy budget balance with equality for a given ω_h. $\omega_h^{BB}(\gamma_l)$ is defined analogously.

The resulting EGT can be compared to first best

$$W^{fb} = \bar{\omega}\bar{v} + (1-\bar{\omega}) * \underline{\gamma}\underline{v} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{\omega}\bar{c}.$$

The result of this comparison will generally depend on the values of the parameters $\bar{v}$, $\underline{v}$, $\underline{c}$, $\bar{c}$, γ_l, ω_h. Note, however, that it is without loss of generality to set $\bar{v} = 1$: Dividing all types by $\bar{v}$ will divide all constraints as well as the objective by $\bar{v}$ and therefore not affect the optimization problem. (Put differently, signals in the optimal information structure will be the previous optimal signals divided by $\bar{v}$. First and second best EGT will be divided by $\bar{v}$ as well.) With this normalization each of the remaining parameters, i.e. $\underline{v}$, $\underline{c}$, $\bar{c}$, γ_l, ω_h, is in the compact set $[0,1]$ and consequently it is easy to numerically search for the parameter constellation in which the ratio of second best and first best EGT is minimal. Note that the just described normalization does not affect the ratio of second best and first best EGT. I computed this ratio numerically for all parameter values on a grid with stepsize 0.01, i.e. all parameter values $\bar{\omega}, \underline{\gamma} \in \{0.01, 0.02, \ldots, 0.99\}$ and $\underline{c} \in \{0.0, 0.01, \ldots, 0.97\}$, $\underline{v} \in \{\underline{c}+0.01, \ldots, 0.98\}$, $\bar{c} \in \{\underline{v}+0.01, \ldots, 0.99\}$ are considered. The lowest ratio was 0.95417 which was achieved at $\bar{\omega} = \underline{\gamma} = 0.04$, $\underline{c} = 0$, $\bar{c} = 0.99$, $\underline{v} = 0.01$. This means that the combination of information and mechanism design can limit the loss due to asymmetric information to less than 5% in a binary type bilateral trade setting. The ratio of first best to second best EGT when using the optimal mechanism but not using information design is a natural comparison point. In this case the lowest EGT ratio equals 0.89189 which was achieved at the same parameter constellation, i.e. $\bar{\omega} = \underline{\gamma} = 0.04$, $\underline{c} = 0$, $\bar{c} = 0.99$, $\underline{v} = 0.01$. This shows that information design can close more than half of the EGT gap left by mechanism design in a binary type bilateral trade setting.

As a final remark, note that in a symmetric setup, i.e. if $\underline{\gamma} = \bar{\omega}$ and $\bar{v} - \bar{c} = \underline{v} - \underline{c}$ hold, both buyer and seller revealing information structures are optimal. Clearly, the player whose information is revealed will have a higher expected payoff in this case as he receives a higher information rent.

6. Conclusion

This paper characterizes the EGT maximizing information structure and mechanism in a bilateral trade setting. A closed form solution is derived for the special case in which the support of the true type distribution is binary. While the derivation is not straightforward the resulting information structure and mechanism are strikingly simple in this binary case: the optimal information structure is fully informative for one player and binary for the other player. The latter player receives either a signal fully revealing that he is a "good type" or a noisy signal. The optimal information structure renders the use of complicated mechanisms unnecessary: The optimal mechanism is deterministic and enforces trade if and only if – conditional on the signals – expected value is above expected costs.

With more general finite type distributions, the optimal information structure is a monotone partition of the type space and the optimal mechanism is deterministic. For type distributions with infinite support, EGT under the optimal information structure can be approximated arbitrarily closely by EGT in finite information structures that are monotone partitions of the type space. Generally, the monotonicity constraint will be slack. If the true type distribution is finite, the number of signals will be finite as well and never exceed two times the number of types.

EGT in the optimal information structure can be interpreted as an upper bound on the EGT achievable in light of asymmetric information by any institutional framework. Consequently, the EGT loss compared to first best can be interpreted as the EGT loss that is fully attributable to information asymmetries. In the binary type setting, this information loss is less than 5% of first best. This is significantly less than the EGT loss without information design (while using the optimal mechanism as in Myerson and Satterthwaite (1983)) which can exceed 10%.

Appendix

A. Optimal mechanism for finite signal distribution

EGT equals

$$\sum_{i=1}^{n}\sum_{j=1}^{m} y(v_i, c_j)(v_i - c_j)\omega_i\gamma_j$$

where $y(v_i, c_j)$ is the probability of trade for v_i and c_j. A buyer of signal v_i has expected utility

$$U(v_i) = \sum_{j=1}^{m}(v_i y(v_i, c_j) - t_B(v_i, c_j))\gamma_j = v_i Y_B(v_i) - T_B(v_i) \tag{7}$$

where the expected transfer $\sum_j t_B(v_i, c_j)\gamma_j$ is denoted by $T_B(v_i)$ and the expected probability of trade is denoted by $Y_B(v_i) = \sum_j y(v_i, c_j)\gamma_j$. Similarly, the expected utility of the seller is

$$\Pi(c_j) = \sum_{i=1}^{n}(t_S(v_i, c_j) - c_j y(v_i, c_j))\omega_j = T_S(c_j) - c_j Y_S(c_j). \tag{8}$$

The goal is to determine the EGT maximizing y and transfer rules t_S and t_B subject to

- the participation constraints

$$U(v_i) \geq 0 \quad \text{for all } v_i \in \{v_1, \ldots, v_n\} \qquad \Pi(c_j) \geq \text{ for all } c_j \in \{c_1, \ldots, c_m\}, \tag{9}$$

- the incentive compatibility constraints

$$v_i Y_B(v_i) - T_B(v_i) \geq v_i Y_B(v_k) - T_B(v_k) \quad \text{for all } v_i, v_k \in \{v_1, \ldots, v_n\}, \tag{IC$_\text{B}$}$$

$$T_S(c_j) - c_j Y_S(c_j) \geq T_S(c_k) - c_j Y_S(c_k) \quad \text{for all } c_j, c_k \in \{c_1, \ldots, c_m\}, \tag{IC$_\text{S}$}$$

- budget balance

$$\sum_{i=1}^{n}\omega_i T_B(v_i) \geq \sum_{j=1}^{m}\gamma_j T_S(c_j). \tag{BB}$$

It is straightforward to show that in this setting every ex ante budget balanced mechanism can be made ex post budget balanced in the sense that starting from an ex ante budget balanced mechanism one can manipulate the transfer rules (without changing the decision rule y and therefore without changing EGT) in a way that the new mechanism satisfies ex post budget balance (and IC as well as IR constraints are nor affected). For this reason, it is without loss of generality to use the simpler ex ante budget balance condition instead of its ex post version. This standard result and its proof are given here for completeness:

Lemma 9. *Take a direct mechanism (y, t_S, t_B) that satisfies* (9), (IC$_\mathrm{B}$), (IC$_\mathrm{S}$) *and 1. Then there is a direct mechanism $(y, \tilde{t}_S, \tilde{t}_B)$ that satisfies* (9), (IC$_\mathrm{B}$), (IC$_\mathrm{S}$) *and the ex post budget balance constraint $\tilde{t}_S(v_i, c_j) = \tilde{t}_B(v_i, c_j)$ for all $v_i \in \{v_1, \ldots, v_n\}$ and $c_j \in \{c_1, \ldots, c_m\}$.*

Proof. If 1 is satisfied with strict inequality, reducing t_B uniformly will keep all constraints satisfied and not affect EGT. Hence, it is without loss of generality to assume in the following that 1 holds with equality under (y, t_S, t_B).

With a slight abuse of notation denote by $T_S(v_i) = \sum_{j=1}^{m} t_s(v_i, c_j)\gamma_j$ the expected transfer of the seller conditional on the buyer type being v_i. Now define the new payment rules

$$\begin{aligned}\tilde{t}_B(v_i, c_j) &= t_B(v_i, c_j) + [t_S(v_i, c_j) - t_B(v_i, c_j)] - [T_S(v_i) - T_B(v_i)] \\ \tilde{t}_S(v_i, c_j) &= t_S(v_i, c_j) - [T_S(v_i) - T_B(v_i)] \,.\end{aligned}$$

Clearly, $\tilde{t}_S(v_i, c_j) = \tilde{t}_B(v_i, c_j)$ and therefore ex post budget balance holds. Furthermore, $\tilde{T}_B(v_i) = T_B(v_i)$ for all v_i and similarly $\tilde{T}(c_j) = T(c_j)$ for all c_j by the assumption that (y, t_S, t_B) is ex ante budget balanced. As y – and therefore Y_S and Y_B – did not change, this implies that $(y, \tilde{t}_S, \tilde{t}_B)$ satisfies (9), (IC$_\mathrm{B}$), (IC$_\mathrm{S}$) because (y, t_S, t_B) did. □

In particular, this means that the objective and all constraints can be expressed in terms of interim transfers T_B and T_S or alternatively in terms of interim rents U and Π. The following lemma gives a simple characterization of incentive compatibility for the discrete case.

Lemma 10. (IC$_\mathrm{B}$) *is satisfied if and only if Y_B is increasing and*

$$U(v_i) = U(v_{i-1}) + \tilde{Y}_B(v_{i-1})(v_i - v_{i-1}) \quad \textit{for} \quad i = 2, \ldots, n \tag{10}$$

where $Y_B(v_{i-1}) \leq \tilde{Y}_B(v_{i-1}) \leq Y_B(v_i)$. (IC$_\mathrm{S}$) *is satisfied if and only if Y_S is decreasing and*

$$\Pi(c_j) = \Pi(c_{j+1}) + \tilde{Y}_S(c_j)(c_{j+1} - c_j) \tag{11}$$

where $Y_S(c_j) \geq \tilde{Y}_S(c_j) \geq Y_S(c_{j+1})$.

Proof of lemma 10: *If:* Let (10) hold and Y_B be increasing. Take $i > k$. Iterating (10), yields

$$U(v_i) = U(v_k) + \sum_{j=k}^{i-1} \tilde{Y}_B(v_j)(v_{j+1} - v_j). \tag{12}$$

As $\tilde{Y}_B(v_j) \geq Y_B(v_j)$ and Y_B is increasing, this implies

$$\begin{aligned} U(v_i) &\geq U(v_k) + \sum_{j=k}^{i-1} Y_B(v_k)(v_{j+1} - v_j) \\ &= U(v_k) + Y_B(v_k)(v_i - v_k). \end{aligned}$$

Hence, (IC_B) is satisfied for v_i and v_k. Similarly starting from (12), $\tilde{Y}_B(v_j) \leq \tilde{Y}_B(v_{j+1})$ and Y_B being increasing implies

$$U(v_i) \leq U(v_k) + \sum_{j=k}^{i-1} Y_B(v_i)(v_{j+1} - v_j)$$

and therefore $U(v_k) \geq U(v_i) + Y_B(v_i)(v_k - v_i)$ which means that (IC_B) is satisfied for v_k and v_i.

Only if: Let (IC_B) be satisfied. For $k = i - 1$, (IC_B) is equivalent to $U(v_i) - U(v_{i-1}) \geq Y_B(v_{i-1})(v_i - v_{i-1})$. Using the incentive constraint that v_{i-1} does not want to misrepresent as v_i, (IC_B) can be rearranged to $U(v_{i-1}) - U(v_i) \geq Y_B(v_i)(v_{i-1} - v_i)$. Taking these two inequalities together gives

$$Y_B(v_i) \geq \frac{U(v_i) - U(v_{i-1})}{v_i - v_{i-1}} \geq Y_B(v_{i-1}).$$

Hence, Y_B is increasing and (10) holds with $\tilde{Y}_B(v_{i-1}) = [U(v_i) - U(v_{i-1})]/[v_i - v_{i-1}]$.

The proof for the seller is analogous. □

(10) and (11) can be rewritten as[14]

$$\begin{aligned} U(v_i) &= U(v_1) + \sum_{k=1}^{i-1} \tilde{Y}_B(v_k)(v_{k+1} - v_k) \\ \Pi(c_j) &= \Pi(c_m) + \sum_{k=j}^{m-1} \tilde{Y}_S(c_k)(c_{k+1} - c_k). \end{aligned}$$

[14]Here I use the notational convention that $\sum_{k=j}^{0} \cdots = 0$ for any $j = 1, 2 \ldots$.

This allows to rewrite the budget balance constraint (BB) as

$$- U(v_1) + \sum_{i=1}^{n} \omega_i \left[v_i Y_B(v_i) - \sum_{k=1}^{i-1} \tilde{Y}_B(v_k)(v_{k+1} - v_k) \right]$$
$$\geq \Pi(c_m) + \sum_{j=1}^{m} \gamma_j \left[c_j Y_S(c_j) + \sum_{k=j}^{m-1} \tilde{Y}_S(c_k)(c_{k+1} - c_k) \right]$$

which is equivalent to[15]

$$\sum_{i=1}^{n} \left[\omega_i Y_B(v_i) v_i - (v_{i+1} - v_i) \tilde{Y}_B(v_i)(1 - W_i) \right]$$
$$\geq U(v_1) + \Pi(c_m) + \sum_{j=1}^{m} \left[\gamma_j Y_S(c_j) c_j + (c_{j+1} - c_j) \tilde{Y}_S(c_j) \Gamma_j \right]$$

where $W_i = \sum_{k=1}^{i} \omega_k$ and $\Gamma_j = \sum_{k=1}^{j} \gamma_k$. In order to relax this constraint, it is best to choose $U(v_1) = \Pi(c_m) = 0$ and $\tilde{Y}_S(c_j) = Y_S(c_{j+1})$ (recall that Y_S is decreasing and that $Y_S(c_j) \geq \tilde{Y}_S(c_j) \geq Y_S(c_{j+1})$) as well as $\tilde{Y}_B(v_i) = Y_B(v_i)$ (recall that Y_B is increasing and that $Y_B(v_{i+1}) \geq \tilde{Y}_B(v_i) \geq Y_B(v_i)$). Note that none of these variables is part of the objective (2) and therefore these choices are indeed optimal. With these choices the budget balance constraint can be written as

$$\sum_{i=1}^{n} [\omega_i Y_B(v_i) v_i - (v_{i+1} - v_i) Y_B(v_i)(1 - W_i)] \geq \sum_{j=1}^{m} [\gamma_j Y_S(c_j) c_j + (c_{j+1} - c_j) Y_S(c_{j+1}) \Gamma_j] \tag{13}$$

which is equivalent to (C).

Neglecting the monotonicity constraints on Y_S and Y_B for now, the mechanism design problem becomes maximizing (2) subject to (C). Hence, the optimal decision rule y must maximize the Lagrangian

$$\mathcal{L}(y) = \sum_{i=1}^{n} \sum_{j=1}^{m} y(v_i, c_j) \omega_i \gamma_j \left[(1 + \lambda) v_i - \lambda (v_{i+1} - v_i) \frac{1 - W_i}{\omega_i} \right.$$
$$\left. - (1 + \lambda) c_j - \lambda (c_j - c_{j-1}) \frac{\Gamma_{j-1}}{\gamma_j} \right]$$

where $\lambda \geq 0$ is the Lagrange parameter of the budget balance constraint. As the

[15] Define $v_{n+1} = v_n$, $c_0 = c_1$ and $c_{m+1} = c_m$ for notational convenience and similarly $\tilde{Y}_b(v_{n+1}) = \tilde{Y}_S(c_{m+1}) = 0$.

Lagrangian is linear in y, the optimal decision rule is

$$y^*(v_i, c_j) \begin{cases} = 1 & \text{if } v_i - (v_{i+1} - v_i)\frac{\lambda}{1+\lambda}\frac{1-W_i}{\omega_i} > c_j + (c_j - c_{j-1})\frac{\lambda}{1+\lambda}\frac{\Gamma_{j-1}}{\gamma_j} \\ \in [0,1] & \text{if } v_i - (v_{i+1} - v_i)\frac{\lambda}{1+\lambda}\frac{1-W_i}{\omega_i} = c_j + (c_j - c_{j-1})\frac{\lambda}{1+\lambda}\frac{\Gamma_{j-1}}{\gamma_j} \\ = 0 & \text{else.} \end{cases}$$

.

B. Correlated signals

In the bilateral trade setup of this paper it is straightforward to show that first best EGT are achievable if one considers correlated information structures. To this end, consider a signal structure that maps each pair of types (v, c) to itself, i.e. each player receives a signal equal to the true type vector (v, c). Amend this signal structure with the mechanism

$$y((v_B, c_B), (v_S, c_S)) = \begin{cases} 1 & \text{if } v_B = v_S \geq c_B = c_S \\ 0 & \text{else} \end{cases}$$

$$t_B((v_B, c_B), (v_S, c_S)) = \begin{cases} (v_B + c_B)/2 & \text{if } v_B = v_S \geq c_B = c_S \\ 0 & \text{else} \end{cases}$$

and $t_S((v_B, c_B), (v_S, c_S)) = t_B((v_B, c_B), (v_S, c_S))$ where the first (second) argument in y, t_B and t_S is the reported buyer (seller) signal. It is straighforward to see that this mechanism is incentive compatible and satisfies the participation constraint. Most importantly, it achieves first best EGT by essentially eliminating the information asymmetry between buyer and seller.

C. Proofs of results in the text

Proof of lemma 1: Suppose to the contrary that all seller types are pooled on one signal $\mathbb{E}[c]$. In this case, the optimal mechanism is clearly a fixed price mechanism with price equal to $\mathbb{E}[c]$. Consequently, the optimal information structure for the buyer is without loss of generality binary: One signal v_l for all types below $\mathbb{E}[c]$ and one signal v_h for all types above $\mathbb{E}[c]$. By assumption 1, v_h has positive probability mass denoted by ω_h. The argument now depends on whether v_l has positive probability mass.

As a first case assume that v_l has positive probability. I will change now the seller information structure and the mechanism in two steps and show that a EGT increasing budget balanced improvement exists. In the first step, change the information structure of the seller to an information structure with two signals $c_l =$

v_l and $c_h \in (\mathbb{E}[c], v_h)$ while maintaining the mechanism $y(v_h, \cdot) = 1$ and $y(v_l, \cdot) = 0$. By assumption 1, such an information structure in which both c_l and c_h have positive probability exists.[16] Note that EGT are the same as before because the trading probabiltiy between any two types have not changed. Furthermore, (C) can be written as $\omega_h(v_h - c_h) > 0$, i.e. (C) is slack. In a second step, increase $y(v_l, c_l)$ from 0 to $\varepsilon > 0$ where ε is chosen small enough to keep (C), which reads $\omega_h(v_h - c_h) - \varepsilon\gamma_l(\omega_h(v_h - c_l) - v_l + c_l) \geq 0$, slack. As $v_l = c_l$, EGT are again unchanged. In a final step, change the seller's information structure such that γ_l, the probabiltiy of receiving the low signal, stays the same but $c_l = v_l - \varepsilon'$ and $c_h \in (\mathbb{E}[c], v_h)$ which is again possible by assumption 1 for $\varepsilon' > 0$ small enough. As $y(v_l, c_l) = \varepsilon \in (0, 1)$, this increases EGT. For $\varepsilon' > 0$ small enough (C) is not violated as it is continuous in ε' and was slack for $\varepsilon' = 0$. This establishes an information structure and mechanism satisfying budget balance and yielding strictly higher EGT than the initial structure in which the seller's types were pooled.

As second case assume that v_l has zero probability mass, i.e. $\mathbb{E}[c] \leq \min supp(H_B)$ and both seller and buyer types are pooled on a single signal each in the supposedly optimal information structure.[17] The optimal mechanism clearly enforces trade with probability 1 in this case. I will change the signal structures in several steps maintaining budget balance in each step and (weakly) increasing EGT in each step. By assumption 1, there exists an $s \in [\min supp(H_S), \max supp(H_B)]$ and an $\varepsilon > 0$ such that $H_B(s) > \varepsilon$ and $1 - H_S(s) > \varepsilon$. In a first step, change both players information structure to binary signals such that signal $c_h = v_l = s$ is sent with probability ε and the signals $c_l < c_h$ and $v_h > v_l$ are sent with probability $1 - \varepsilon$ (where c_l and v_h are chosen such that the expected value equals the expected value of the type distribution; by the definition of s and ε such a distribution is feasible). The new information structure leads to the same EGT when maintaining trade with probability 1 and is clearly budget balanced as a fixed price mechanism with price s is possible. In a second step, change the mechanism by setting $y(v_l, c_h) = 0$. As $v_l = c_h$, this does not affect EGT and as the change relaxes (C), this constraint holds now with strict inequality. In a final step, increase c_h slightly and decrease c_l slightly while both signals are still sent with probabilities ε and $1-\varepsilon$ (the decrease in c_l is, of course, chosen such that the expected value is maintained). This is feasible as $1 - H_S(c_h) = 1 - H_S(s) > \varepsilon$ by

[16]For example, let $\gamma_l = H_S(v_l)/2$ where $H_S(v_l) > 0$ by assumption 1 as $v_l \geq \min supp(H_B) > \min supp(H_S)$.

[17]An argument analogous to the first case establishes also that $\mathbb{E}[v] \geq \max supp(H_S)$ in this case.

the definition of ε. Since (C) is continuous in signals, a sufficiently small change will not violate this constraint. Furthermore, EGT are strictly increased as costs conditional on trade decreases – due to $Y_S(c_h) < Y_S(c_l)$ – and the probability of trade is unaffected. $\square$

Proof of lemma 2: Suppose to the contrary $Y_B(v_i) = Y_B(v_{i+1})$ for some $i \in \{1, \ldots, n-1\}$ in the optimal mechanism under the optimal information structure. In case the monotonicity constraint binds for more than two signals, let v_i be the lowest signal for which it binds. Now consider an information structure in which signals v_i and v_{i+1} are merged, that is, every type v that got either signal v_i or v_{i+1} will now get signal

$$\tilde{v} = \frac{\omega_i}{\omega_i + \omega_{i+1}} v_i + \frac{\omega_{i+1}}{\omega_i + \omega_{i+1}} v_{i+1}$$

and nothing changes for other types. Adapt the decision rules y by letting

$$\tilde{y}(\tilde{v}, c_j) = \frac{\omega_i}{\omega_i + \omega_{i+1}} y(v_i, c_j) + \frac{\omega_{i+1}}{\omega_i + \omega_{i+1}} y(v_{i+1}, c_j).$$

Note that this construction implies that $\tilde{Y}_S = Y_S$ and $\tilde{Y}_B(v_k) = Y_B(v_k)$ for all $k \in \{1, \ldots, i-1, i+2, \ldots, n\}$ and in particular $\tilde{Y}_B(\tilde{v}) = Y_B(v_i) = Y_B(v_{i+1})$. The objective (2) which can be written as $\sum_i \omega_i Y_B(v_i) v_i - \sum_j \gamma_j Y_S(c_j) c_j$ is therefore unchanged by the merging of signals. However, constraint (C) is strictly relaxed by the merging of signals: Note that (C) can be written as

$$\left\{ \sum_{i=1}^{n} Y_B(v_i)\omega_i \left[v_i - (v_{i+1} - v_i)\frac{1 - W_i}{\omega_i} \right] \right\} - \left\{ \sum_{j=1}^{m} Y_S(c_j)\gamma_j \left[c_j + (c_{j+1} - c_j)\frac{\Gamma_j}{\gamma_j} \right] \right\} \geq 0.$$

The merging of types affects only the two terms for v_i and v_{i+1} as Y_S and Y_B for other signals were not affected. Hence, the relevant two terms are (using the notation $\tilde{\omega} = \omega_i + \omega_{i+1}$)

$$
\begin{aligned}
& -Y_B(v_{i-1})v_i(1-W_{i-1}) + Y_B(v_i)\left[\omega_i v_i - (v_{i+1}-v_i)(1-W_i)\right] \\
& +Y_B(v_{i+1})\left[\omega_{i+1}v_{i+1} - (v_{i+2}-v_{i+1})(1-W_{i+1})\right] \\
= \;& -Y_B(v_{i-1})v_i(1-W_{i-1}) + \tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+1}-v_i)(1-W_i) - (v_{i+2}-v_{i+1})(1-W_{i+1})\right] \\
= \;& -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + Y_B(v_{i-1})(\tilde{v}-v_i)(1-W_{i-1}) \\
& +\tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+1}-v_i)(1-W_{i+1}) - (v_{i+2}-v_{i+1})(1-W_{i+1})\right] - \tilde{Y}_B(\tilde{v})(v_{i+1}-v_i)\omega_{i+1} \\
= \;& -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + Y_B(v_{i-1})(\tilde{v}-v_i)(1-W_{i+1}+\omega_i+\omega_{i+1}) \\
& +\tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+2}-v_i)(1-W_{i+1})\right] - \tilde{Y}_B(\tilde{v})(v_{i+1}-v_i)\omega_{i+1} \\
= \;& -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + Y_B(v_{i-1})(\tilde{v}-v_i)(1-W_{i+1}) + Y_B(v_{i-1})(v_{i+1}-v_i)\omega_{i+1} \\
& +\tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+2}-\tilde{v})(1-W_{i+1})\right] - \tilde{Y}_B(\tilde{v})(v_{i+1}-v_i)\omega_{i+1} - \tilde{Y}_B(\tilde{v})(\tilde{v}-v_i)(1-W_{i+1}) \\
= \;& -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + \tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+2}-\tilde{v})(1-W_{i+1})\right] \\
& +(Y_B(v_{i-1}) - \tilde{Y}_B(\tilde{v}))(\tilde{v}-v_i)(1-W_{i+1}) + (Y_B(v_{i-1}) - \tilde{Y}_B(\tilde{v}))(v_{i+1}-v_i)\omega_{i+1} \\
< \;& -Y_B(v_{i-1})\tilde{v}(1-W_{i-1}) + \tilde{Y}_B(\tilde{v})\left[\tilde{v}\tilde{\omega} - (v_{i+2}-\tilde{v})(1-W_{i+1})\right]
\end{aligned}
$$

where the first equality uses $Y_B(v_i) = Y_B(v_{i+1}) = \tilde{Y}_B(\tilde{v})$ and the definition of $\tilde{v}$, the inequality uses $\tilde{Y}_B(\tilde{v}) = Y_B(v_i) > Y(v_{i-1})$ (recall that i was the lowest pooled type). Note that the term we end up with is exactly the term referring to $\tilde{v}$ in (C) under the modified $\tilde{y}$. Consequently, the merging of signals strictly relaxed (C) without affecting the objective. If (C) binds, this contradicts the optimality of y. If (C) does not bind, then the information structure after the merging of types is coarser and also a solution as it is feasible, satisfies budget balance and yields the same EGT as the initial information structure.

The proof for the seller is analogous. □

Proof of proposition 1: We show the result for the buyer. Suppose by way of contradiction that the optimal $(v_i)_{i=1}^n$ and $(\omega_i)_{i=1}^n$ do not form a monotone partition (up to a measure zero set). This implies that there exists some $i \in \{1, \ldots, n\}$ and a set of true valuation types N_i with some mass $\eta > 0$ that receives signal v_i and a set of true valuation types N_{i+1} with mass $\eta > 0$ that receives signal v_{i+1} such that $\mathbb{E}[v|v \in N_i] > \mathbb{E}[v|v \in N_{i+1}]$. We will return to these sets later.

Consider for now the optimization problem of maximizing EGT subject to (C): Maximize EGT, i.e. $\sum_i \sum_j \omega_i \gamma_j (v_i - c_j) y(v_i, c_j)$, over y, $(v_i)_{i=1}^n$, $(c_j)_{j=1}^m$, ω_i and γ_j subject to (C). Let the domain for y be $[0,1]$ and the domain for $(\omega_i)_{i=1}^n$, $(v_i)_{i=1}^n$ is the set of all distributions such that F is a mean preserving spread of these distributions. Respectively, the domain of $(\gamma_j)_{j=1}^m$, $(c_j)_{j=1}^m$ is such that G is a mean preserving spread of these distributions. Note that incentive compatibility and participation constraints will be automatically satisfied by the solution due to substituting the expressions from lemma 10 and participation contraints into the budget constraint in order to obtain (C). (By lemma 2 the monotonicity constraint is slack.) That is, the solution to this program will be the optimal information structure and mechanism if the number of buyer (seller) signals is restricted to no more than n (m).[18] Writing the Lagrangian for this optimization problem with Lagrange parameters λ for constraint (C) yields:

$$\mathcal{L} = \sum_{i=1}^{n}\sum_{j=1}^{m}\Bigg\{ y(v_i, c_j)\omega_i\gamma_j \Bigg[(1+\lambda)v_i - \lambda(v_{i+1} - v_i)\frac{1 - W_i}{\omega_i} \\ -(1+\lambda)c_j - \lambda(c_j - c_{j-1})\frac{\Gamma_{j-1}}{\gamma_j}\Bigg]\Bigg\}$$

A solution to this finite-dimesional problem exists by the Weierstrass theorem as the feasible set is compact and non-empty and the objective is continuous. Consider $\mathcal{L}$ evaluated at the solution values for y, $(\omega_i)_{i=1}^n$, $(\gamma_j)_{j=1}^m$ and $(c_j)_{j=1}^m$. Given that, the optimal values for $(v_i)_{i=1}^n$ have to maximize $\mathcal{L}$ (within the feasible set of v_i, i.e. all those $(v_i)_{i=1}^n$ that yield together with $(\omega_i)_{i=1}^n$ a distribution such that F is a mean preserving spread of it). Now consider the following family of buyer valuation distributions indexed by ε which I denote by $(\tilde{v}_i)_{i=1}^n$: Fix all valuations apart from some $\tilde{v}_i$ and $\tilde{v}_{i+1}$ at their optimal levels (i.e. at the values that are part of the solution of the maximization problem above) and let

$$\begin{aligned}\tilde{v}_i(\varepsilon) &= \frac{(\omega_i - \varepsilon)v_i + \varepsilon v_{i+1}}{\omega_i} \\ \tilde{v}_{i+1}(\varepsilon) &= \frac{(\omega_{i+1} - \varepsilon)v_{i+1} + \varepsilon v_i}{\omega_{i+1}}\end{aligned}$$

where v_i and v_{i+1} are the solution values in the maximization problem above. As $v_i(0) = v_i$ and $v_{i+1}(0) = v_{i+1}$, the auxiliary maximization problem of maximizing $\mathcal{L}$ over ε (where all variables apart from $\tilde{v}_i$ and $\tilde{v}_{i+1}$ are fixed at their optimal solution) must be solved by $\varepsilon = 0$ (if the information structure is feasible for ε in an open neighborhood around 0). The corresponding derivative of $\mathcal{L}$ with respect

[18] Strictly speaking one should also add constraints enforcing $v_{i+1} - v_i \geq 0$ and $c_{j+1} \geq c_j$ which will, however, not change the argument below and only clutter notation further.

to ε is

$$\frac{d\mathcal{L}}{d\varepsilon} = (v_{i+1} - v_i)\left[Y_B(v_i)(1 + \lambda + \lambda(1 - W_i)/\omega_i) - Y_B(v_{i-1})\lambda(1 - W_{i-i})/\omega_i\right]$$
$$- (v_{i+1} - v_i)\left[Y_B(v_{i+1})(1 + \lambda + \lambda(1 - W_{i+1})/\omega_{i+1}) - Y_B(v_i)\lambda(1 - W_i)/\omega_{i+1}\right] \tag{14}$$

Note that the derivative does not depend on ε, i.e. $\mathcal{L}$ in the auxiliary maximization problem is linear in ε. It is straightforward to see that $(\tilde{v}_i)_{i=1}^n$ is feasible for $\varepsilon \geq 0$ if $\varepsilon \geq 0$ is not too high. (Essentially $\tilde{v}_i$ and $\tilde{v}_{i+1}$ use the optimal information structure which is feasible and then swap the signal for ε of those types receiving signals v_i and v_{i+1} in the optimal information structure. Clearly, this does not change ω_i or ω_{i+1} and yields a new feasible information structure.) I will now show that $(\tilde{v}_i)_{i=1}^n$ are also feasible for $\varepsilon < 0$ (not too far from 0) if the optimal information structure is not a monotone partition. After ruling out that the slope of $\mathcal{L}$ in ε is zero, this will complete the proof as feasibility for ε in an open interval around 0 means that $\varepsilon = 0$ cannot maximize the linear $\mathcal{L}$ in the auxiliary problem. This contradiction establishes that the optimal information structure (for the buyer) must be a monotone partition.

To see that $\varepsilon < 0$ is feasible, consider changing the information structure by swapping the signal of mass $\tau < \eta$ in N_i and N_{i+1}, i.e mass $\tau < \eta$ of the types in N_i receives signal v_{i+1} (instead of v_i) and mass τ in N_{i+1} receives signal v_i (instead of v_{i+1}). This is clearly feasible and does not change ω_i or ω_{i+1} but the expected valuation when receiving signals v_i or v_{i+1} changes to

$$\begin{aligned}\tilde{v}_i(\tau) &= \frac{\omega_i v_i - \tau\left(\mathbb{E}[v|v \in N_i] - \mathbb{E}[v|v \in N_{i+1}]\right)}{\omega_i} \\ \tilde{v}_{i+1}(\tau) &= \frac{(\omega_{i+1} v_{i+1} + \tau\left(\mathbb{E}[v|v \in N_i] - \mathbb{E}[v|v \in N_{i+1}]\right)}{\omega_{i+1}}.\end{aligned}$$

Choosing $\tau = -\varepsilon(v_{i+1} - v_i)/\left(\mathbb{E}[v|v \in N_i] - \mathbb{E}[v|v \in N_{i+1}]\right)$ yields $\tilde{v}_i(\varepsilon)$ and $\tilde{v}_i(\varepsilon)$ for negative ε.

The last step is to rule out that $\mathcal{L}$ has slope 0 in ε (when fixing all variables apart from $\tilde{v}_i(\varepsilon)$ and $\tilde{v}_{i+1}(\varepsilon)$ at their optimal values). To get a contradiction suppose this was the case and note that this is only possible if $\lambda \neq 0$, see (14) and recall that $Y_B(v_{i+1}) > Y_B(v_i)$ by lemma 2. Then there exists an $\varepsilon' > 0$ such that

$$\tilde{v}_i(\varepsilon') - (\tilde{v}_{i+1}(\varepsilon') - \tilde{v}_i(\varepsilon'))\frac{\lambda}{1+\lambda}\frac{1 - W_i}{\omega_i} = \tilde{v}_{i+1}(\varepsilon') - (v_{i+2} - \tilde{v}_{i+1}(\varepsilon'))\frac{\lambda}{1+\lambda}\frac{1 - W_{i+1}}{\omega_{i+1}}, \tag{15}$$

i.e. the two signals have the same virtual valuation.[19] $\mathcal{L}$ evaluated for ε' is the same as when evaluated at the optimal solution by the assumption that its derivative in ε is zero. As a next step, change $y(v_i, \cdot)$ and $y(v_{i+1}, \cdot)$ by assigining the average trading probability, i.e. $\tilde{y}(\tilde{v}_i, c_j) = \tilde{y}(\tilde{v}_{i+1}, c_j) = y(v_i, c_j)\omega_i/(\omega_i + \omega_{i+1}) + y(v_{i+1}, c_j)\omega_{i+1}/(\omega_i + \omega_{i+1})$ for all $j = 1, \ldots, m$. As both $\tilde{v}_i(\varepsilon')$ and $\tilde{v}_{i+1}(\varepsilon')$ have the same virtual valuation and as $\mathcal{L}$ is linear in y, this does not change the value of $\mathcal{L}$. Finally, note that due to the argument in the proof of lemma 2, merging the two signals $\tilde{v}_i(\varepsilon')$ and $\tilde{v}_{i+1}(\varepsilon')$ into one signal will not affect EGT but relaxes (C). Hence, such a merging of signals will strictly increase $\mathcal{L}$. But this implies that $(v_i, v_{i+1}, y(v_i, \cdot), y(v_{i+1}, \cdot))$ do not jointly maximize $\mathcal{L}$ in an auxilliary problem in which we fix all other variables at their optimal values. This, however, contradicts the optimality of $(v_i, v_{i+1}, y(v_i, \cdot), y(v_{i+1}, \cdot))$.

The argument for the seller is analogous. □

Proof of lemma 3: Consider the hypothetical problem of maximizing EGT subject to (C) being violated by no more than η (through the choice of an information structure and mechanism). Denote the by $W^*(\eta)$ the value of this maximization problem (more formally, the supremum of EGT achievable by information structures and mechanisms that do not violate the ex ante budget balance constraint by more than η). As both EGT and (C) are continuous, W^* is also continuous. Let $\tilde{\eta} < 0$ be such that $W^*(0) - W^*(\tilde{\eta}) < \varepsilon/3$. (Note that a negative η indicates a stricter constraint.)

Define the set of distributions $\mathcal{F}_\kappa$ as the set of distributions with cdfs F_κ such that (i) $\mathbb{E}_{F_\kappa}[v] \leq \mathbb{E}_{H_B}[v] - \kappa$ and (ii) $\int_{-\infty}^{x} F_\kappa(v)\,dv \leq \int_{-\infty}^{x} H_B(v+\kappa)\,dv - \kappa$ for all $x \in (-\infty, \max supp(H_B) - \kappa]$. Similarly, define the set $\mathcal{G}$ as the set of distributions with cdfs G_κ such that (i) $\mathbb{E}_{G_\kappa}[c] \geq \mathbb{E}_{H_S}[c] + \kappa$ and (ii) $\int_{-\infty}^{x} G_\kappa(c)\,dc \leq \int_{-\infty}^{x} H_S(c+\kappa)\,dc - \kappa$ for all $x \in (-\infty, \max supp(H_S) - \kappa]$. Note that $\mathcal{F}_0$ and $\mathcal{G}_0$ are the feasible sets of distributions in the EGT maximization problem of this paper as the set of mean preserving spreads of a distribution equals the set of distributions that have the same mean while also second order stochastically dominating the distribution, see Mas-Colell et al. (1995, ch. 6.D).

Consider now the problem of maximizing EGT subject to (C) being violated by no more than $\tilde{\eta}$ over the sets $\mathcal{F}_\kappa$ and $\mathcal{G}_\kappa$. Let F and G denote an information structure such that under this information structure and the optimal mechanism

[19] To be precise, such an ε' exists as the left hand side (LHS) of (15) is strictly below RHS for $\varepsilon' = 0$, LHS is strictly increasing in ε' while RHS is strictly decreasing in ε' and both LHS and RHS are continuous in ε'. Furthermore, $\tilde{v}_i(\varepsilon') = \tilde{v}_{i+1}(\varepsilon')$ if $\varepsilon' = (\omega_{i+1}\omega_i(v_{i+1} - v_i)/(v_{i+1}\omega_i - v_i\omega_{i+1} + i + 1\omega_{i+1} - v_i\omega_i))$ and RHS<LHS for this ε'. Consequently, the intermediate value theorem implies that an ε' exists at which LHS=RHS.

(i) (C) is violated by at most $\tilde{\eta}$, (ii) EGT are above $W^*(\tilde{\eta}) - \varepsilon/3$ and (iii) $F \in \mathcal{F}_{\tilde{\kappa}}$ and $G \in \mathcal{G}_{\tilde{\kappa}}$ for some $\tilde{\kappa} > 0$. Such F, G and $\tilde{\kappa}$ exist by the definition of $\tilde{\eta}$ and as the conditions defining $\mathcal{F}_\kappa$ and $\mathcal{G}_\kappa$ are continuous in κ (while EGT and (C) are continuous in signals).

Approximate (F, G) by a series of distributions $(F_n, G_n)_{n=1}^{\infty}$ such that (i) the support of F_n and G_n have at most n elements and (ii) $F_n \to F$ almost everywhere and $G_n \to G$ almost everywhere. Then F_n (G_n) converges to F (G) weakly and by the Helly-Bray theorem EGT and (C) under (F_n, G_n) converge to the corresponding values under (F, G).[20] Therefore for some sufficiently high n^* EGT under (F_{n^*}, G_{n^*}) are above $W^*(\tilde{\eta}) - 2\varepsilon/3 > W^*(0) - \varepsilon$ and (C) is violated by at most $\tilde{\eta}$. But this implies – by $\tilde{\eta} < 0$ – that under the finite information structure (F_{n^*}, G_{n^*}) EGT above $W^*(0) - \varepsilon$ are achievable without violating (C). Finally, define $F^*_{n^*}$ by "shifting F_{n^*} up" such that $F^*_{n^*}$ has expected value $\mathbb{E}_{H_B}[v]$, i.e. $F^*_{n^*}(x) = F_{n^*}(x - \mathbb{E}_{H_B}[v] + \mathbb{E}_{F_{n^*}}[v])$ and note that the definition of $\mathcal{F}_{\tilde{\kappa}}$ implies $\mathbb{E}_{H_B}[v] - \mathbb{E}_{F_{n^*}}[v] > 0$ (for n^* sufficiently high). Similarly, define $G^*_{n^*}(x) = F_{n^*}(x + \mathbb{E}_{H_S}[c] - \mathbb{E}_{G_{n^*}}[c])$. Note that shifting the distribution of buyer (seller) valuations up (down) by a constant, increases EGT and relaxes the budget balance constraint, see (C). Consequently, EGT under $(F^*_{n^*}, G^*_{n^*})$ are above $W(0) - \varepsilon$. Furthermore, H_B is a mean preserving spread of $F^*_{n^*}$ by the definition of $\mathcal{F}_{\tilde{\kappa}}$ and similarly H_S is a mean preserving spread of $G^*_{n^*}$. Consequently, EGT of at least $W(0) - \varepsilon$ can be achieved by a feasible finite information structure. □

Proof of proposition 2: The proof is by contradiction, i.e. I show that any information structures and mechanism such that $y(v_i, c_j) \in (0, 1)$ are not jointly optimal. To do so consider the problem of maximizing the Lagrangian (3) over y, signals and probabilities. Optimality requires that there is no feasible information structure and mechanism achieving a higher Lagrangian value than the optimal mechanism and information structure. For now, assume that the Lagrange parameter $\lambda \neq 0$. The proof exploits the following intermediate result in a number of ways:

Intermediate Result: In any information structure and mechanism maximizing the Lagrangian, $y(v_i, \cdot) \neq y(v_{i+1}, \cdot)$ for any two buyer signals v_i and v_{i+1} if $\lambda \neq 0$. Similarly, $y(\cdot, c_j) \neq y(\cdot, c_{j+1})$ for any two seller signals if $\lambda \neq 0$.[21]

Proof of the intermediate result: Suppose otherwise, i.e. $y(v_i, \cdot) = y(v_{i+1}, \cdot)$.

[20] We use the same mechanism as under (F, G) here. For completeness, define $y(v, c) = sup_{v' < v, c' > c} y(v', c')$ for all (v, c) not in the support of (F, G) (and let $y(v, c) = 0$ if $y(v', c')$ is not defined for any $v' < v$ and $c' > c$). This ensures the monotonicity of Y_S and Y_B.

[21] Even if $\lambda = 0$ the merging of types works but does not give a strict increase in the Lagrangian, i.e. there is always an optimal mechanism and information structure in which the result is true.

Now consider an alternative information structure in which the two types v_i and v_{i+1} are "merged", i.e. $\tilde{v} = (\omega_i v_i + \omega_{i+1} v_{i+1})/(\omega_i + \omega_{i+1})$ and $\tilde{\omega} = \omega_i + \omega_{i+1}$ and $y(\tilde{v}, \cdot) = y(v_i, \cdot)$ while all other variables remain as in the supposedly optimal mechanism. Clearly, EGT are not affected by the merging of types. However, due to the same steps as in the proof of lemma 2 the (C) is relaxed by the merging of types. Consequently, the value of the Lagrangian (3) is increased which contradicts the optimality of the original information structure. The proof for the seller is analogous. □

Suppose to the contrary of proposition 2 that $y(v_i, c_j) \in (0, 1)$. Note that this implies that the derivative of $\mathcal{L}$ with respect to $y(v_i, c_j)$ equals zero as $\mathcal{L}$ is linear in $y(v_i, c_j)$. Hence, changing $y(v_i, c_j)$ to either 0 or 1 does not affect the value of the Lagrangian. If such a change results in two adjacent types having the same mechanism y, the intermediate result above implies that optimality is violated as there exists another information structure leading to a strictly higher value of the Lagrangian.

To see that such a change leads to two adjacent types having the same mechanism y, note first that by the monotonicity of the virtual valuation $y(v_i, c_j) < 1$ implies $y(v_i, c_k) = 0$ for all $k > j$ and $y(v_l, c_j) = 0$ for all $l < i$. Furthermore, $y(v_i, c_j) > 0$ implies $y(v_i, c_k) = 1$ for all $k < j$ and $y(v_l, c_j) = 1$ for all $l > i$; see table 2 for an illustration. This implies that if $y(v_{i+1}, c_{j+1}) = 1$, then after changing $y(v_i, c_j)$ to zero $y(\cdot, c_j) = y(\cdot, c_{j+1})$. If, however, $y(v_{i+1}, c_{j+1}) = 0$, then after changing $y(v_i, c_j)$ to 1 $y(v_i, \cdot) = y(v_{i+1}, \cdot)$. If, $y(v_{i+1}, c_{j+1}) \in (0, 1)$, then changing $y(v_{i+1}, c_{j+1})$ to zero and $y(v_i, c_j)$ to 1 will not affect the value of the Lagrangian but then again $y(v_i, \cdot) = y(v_{i+1}, \cdot)$. Finally, observe that if $i = n$ or $j = m$ (and therefore there is not v_{i+1} and c_{j+1}) similar steps can be undertaken with v_{i-1} and c_{j-1} instead of v_{i+1} and c_{j+1}.

	$\cdots$	c_{j-1}	c_j	c_{j+1}	$\cdots$
$\vdots$			$\vdots$	$\vdots$	$\cdots$
v_{i-1}			0	0	$\cdots$
v_i	$\cdots$	1	$y(v_i, c_j)$	0	$\cdots$
v_{i+1}	$\cdots$	1	1		
$\vdots$	$\cdots$	$\vdots$	$\vdots$		

Table 2: Implications of strictly monotone virtual valuation and $y(v_i, c_j) \in (0, 1)$

Finally, consider $\lambda = 0$. In this case, $y(v_i, c_j) \in (0, 1)$ implies $v_i = c_j$ by (4). Hence, all the steps above (in the $\lambda \neq 0$ case) will maintain the Lagrangian value and therefore EGT while – through the merging of types – strictly relax constraint

(C). The resulting information structure and mechanism would then be optimal while (C) would be slack. □

Proof of lemma 5: By proposition 2, $y(v_l, c_l)$ and $y(v_h, c_h)$ are in $\{0, 1\}$. To show that trade takes place if and only if expected value is above expected cost note that (4) implies the "only if" part. For "if" consider first the case where either $y(v_l, c_l) = 0$ or $y(v_h, c_h) = 0$ (or both). In these cases, the optimal mechanism is a fixed price mechanism in which the fixed price can be chosen either $t = v_h$ or $t = c_l$ and clearly the result holds. The only remaining case is $y(v_h, c_h) = y(v_l, c_l) = 1$ and it remains to show $c_h > v_l$ in this case. Consider to the contrary $v_l \geq c_h$. But in this case a fixed price contract at price $t = c_h$ and trade with probability 1 would (weakly) increase EGT while being budget balanced, incentive compatible and satisfying the participation constraints. As in this case trade takes place regardless of signal, this outcome can be achieved by a totally uninformative information structure (i.e. one signal per player). The optimality of such a signal structure is, however, ruled out by lemma 5. □

Proof of lemma 6: The proofs will be by contradiction, i.e. I will show an improvement in EGT if the properties do not hold. First, suppose $y(v_l, c_h) > 0$. Note that this implies $y(v_i, c_j) = 1$ for all $(v_i, c_j) \neq (v_l, c_h)$ by monotonicity of the virtual valuation. By (4), $y(v_l, c_h) > 0$ implies $v_l \geq c_h$ (with strict inequality if either $v_h > v_l$ or $c_l < c_h$ have positive probability mass) and therefore EGT would be (weakly) higher if $y(v_l, c_h) = 1$, i.e. EGT would be higher if all buyer types bought from all seller types. As $v_l \geq c_h$ implies $\mathbb{E}[v] \geq \mathbb{E}[c]$ (again with strict inequality if either $v_h > v_l$ or $c_l < c_h$ have positive probability mass), trade with probability 1 is feasible by an information structure that sends signal $\mathbb{E}[v]$ to all buyers and $\mathbb{E}[c]$ to all sellers paired with a fixed price mechanism (where the fixed price is in $[\mathbb{E}[c], \mathbb{E}[v]]$.) As this information structure is not optimal by lemma 1, there is no optimal information structure in which $y(v_l, c_h) > 0$.

Second, suppose $y(v_h, c_l) < 1$. By the monotonicity of the virtual valuation, this implies $y(v_i, c_j) = 0$ for all $(v_i, c_j) \neq (v_h, c_l)$. By (4), $y(v_h, c_l) < 1$ implies $v_h \leq c_l$. EGT in this mechanism and information structure are therefore at most zero. Hence, it remains to show that there is an alternative information structure and mechanism yielding strictly positive EGT. By assumption 1, there exists a fixed price t such that the probability that $v \geq t$ as well as the probability that $c \leq t$ is strictly positive. Consider now the information structure that sends a high signal to buyers with valuation weakly above t and a low signal otherwise. Similarly, let the signal for sellers with $c \leq t$ be low and high otherwise. Pair this information structure with a mechanism enforcing trade if and only if the buyers signal is high and the sellers signal is low at price t. Clearly, this mechanism is incentive compatible, budget balanced, satisfies participation constraints and yields strictly positive EGT. □

Proof of corollary 2: As the optimal information structure is a monotone partition as a consequence of proposition 1, its support could have at most three elements in which case two of these elements would also be elements of the support of the true type distribution. The following result, stated as a separate lemma, rules this possibility out and therefore the support of the optimal signal structure can have at most two elements.

Lemma 11. *Let the true type distribution of buyer valuations H_B be discrete and let its support be $\{\hat{v}_1, \hat{v}_2, \dots\}$. If $\hat{v}_i$ and $\hat{v}_{i+1}$ are in the support of the optimal signal distribution with at most n (m) buyer (seller) signals and constraint (C) binds, then the optimal information structure assigns zero probability to all signals in $(\hat{v}_i, \hat{v}_{i+1})$.*
If constraint (C) does not bind, then either the optimal information structure assigns zero probability to all signals in $(\hat{v}_i, \hat{v}_{i+1})$ or there exists an optimal information structure with less than n (m) buyer (seller) types.
(An analogous result holds for the seller.)

Proof of lemma 11: Suppose otherwise, i.e. let the optimal information structure put positive probability on types $v_{-i} < v_i < v_{i+1}$ and let v_{i-1} and v_{i+1} be neighboring elements in the support of H_B. Denote the corresponding probabilities in the optimal information structure by ω_{i-1}, ω_i and ω_{i+1}. We will consider the following alternative distributions indexed by ε:

$$\begin{aligned} \tilde{\omega}_{i-1}(\varepsilon) &= \omega_{i-1} - \varepsilon \frac{v_{i+1} - v_i}{v_{i+1} - v_{i-1}} \\ \tilde{\omega}_i(\varepsilon) &= \omega_i + \varepsilon \\ \tilde{\omega}_{i+1}(\varepsilon) &= \omega_{i+1} - \varepsilon \frac{v_i - v_{i-1}}{v_{i+1} - v_{i-1}}. \end{aligned}$$

(All other variables, e.g. cost types probabilities of trade and other valuation types, are fixed at their optimal levels.) Note that the expected valuation is not affected by changes in ε and as v_{i-1} and v_{i+1} are *neighboring* elements of the true valuation support positive as well as negative ε are feasible (if not too large in absolute value).

Now consider the Lagrangian $\mathcal{L}$ of the maximization problem maximizing EGT over ε subject to (C) (fixing all other variables at their optimal level). From the definition $\tilde{\omega}_{i-1}$, $\tilde{\omega}_i$ and $\tilde{\omega}_{i+1}$, it is clear the $\mathcal{L}$ is linear in ε. As ω_{i-1}, ω_i and ω_{i+1}

are by assumption part of the optimal solution, $\mathcal{L}$ has to be maximized by $\varepsilon = 0$. As $\mathcal{L}$ is linear in ε and as ε in an open interval around 0 are feasible, this can only be the case if the derivative of $\mathcal{L}$ with respect to ε is zero everywhere. In the following it is shown that this is not possible if (C) binds.

Suppose the derivative of $\mathcal{L}$ with respect to ε is zero everywhere. For $\varepsilon = 0$, we have $VV(v_{i-1}, 0) < VV(v_i, 0) < VV(v_{i+1}, 0)$ by lemma 2 (where $VV(v_i, \varepsilon)$ denotes the virtual valuation of v_i for a given ε). As ε increases the virtual valuations change as $\tilde{\omega}_{i-1}$ and $\tilde{\omega}_{i+1}$ decrease while ω_i increases. Denote by $\varepsilon' > 0$ the lowest ε such that (at least) one of the following conditions is met

- $VV(v_i, \varepsilon) = VV(v_{i+1}, \varepsilon)$
- $\tilde{\omega}_{i-1}(\varepsilon) = 0$.

For concreteness, let the first condition be met at ε', i.e. $VV(v_i, \varepsilon') = VV(v_{i+1}, \varepsilon')$. Note that the value of $\mathcal{L}$ at $\varepsilon = \varepsilon'$ is the same as at $\varepsilon = 0$ as the derivative of $\mathcal{L}$ with respect to ε is supposed to be zero. As a next step (which will again not change $\mathcal{L}$), change $y(v_i, \cdot)$ and $y(v_{i+1}, \cdot)$ to $\tilde{y}(v_i, c_j) = \tilde{y}(v_{i+1}, c_j) = y(v_i, c_j)\omega_i/(\omega_i + \omega_{i+1}) + y(v_{i+1}, c_j)\omega_{i+1}/(\omega_i + \omega_{i+1})$ for $j = 1, \ldots, m$. This change will not affect $\mathcal{L}$ as $\mathcal{L}$ is linear in $y(v_i, c_j)$ with slope equal to the virtual valuation (plus a term that is constant across buyer signals and therefore unaffected) and both $\tilde{v}_i$ and $\tilde{v}_{i+1}$ had the same virtual valuation. As a last step, note that – following the proof of lemma 2 – merging types v_i and v_{i+1} to $v_i\omega_i/(\omega_i + \omega_{i+1}) + v_{i+1}\omega_{i+1}/(\omega_i + \omega_{i+1})$ with probability $\tilde{\omega}_i(\varepsilon') + \tilde{\omega}_{i+1}(\varepsilon')$ will not affect EGT but relax (C), see the proof of lemma 2. Hence, the value of $\mathcal{L}$ increases due to this change. However, this contradicts that at the optimal solution $\mathcal{L}$ is maximized by the "optimal" values v_{i-1}, v_i, v_{i+1} and ω_{i-1}, ω_i, ω_{i+1} (holding all other variables at their optimal values).

If the other conditions is met at ε', i.e. $\tilde{\omega}_{i-1}(\varepsilon') = 0$, the last step of the proof is similar. If $\tilde{\omega}_{i-1}(\varepsilon') = 0$, eliminating v_{i-1} will strictly increase $\mathcal{L}$ (as v_i's incentive compatibility constraint is strictly relaxed).

Finally, consider the case where (C) does not bind. Then the merging of signals in the steps above established that there is an information structure that (i) yields the same EGT, (ii) does not violate (C) and (iii) uses less signals than the initial optimal information structure. □

Proof of lemma 4: For concreteness let $n \leq m$ and assume that(C) holds with inequality under the optimal signal structure with at most n (m) buyer (seller) signals. Denote the solution to this problem by the optimal cutoffs for the buyer

$(\underline{s}, k_1, \ldots, k_{n-1}, \bar{s})$ and the seller $(\underline{s}, g_1, \ldots, g_{m-1}, \bar{s})$ where $\underline{s}$ and $\bar{s}$ are the minimum and maximum of the common support of H_S and H_B. Denote the trade probabilities under the information structure given by the cutoffs $(\underline{s}, k_1, \ldots, k_{n-1}, \bar{s})$ and $(\underline{s}, g_1, \ldots, g_{m-1}, \bar{s})$ by $y^*_{n,m}$.

To show that EGT is higher if one more signal is allowed, I will introduce an additional cutoff into either the buyer's or the seller's signal structure and show that this increases EGT without violating (C). To do so, I will distinguish two cases: First, the highest seller type sells with zero probability in $y^*_{n,m}$ and, second, the highest seller type sells with positive probability.

First, $y^*_{n,m}(v_n, c_m) > 0$. Then consider the cutoffs $(\underline{s}, g_1, \ldots, g_{m-1}, \bar{s} - \varepsilon, bars)$ for the seller while the cutoffs for the buyer remain unchanged. Amend y^*n, m with $y(v_i, c_{m+1}) = 0$ for all v_i. For $\varepsilon = 0$ (implying $Y_S(c_{m+1}) = 0$), the information structure and also the balanced budget constraint (BB) are unchanged and therefore EGT is the same as above. As the balanced budget constraint is continuous in ε and held with inequality for $\varepsilon = 0$, it will still hold for $\varepsilon > 0$ small enough. Clearly, welfare is higher in the new information structure (for $\varepsilon > 0$ small enough) as inefficient trades between $v_n < \bar{s}$ and sellers with a type in $[\bar{s} - \varepsilon, \bar{s}]$ are avoided. (By the assumption that H_S is continuous with an interval as support, this event has positive probability.)

Second, $y^*_{n,m}(v_n, c_m) = 0$. Then consider the cutoffs $(\underline{s}, k_1, \ldots, k_{n-1}, \bar{x} - \varepsilon, \bar{s})$ for the buyer while the cutoffs for the seller remain unchanged. Amend y^*n, m with $y(v_{n+1}, c_j) = 1$ for all c_j. For $\varepsilon = 0$ (implying $\omega_{n+1} = 0$), the information structure and also the balanced budget constraint (BB) are unchanged and therefore EGT is the same as above. As the balanced budget constraint is continuous in ε and held with inequality for $\varepsilon = 0$, it will still hold for $\varepsilon > 0$ small enough. Clearly, welfare is higher in the new information structure (for $\varepsilon > 0$ small enough) as efficient trades between buyers with types in $[\bar{s} - \varepsilon, \bar{s}]$ and sellers with signal $c_m < \bar{s}$ are enabled. □

Proof of lemma 7: By lemmas 6 and 5, the only other possibilities are (i) $y(v_l, c_l) = 0 = y(v_l, c_h)$ while $y(v_h, c_l) = 1 = y(v_h, c_h)$, (ii) $y(v_l, c_l) = 1 = y(v_h, c_l)$ while $y(v_l, c_h) = 0 = y(v_h, c_h)$ and (iii) $y(v_l, c_l) = 0 = y(v_h, c_h) = y(v_l, c_h)$ while $y(v_h, c_l) = 1$. In (i) costs are not decision relevant and therefore it is without loss to have only one cost signal. In (ii) valuations are not decision relevant and it is without loss to have only one valuation signal. In both cases, the optimality of a single signal would contradict lemma 1. Therefore, only case (iii) remains to be ruled out which is done next.

Suppose, contrary to the lemma, that $y(v_l, c_l) = 0 = y(v_h, c_h) = y(v_l, c_h)$ while

$y(v_h, c_l) = 1$, which means that trade occurs only between the high valuation and the low cost type, note that by lemma 5 $v_l \leq c_l$ and $v_h \leq c_h$. This immediately implies that $c_l > \underline{c}$ and $v_h < \bar{v}$ by assumption 1 and therefore $c_h = \bar{c}$ and $v_l = \underline{v}$ by corollary 2. The next step is to show $v_h = \bar{c}$. By lemma 5, $y(v_h, c_h) = 0$ implies $v_h \leq c_h = \bar{c}$. If $v_h < \bar{c}$, then increasing the probability that a $\bar{c}$ type receives a c_l signal by $\varepsilon > 0$ will improve EGT as it reduces the probability of inefficient trade. The resulting information structure is clearly feasible for $\varepsilon > 0$ sufficiently small and budget balance still holds as a fixed price mechanism can be used. Hence, $v_h = \bar{c}$ has to hold. An analogous argument establishes $c_l = \underline{v}$. Note that as a consequence there are no gains from trade between a $\underline{v}$ type receiving signal v_h and a seller of signal c_l. I will now change first the information structure and then the mechanism to achieve higher EGT thereby contradicting the optimality of the original information structure and mechanism. First, EGT do not change if the buyer receives a fully informative signal (while holding the seller's information structure and y fix) because of the previous observation that there are zero gains from trade between a $\underline{v}$ type receiving v_h and a seller with signal c_l. But as $\bar{v} > \bar{c} \geq c_h$, EGT can be strictly increased from there by changing the mechanism y by setting $y(v_h, c_h) = 1$ instead of $y(v_h, c_h) = 0$. Again budget balance holds as the resulting mechanism can be implemented by a fixed price mechanism with price $t = c_h$.

Therefore, $y(v_l, c_l) = 1 = y(v_h, c_h)$ which implies that trade happens unless the cost signal is high and the valuation signal is low. I will hold the mechanism, i.e. y, fixed for the remainder of the proof and first focus on the buyer showing that $v_h = \bar{v}$ in the optimal information structure. By way of contradiction suppose $v_h < \bar{v}$ and note that by corollary 2 this implies $v_l = \underline{v}$. As $v_h < \bar{v}$, some buyers with true valuation $\underline{v}$ receive the signal v_h. Consider now moving ε of these buyers to signal v_l. Put differently, the following information structures are feasible for small $\varepsilon > 0$:

$$\tilde{v}_l(\varepsilon) = \underline{v} \qquad \tilde{v}_h(\varepsilon) = \frac{\omega_h - \varepsilon - \bar{\omega}}{\omega_h - \varepsilon}\underline{v} + \frac{\bar{\omega}}{\omega_h - \varepsilon}\bar{v} \qquad \tilde{\omega}_h(\varepsilon) = \omega_h - \varepsilon \qquad \tilde{\omega}_l(\varepsilon) = 1 - \omega_h + \varepsilon$$

where $\bar{\omega}$ is the share of $\bar{v}$ in the true buyer type distribution. Note that the original information structure is obtained for $\varepsilon = 0$. Constraint (C) in the binary case (with y fixed as above) can be written as

$$\gamma_l \tilde{v}_l(\varepsilon) + (1 - \gamma_l)\tilde{\omega}_h(\varepsilon)\tilde{v}_h(\varepsilon) - \tilde{\omega}_h(\varepsilon)c_h - \gamma_l(1 - \tilde{\omega}_h(\varepsilon))c_l \geq 0.$$

The derivative of the left hand side of this condition with respect to ε is $c_h - \underline{v} + \gamma_l(\underline{v} - c_l)$ which is positive as $c_h \geq \underline{v}$ and $\underline{v} \geq c_l$ by lemmas 6 and 5. Clearly, EGT are strictly increasing in ε as well as the moved types with valuation $\underline{v}$ no longer trade inefficiently with high cost sellers. This implies that EGT are strictly higher for $\varepsilon > 0$ while (C) is not violated and thereby optimality of the original information structure is contradicted. Hence, $v_h = \bar{v}$ has to hold in the optimal information structure.

The proof for $c_l = \underline{c}$ in the optimal information structure is analogous. □

Proof of lemma 8: See appendix D below.

Proof of proposition 3: See appendix D below.

D. Derivations binary type distribution

By lemmas 6 and 7, $y(v_h, c_h) = y(v_l, c_l) = y(v_h, c_l) = 1$ while $y(v_l, c_h) = 0$ and $v_h = \bar{v}$ while $c_l = \underline{c}$. Let $\bar{\omega}$ ($\underline{\gamma}$) be the share of high (low) types in H_B (H_S). Then the optimization problem can be formulated in terms of the variables $\omega_h \in [0, \bar{\omega}]$ and $\gamma_l \in [0, \underline{\gamma}]$ and

$$\begin{aligned} v_l &= \frac{\bar{\omega} - \omega_h}{1 - \omega_h}\bar{v} + \frac{1 - \bar{\omega}}{1 - \omega_h}\underline{v} \\ c_h &= \frac{\underline{\gamma} - \gamma_l}{1 - \gamma_l}\underline{c} + \frac{1 - \underline{\gamma}}{1 - \gamma_l}\bar{c}. \end{aligned}$$

Constraint (C) can be written as

$$BB(\omega_h, \gamma_l) = \gamma_l \frac{\bar{\omega} - \omega_h}{1 - \omega_h}\bar{v} + \gamma_l \frac{1 - \bar{\omega}}{1 - \omega_h}\underline{v} + (1 - \gamma_l)\omega_h \bar{v} - \omega_h \frac{\underline{\gamma} - \gamma_l}{1 - \gamma_l}\underline{c} - \omega_h \frac{1 - \underline{\gamma}}{1 - \gamma_l}\bar{c} - \gamma_l(1 - \omega_h)\underline{c} \geq 0.$$

The objective, EGT, equals

$$W(\omega_h, \gamma_l) = (\omega_h \underline{\gamma} + (\bar{\omega} - \omega_h)\gamma_l)(\bar{v} - \underline{c}) + \gamma_l(1 - \bar{\omega})(\underline{v} - \underline{c}) + \omega_h(1 - \underline{\gamma})(\bar{v} - \bar{c}).$$

As $W(\omega_h, \gamma_l)$ is strictly increasing in both variables, BB holds with equality if and only if $BB(\bar{\omega}, \underline{\gamma}) < 0$: If BB held with inequality, increasing either γ_l or ω_h by a sufficiently small amount would increase EGT without violating BB.

Note at this point that it is possible to normalize the problem as described in the main text: the maximizing ω_h and γ_l in the original problem equal the maximizing choices in the normalized problem in which $\bar{v}^{normal} = 1$, $\underline{v}^{normal} = \underline{v}/\bar{v}$, $\underline{c}^{normal} = \underline{c}/\bar{v}$ and $\bar{c}^{normal} = \bar{c}/\bar{v}$. First/second best EGT in the original problem equals first/second best EGT in the normalized problem times $\bar{v}$. This is true as W, W^{fb}, and BB are linear in the types $\bar{v}$, $\underline{v}$, $\bar{c}$ and $\underline{c}$.

Solving the BB condition (holding with equality) for ω_h yields[22]

$$\omega_h^{BB}(\gamma_l) = \frac{1}{2}\left(1 + \frac{\gamma_l(\bar{v} - \underline{c})}{\frac{1-\underline{\gamma}}{1-\gamma_l}(\bar{c} - \underline{c}) - (1-\gamma_l)(\bar{v} - \underline{c})}\right)$$
$$- \sqrt{\frac{1}{4}\left(1 + \frac{\gamma_l(\bar{v} - \underline{c})}{\frac{1-\underline{\gamma}}{1-\gamma_l}(\bar{c} - \underline{c}) - (1-\gamma_l)(\bar{v} - \underline{c})}\right)^2 - \frac{\gamma_l\bar{\omega}(\bar{v} - \underline{v}) + \gamma_l(\underline{v} - \underline{c})}{\frac{1-\underline{\gamma}}{1-\gamma_l}(\bar{c} - \underline{c}) - (1-\gamma_l)(\bar{v} - \underline{c})}}$$

while solving the BB condition (holding with equality) for γ_l yields

$$\gamma_l^{BB}(\omega_h) = \frac{1}{2}\left(1 + \frac{\omega_h(\bar{v} - \underline{c})}{\frac{1-\bar{\omega}}{1-\omega_h}(\bar{v} - \underline{v}) - (1-\omega_h)(\bar{v} - \underline{c})}\right)$$
$$- \sqrt{\frac{1}{4}\left(1 + \frac{\omega_h(\bar{v} - \underline{c})}{\frac{1-\bar{\omega}}{1-\omega_h}(\bar{v} - \underline{v}) - (1-\omega_h)(\bar{v} - \underline{c})}\right)^2 - \frac{\omega_h(\bar{v} - \bar{c}) + \omega_h\underline{\gamma}(\bar{c} - \underline{c})}{\frac{1-\bar{\omega}}{1-\omega_h}(\bar{v} - \underline{v}) - (1-\omega_h)(\bar{v} - \underline{c})}}.$$

$\omega_h^{BB}(\gamma_l)$ can be plugged into W in order to get a one-dimensional optimization problem over $\gamma_l \in [\gamma_l^{BB}(\bar{\omega}), \underline{\gamma}]$. I numerically verified that the resulting objective function is convex in γ_l (under the assumption that $BB(\bar{\omega}, \underline{\gamma}) < 0$).[23] Consequently the solution is either

- $\gamma_l = \gamma_l^{BB}(\bar{\omega})$ and therefore $\omega_h = \bar{\omega}$ or
- $\gamma_l = \underline{\gamma}$ and therefore $\omega_h = \omega_h^{BB}(\underline{\gamma})$.

Put differently, one player receives a perfectly informative signal and the other player a noisy signal. For concreteness, the relevant values $\gamma_l^{BB}(\bar{\omega})$ and $\omega_h^{BB}(\underline{\gamma})$ are given explicitly:

$$\begin{aligned}
\gamma_l^{BB}(\bar{\omega}) &= \frac{1}{2}\left(1 + \frac{\bar{\omega}(\bar{v} - \underline{c})}{\bar{\omega}\bar{v} - \underline{v} + (1-\bar{\omega})\underline{c}}\right) \\
&\quad - \sqrt{\frac{1}{4}\left(1 + \frac{\bar{\omega}(\bar{v} - \underline{c})}{\bar{\omega}\bar{v} - \underline{v} + (1-\bar{\omega})\underline{c}}\right)^2 - \frac{\bar{\omega}(\bar{v} - \bar{c}) + \bar{\omega}\underline{\gamma}(\bar{c} - \underline{c})}{\bar{\omega}\bar{v} - \underline{v} + (1-\bar{\omega})\underline{c}}} \\
\omega_h^{BB}(\underline{\gamma}) &= \frac{1}{2}\left(1 + \frac{\underline{\gamma}(\bar{v} - \underline{c})}{\bar{c} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{v}}\right) \\
&\quad - \sqrt{\frac{1}{4}\left(1 + \frac{\underline{\gamma}(\bar{v} - \underline{c})}{\bar{c} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{v}}\right)^2 - \frac{\underline{\gamma}\bar{\omega}(\bar{v} - \underline{v}) + \underline{\gamma}(\underline{v} - \underline{c})}{\bar{c} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{v}}}.
\end{aligned}$$

[22]The second solution of the quadratic equation is above 1 – as $\frac{\gamma_l(\bar{v}-\underline{c})}{\frac{1-\underline{\gamma}}{1-\gamma_l}(\bar{c}-\underline{c})-(1-\gamma_l)(\bar{v}-\underline{c})} > 1$ by $\gamma_l \leq \underline{\gamma}$ – and therefore not relevant. Note that there always exists a solution in $(0,1)$ as the budget balance constraint is slack if $\omega_h = 0$.

[23]The code is available on the website of the author (`https://schottmueller.github.io/`).

To determine which of the two solutions yields higher EGT it is simplest to compare for both the difference to first best EGT. As $W(\bar{\omega}, \gamma_l)$ is linear in γ_l this difference can be expressed as

$$W(\bar{\omega}, \underline{\gamma}) - W(\bar{\omega}, \gamma_l^{BB}(\bar{\omega})) = (1-\bar{\omega})(\underline{v} - \underline{c})$$
$$\left(\underline{\gamma} - \frac{1}{2}\left(1 + \frac{\bar{\omega}(\bar{v} - \underline{c})}{\bar{\omega}\bar{v} - \underline{v} + (1-\bar{\omega})\underline{c}}\right) + \sqrt{\frac{1}{4}\left(1 + \frac{\bar{\omega}(\bar{v} - \underline{c})}{\bar{\omega}\bar{v} - \underline{v} + (1-\bar{\omega})\underline{c}}\right)^2 - \frac{\bar{\omega}(\bar{v} - \bar{c}) + \bar{\omega}\underline{\gamma}(\bar{c} - \underline{c})}{\bar{\omega}\bar{v} - \underline{v} + (1-\bar{\omega})\underline{c}}}\right)$$

$$W(\bar{\omega}, \underline{\gamma}) - W(\omega_h^{BB}(\underline{\gamma}), \underline{\gamma})) = (1-\underline{\gamma})(\bar{v} - \bar{c})$$
$$\left(\bar{\omega} - \frac{1}{2}\left(1 + \frac{\underline{\gamma}(\bar{v} - \underline{c})}{\bar{c} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{v}}\right) + \sqrt{\frac{1}{4}\left(1 + \frac{\underline{\gamma}(\bar{v} - \underline{c})}{\bar{c} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{v}}\right)^2 - \frac{\underline{\gamma}\bar{\omega}(\bar{v} - \underline{v}) + \underline{\gamma}(\underline{v} - \underline{c})}{\bar{c} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{v}}}\right).$$

Consequently, $\gamma_l = \gamma_l^{BB}(\bar{\omega})$ and therefore $\omega_h = \bar{\omega}$ in the optimal mechanism if and only if

$$\begin{aligned}
&\frac{(1-\underline{\gamma})(\bar{v} - \bar{c})}{(1-\bar{\omega})(\underline{v} - \underline{c})}\Bigg[\bar{\omega} - \frac{1}{2}\left(1 + \frac{\underline{\gamma}(\bar{v} - \underline{c})}{\bar{c} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{v}}\right) \\
&\qquad + \sqrt{\frac{1}{4}\left(1 + \frac{\underline{\gamma}(\bar{v} - \underline{c})}{\bar{c} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{v}}\right)^2 - \frac{\underline{\gamma}\bar{\omega}(\bar{v} - \underline{v}) + \underline{\gamma}(\underline{v} - \underline{c})}{\bar{c} - \underline{\gamma}\underline{c} - (1-\underline{\gamma})\bar{v}}}\Bigg] \\
&\qquad\qquad \geq \underline{\gamma} - \frac{1}{2}\left(1 + \frac{\bar{\omega}(\bar{v} - \underline{c})}{\bar{\omega}\bar{v} - \underline{v} + (1-\bar{\omega})\underline{c}}\right) \\
&\qquad\qquad + \sqrt{\frac{1}{4}\left(1 + \frac{\bar{\omega}(\bar{v} - \underline{c})}{\bar{\omega}\bar{v} - \underline{v} + (1-\bar{\omega})\underline{c}}\right)^2 - \frac{\bar{\omega}(\bar{v} - \bar{c}) + \bar{\omega}\underline{\gamma}(\bar{c} - \underline{c})}{\bar{\omega}\bar{v} - \underline{v} + (1-\bar{\omega})\underline{c}}}
\end{aligned}$$

and $\gamma_l = \underline{\gamma}$ and therefore $\omega_h = \omega_h^{BB}(\underline{\gamma})$ otherwise.

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Supplementary Material

This section proves lemma 1 using not assumption 1 from the main text but the following support assumption instead:

Assumption 2. *H_S and H_B have the same support and this support is an interval.*

Proof of lemma 1 under assumption 2: Suppose to the contrary that all seller types are pooled on one signal $\mathbb{E}[c]$. In this case, the optimal mechanism is clearly a fixed price mechanism with price equal to $\mathbb{E}[c]$. Consequently, the optimal information structure for the buyer is without loss of generality binary: One signal v_l for all types below $\mathbb{E}[c]$ and one signal v_h for all types above $\mathbb{E}[c]$. By assumption 2, both v_h and v_l have positive probability mass denoted by ω_h and ω_l. Also note that clearly $v_l < \mathbb{E}[c] < v_h$.

I will change now the seller information structure and the mechanism in two steps and show that a EGT increasing budget balanced improvement exists. In the first step, change the information structure of the seller to an information structure with two signals $c_l = v_l$ and $c_h \in (\mathbb{E}[c], v_h)$ while maintaining the mechanism $y(v_h, \cdot) = 1$ and $y(v_l, \cdot) = 0$. By assumption 2 and $v_h > \mathbb{E}[c]$, such an information structure in which both c_l and c_h have positive probability exists.[24] Note that EGT are the same as before because the trading probabiltiy between any two types have not changed. Furthermore, constraint (C) can be written as $\omega_h(v_h - c_h) > 0$, i.e. (C) is slack. In a second step, increase $y(v_l, c_l)$ from 0 to $\varepsilon > 0$ where ε is chosen small enough to keep (C), which reads $\omega_h(v_h - c_h) - \varepsilon\gamma_l(\omega_h(v_h - c_l) - v_l + c_l) \geq 0$, slack. As $v_l = c_l$, EGT are again unchanged. In a final step, change the seller's information structure such that γ_l, the probabiltiy of receiving the low signal, stays the same but $c_l = v_l - \varepsilon'$ and $c_h \in (\mathbb{E}[c], v_h)$ which is again possible by assumption 2 for $\varepsilon' > 0$ small enough. As $y(v_l, c_l) = \varepsilon \in (0, 1)$, this increases EGT. For $\varepsilon' > 0$ small enough (C) is not violated as it is continuous in ε' and was slack for $\varepsilon' = 0$. This establishes an information structure and mechanism satisfying budget balance and yielding strictly higher EGT than the initial structure in which the seller's types were pooled. □

[24] Think of γ_l, i.e. the probability of signal c_l, being very small.