Security-bid Auctions with Information Acquisition*

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Abstract

We study security-bid auctions in which bidders compete for an asset by bidding with securities whose payments are contingent on the asset's realized value and can covertly acquire information at some cost before participating in an auction. We first consider auctions with ordered securities in which the seller restricts the security design to an ordered set and uses a first- or second-price auction. We show that steeper securities give agents lower marginal returns to information and may yield lower revenues. We then study linear mechanisms in which payments linearly depend on the asset's realized value. We show that the revenue-maximizing linear mechanism assigns the asset efficiently, and the winner pays in cash if their expected values are high and pays in stock if their expected values are low. This result implies that the use of cash payment is positively correlated to synergies in mergers and acquisitions. We empirically test this implication and find consistent results.

Keywords: Information Acquisition, Securities, Auctions, Contingent Mechanisms

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1 Introduction

In most of the literature on auctions, two important assumptions are made. First, the amount of information possessed by agents is fixed exogenously. Second, the payment by an agent depends only on his report and not on his realized value. However, in many important settings, these assumptions do not apply. For example, in the auctions for offshore oil and gas leases in the U.S., companies conduct seismic surveys to collect information about the tracts offered for sale before participating in the auctions. Another example is the sale of financial or business assets, in which buyers perform due diligence to investigate the quality and compatibility of the assets before submitting offers. In these examples, information held by agents is not only endogenous, but also costly to acquire. In the example of U.S. auctions for offshore oil and gas leases (see Haile et al. (2010)), 3-D seismic surveys have been used in 80% of wells drilled in Gulf of Mexico by 1996, and it cost $\$100,000$ to examine a 50 square mile 3-D seismic survey in 2000. Similarly, in the sale of a business asset, the legal and accounting costs of performing due diligence often amount to millions of dollars (see Quint and Hendricks (2013) and Bergemann et al. (2009)).

Moreover, in these examples, the ex-post values of the assets are contractible, and the payments by agents can depend on their ex post payoffs. For example, in the auctions for offshore oil and gas leases in the U.S., the winner's payment to the government is a bonus plus a fraction of revenues from any oil or gas extracted. When selling a company to an acquirer or soliciting venture capital, equity and other securities are commonly used.

Earlier studies have established that auctions using cash bids can affect the incentives for agents to acquire information (see, e.g., Stegeman (1996) and Persico (2000)). Surprisingly, few studies have considered auctions using security bids with information acquisition. To the best of my knowledge, the only paper that has studied this question is Gaier et al. (2005) who show that in the pure common value setting, share auctions give agents lower incentives to gather information than cash auctions. In this paper, we study security-bid auctions in the independent private value setting. The feasible set of securities admit many standard sets of securities, including cash and equities.

We consider first auctions with ordered securities. In these auctions, the seller restricts the security design to an ordered set and uses a first- or second-price auction. We show that for either first- or second-price auction, steeper securities give agents lower marginal return to information, and all securities give agents lower marginal return to information than cash.

This paper is closely related to DeMarzo et al. (2005), who also study security-bid auctions. They show that, when the information possessed by agents is exogenous, steeper securities yield higher revenues and all security-bid auctions yield higher revenues than cash auctions. However, this result may not hold with costly information acquisition. The seller's expected revenue is given by the difference between the expected surplus and the information rents accruing to the agents. On the one hand, steeper securities yield lower information rents given the accuracy of information. On the other hand, steeper securities lead to less accurate information, reducing the expected surplus, as the allocation becomes less efficient ex post. As a result, steeper securities may yield lower revenues and security-bid auctions may yield lower revenues than cash auctions as the agents acquire less accurate information.

The above analysis shows that security design can affect the seller's revenue in two opposed ways. A natural question arises: what is the revenue-maximizing security deign with costly information acquisition? To address this question, we study *linear mechanisms* in which the payment by an agent linearly depends on his ex-post payoff. In other words, an agent pays either in cash or in stock. We characterize the revenue-maximizing linear mechanism. The optimal linear mechanism allocates the asset efficiently conditional on the private information ex post. An agent pays if and only if he wins. The winner pays in cash if their expected values are above a threshold and pays in stock if their expected values are below the threshold. The threshold decreases as the marginal cost of acquiring additional information increases.

Intuitively, stock payment reduces the information rent accrued to the agents, but it also reduces the incentive for the agents to acquire information. The optimal payment method needs to balance this trade-off. Roughly speaking, increasing the share of stock payments for low type and high type have a similar impact on the agents' incentives to acquire information, yet any information rent received by low type must also be received by high type. Therefore, the optimal mechanism prescribes a stock payment for low type and a cash payment for high type. The optimal threshold decreases to provide agents stronger incentives towards information acquisition as it becomes marginally more costly to acquire information.

As an application, our results provide a theoretical explanation for the choice of payment methods in mergers and acquisitions based on information acquisition. A key prediction of our model is that the use of cash bids positively associates with merge synergies. We empirically test this prediction using a sample of over seven thousand US merge deals from 1977 to 2020. We use both offer to target stock price premium and abnormal returns around announcement dates as proxies for the synergy value. Consistent with the model prediction, we find that deals paid entirely in cash have higher synergy values than the other merge deals.

The rest of this paper is organized as follows. Section 1.1 discusses related work. Section 2 presents the model. Section 3 studies auctions with ordered securities and compares security designs in terms of agents' incentives to acquire information and the seller's revenue. Section 4 characterizes the revenue-maximizing linear mechanism. Section 5 contains the empirical analysis.

1.1 Other related literature

Standard or efficient auctions using cash bids. First, this paper is related to the literature that studies information acquisition in cash mechanisms. Earlier contributions focus on the commonly used auction formats using cash bids. Matthews (1984a) focuses on first-price auctions with pure common values. Stegeman (1996) finds that both first-price and second-price auctions lead to the same efficient incentive for information acquisition when agents have independent private values. In contrast, Persico (2000) finds that agents have stronger incentives to acquire information under the first-price auction than under the second-price auction when their values are affiliated.

More recently, Bergemann and Välimäki (2002) and Bergemann et al. (2009) study the incentives for agents to collect information in ex post efficient mechanisms. Li (2019) studies the ex ante efficient mechanisms taking information costs into consideration. Different from the above papers, this paper studies information acquisition in auctions using security bids as opposed to cash bids and focuses on the comparison between security designs rather than auction formats.

Revenue-maximizing cash mechanisms. This paper is also related to the literature that studies the revenue-maximizing cash mechanisms with costly information acquisition. The mostly closely related paper is Shi (2012) who considers a similar model but focuses on cash mechanisms. He finds that the optimal monopoly price is always below the standard monopoly price to encourage information acquisition. In contrast to Shi (2012), the allocation is ex post efficient in the optimal linear mechanism, and the seller encourages information acquisition through the choice of payment methods.

Crémer et al. (2009) characterize the optimal mechanism when agents face binary information decisions and the seller can control their access to information. They find that the seller can completely overcome the agents' incentive problems and extract full surplus. Levin and Smith (1994), Ye (2004) and Lu and Ye (2018) model the information cost as an entry cost so that agents' information decisions are observable. In this paper, as in Shi (2012), agents have a continuum of information choices, and their information choices are also their private information.

Contingent mechanisms. This paper is broadly related to the literature that studies contingent mechanisms (among them Hansen (1985) and DeMarzo et al. (2005)). Che and Kim (2010) add to the analysis of DeMarzo et al. (2005) a caveat – that a higher return requires a higher cost. They find that steeper securities are more vulnerable to adverse selection, and may yield lower expected revenue, than flatter ones. Sogo et al. (2016) extend DeMarzo et al. (2005) to a setting in which it is costly to participate in the security-bid auction and potential bidders know their private valuations when deciding whether to enter. They find that auctions with steeper securities also attract more entry, further enhancing the revenues from such auctions. Liu and Bernhardt (2019) study the revenue-maximizing equity auctions when bidder's valuations and opportunity costs are private information. In the above papers, the private information held by agents is assumed to be exogenous. By contrast, this paper studies the environments in which agents can covertly acquire information at some cost.

Payment method in takeover auctions. Finally, this paper is related to the literature on the payment method choice in corporate takeovers.

Eckbo et al. (1990) and Fishman (1989) explore the role of two-sided asymmetric information in the acquirer's choice of payment method. Eckbo et al. (1990) identify a separating equilibrium in which the value of the acquirer is revealed by their choice of payment method. The empirical results of Eckbo et al. (1990) are consistent with their theoretical implication: the average announcement-month bidder abnormal returns are on average highest in all-cash offers, lowest in all-stock offers, and with mixed cash-stock offers in between. In Fishman (1989), there is more than one potential bidder and a cash bid serves to preempt potential competition from rival bidders. Thus, in equilibrium bidders with positive information make cash bids, while bidders with less positive information make bids with payment in the debt security.

Gorbenko and Malenko (2018) study the link between financially constraints on the side of bidders and its decision on whether to bid in cash or in stock. They show that the use of cash as means of payment is positively associated with synergies and the acquirer's gains from the deal and negatively associated with financial constraints.

In Shleifer and Vishny (2003) and Rhodes-Kropf and Viswanathan (2004), the payment method choice is driven by stock market misvaluation or bidder opportunism. Intuitively, if the bidder stock is overvalued, the acquirer will be tempted to use stock as the payment method to capitalize on this overvaluation. The bidder opportunism hypothesis implies that it is less likely the bidder will succeed in paying the target with overpriced bidder stock when the target is better informed about the bidder. Eckbo et al. (2018) test this hypothesis by estimating the probability that the deal is paid in stock as a function of empirical proxies indicating how well informed the target is about the bidder. They find that the likelihood that the deal is paid in stock increases in the target information proxies, rejecting the bidder opportunism hypothesis.

2 Model

There are n agents, indexed by $i \in \{1, \cdots, n\}$, who compete for an asset. The value of the asset to agent i is θ_i, which is unknown to all agents or to the seller initially. Each agent has a quasi-liner utility. If agent i receives the asset with probability $q_i \in [0, 1]$ and pays $s_i \in \mathbb{R}$, then his ex post payoff is $q_i\theta_i - s_i$. The seller's reservation value is zero.

Initially, agents know only $\{\theta_i\}$ are independently drawn from a common cumulative distribution F with support $\Theta := [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_+$. The distribution F has a continuous and positive density function f. Agent i can *covertly* acquire a signal $x_i \in \mathbb{R}$ regarding θ_i by choosing a joint distribution of (θ_i, x_i) from a family of joint distributions $\{G(\theta_i, x_i; \alpha_i)\}$, indexed by their *accuracy* $\alpha_i \in \mathbb{A} := [\underline{\alpha}, \overline{\alpha}]$. For each $\alpha \in \mathbb{A}$, we also refer to $G(\cdot, \cdot; \alpha)$ as an *information structure*. For all $\alpha \in \mathbb{A}$, $G(\cdot, \cdot; \alpha)$ admits the same marginal distribution of θ as the prior, and has a continuous and positive density function g. Assume, without loss of generality, that the marginal distribution of x follows a uniform distribution on $[0, 1]$. We slightly abuse notation by using θ and x to denote both the random variables and their realizations. For all $\alpha \in \mathbb{A}$, θ and x are strictly affiliated:

Definition 1 (Milgrom and Weber (1982)) *For all $\alpha \in \mathbb{A}$, the two random variables θ and x are strictly* affiliated*: for all $\theta' > \theta$ and $x' > x$,*

$$g(\theta', x'; \alpha)g(\theta, x; \alpha) > g(\theta', x; \alpha)g(\theta, x'; \alpha). \tag{1}$$

By Lemma 4 in the appendix, (θ, x) satisfies the *strict monotone likelihood ratio property*, i.e., $g(x|\theta'; \alpha)/g(x|\theta; \alpha)$ is strictly increasing in x if $\theta' > \theta$. This means that the private signal x_i is "good news" about the asset value θ_i.

A signal with higher α is more accurate (in the sense defined shortly). Let $C(\alpha)$ denote the cost of acquiring a signal with accuracy α. As is standard in the literature, we assume that C is non-negative, non-decreasing, continuously differentiable and convex.

2.1 Timing

The game proceeds in the following way. The seller announces a mechanism. After observing the mechanism, the agents simultaneously make their information choices, $\{\alpha_i\}$, and observe their realized signals, $\{x_i\}$. Then, the agents simultaneously decide whether to participate in the mechanism. All participating agents report their private information. Finally, an outcome is realized.

The payoff structure, the timing of the game, the information technology and the prior distribution are common knowledge.

2.2 Contingent mechanisms

In this paper, we focus on the case that $\{\theta_i\}$ are contractible and on contingent mechanisms in which agent i's payment can be contingent on his true type θ_i, i.e., a *security*. A security can be described by a function $s(\theta)$. A security $s(\theta)$ is a cash payment if it is independent of θ. We make the following monotonicity assumption on the set of feasible securities which includes cash as a special case.

Definition 2 *The function $s(\theta)$ is a feasible* security *if both $s(\theta)$ and $\theta - s(\theta)$ are non-decreasing.*

Monotonicity ensures that the equilibrium outcome is efficient. It is also satisfied by almost all securities used in practice. For example, the feasible set of securities admit the following standard sets of securities:

- Equity: The seller receives some fraction $r \in [0, 1]$ of the future cash flow θ. Then, the seller gets $s(\theta) = r\theta$ and the buyer gets $\theta - s(\theta) = (1 - r)\theta$.
- Debt: The seller is promised a face value $d \geq \underline{\theta}$, secured by the asset. Then, the seller gets $s(\theta) = \min\{d, \theta\}$ and the buyer gets $\theta - s(\theta) = \max\{\theta - d, 0\}$.
- Convertible debt: The seller is promised a face value $d \geq \underline{\theta}$, secured by the asset, or a fraction $r \in [0, 1]$ of θ. Then, the seller gets $s(\theta) = \max\{\min\{d, \theta\}, r\theta\}$ and the buyer gets $\theta - s(\theta) = \min\{\max\{\theta - d, 0\}, (1 - r)\theta\}$.
- Levered equity: The seller receives a fraction $r \in [0, 1]$ of θ after the face value $d \geq \underline{\theta}$ is paid. Then, the seller gets $s(\theta) = r \max\{\theta - d, 0\}$ and the buyer gets $\theta - s(\theta) = (1 - r) \max\{\theta - d, 0\} + \min\{\theta, d\}$.
- Call option: The seller receives a call of the firm at the strike price k. Then, the seller gets $s(\theta) = \max\{\theta - k, 0\}$ and the buyer gets $\theta - s(\theta) = \min\{k, \theta\}$.

If we think of the asset as the "rights to a project" as in DeMarzo et al. (2005), and the winner must make an initial investment $X > \underline{\theta}$ in order to generate a future cash follow θ, then a security defined in DeMarzo et al. (2005) satisfies $S(\theta) = s(\theta) - X$. If both $s(\theta)$ and $\theta - s(\theta)$ are non-decreasing, then $S(\theta)$ and $\theta - S(\theta)$ are also non-decreasing. In addition, DeMarzo et al. (2005) assume that $0 \leq S(\theta) \leq \theta$ (or equivalently $X \leq s(\theta) \leq \theta + X$), which rules out any negative cash payments. DeMarzo et al. (2005) interpret $s(\theta) \leq \theta + X$ as a limited liability constraint for the buyer and $s(\theta) \geq X$ as a limited liability constraint for the seller. They show that this constraint can arise if the initial investment X is not verifiable.

In this paper, we focus on two classes of contingent mechanisms: (i) standard first- and second-price auctions with ordered securities and (ii) mechanisms in which the payment is a linear security.

3 Auctions with ordered securities

In this section, we focus on security-bid auctions. In a security-bid auction, the seller restricts the bids to a well-ordered set of securities, and uses a standard auction format, such as a first- or second-price auction, to allocate the asset and determine the payment. Without restrictions on the set of admissible securities, ranking different securities is hard and will depend upon the seller's belief. We impose the following requirements on the set of admissible bids as in DeMarzo et al. (2005):

Definition 3 *The function $s(\sigma,\theta)$ for $\sigma \in [\sigma_0,\sigma_1]$ defines an ordered set of securities if:*

1. *$s(\sigma,\cdot)$ is a feasible security.*
2. *For all $\alpha \in \mathbb{A}$ and $x \in [0,1]$, $\mathbb{E}[s_\sigma(\sigma,\theta)|x;\alpha] > 0$.*
3. *$\underline{\theta} - s(\sigma_0,\underline{\theta}) \geq 0$ and $\overline{\theta} - s(\sigma_1,\overline{\theta}) \leq 0$.*

We also use $\mathscr{S} := \{s(\sigma,\cdot)\}_{\sigma\in[\sigma_0,\sigma_1]}$ to denote an ordered set of securities. The second condition in Definition 3 says that for any information structure and any realized signal, the seller's expected revenue, $\mathbb{E}[s(\sigma,\theta)|x;\alpha]$, is strictly increasing in σ. Thus, a higher σ corresponds to a higher bid. The third condition ensures that the range of bids is sufficiently large so that every agents earn a non-negative payoff by bidding the lowest bid and no agent earns a positive payoff by bidding the highest bid. Examples of ordered sets of securities include sets of cash payments, (levered) equity and (convertible) debt indexed by equity share or face value, and call options indexed by strike price.

Given an ordered set of securities, it is nature to generalize the standard first- and second-price auctions using cash bids to our setting using security bids:

First-price auction: Each agent submits a security. The agent who submitted the highest security (highest σ) wins and pays according to his security. Ties are randomly broken.

Second-price auction: Each agent submits a security. The agent who submitted the highest security (highest σ) wins and pays the second highest security (second highest σ). Ties are randomly broken.

For tractability, we restrict our attention to symmetric equilibria in which all agents make the same information choice, i.e., $\alpha_i = \alpha$ for all i.

3.1 Equilibrium with exogenous information

Consider first the situation in which each agent's private information is fixed and symmetric: $\alpha_i = \alpha$ for all i. We make the following assumption on the feasible sets of securities to rule out a solution as in Crémer (1987).

Assumption 1 $\chi(\theta) \equiv \theta \notin \mathscr{S}$.

Theoretically, if $\chi \in \mathscr{S}$, then it is an equilibrium in the first- and second-price auctions that all agents bid χ irrespective of their realized signals and get 0. This trivializes the information acquisition problem. Assumption 1 is typically made in the security design literature. It is weaker than the assumptions made by DeMarzo et al. (2005), who assume that $X \leq s(\theta) \leq \theta + X$ for some $X > \underline{\theta}$. To see this, note that $s(\underline{\theta}) \geq X > \underline{\theta}$, which implies that $\chi \notin \mathscr{S}$.

By the standard argument we have the following characterization of the equilibrium in a second-price security-bid auction.

Lemma 1 *Suppose Assumption 1 holds. Given $\alpha_i = \alpha$ for all i, the unique equilibrium in weakly undominated strategies in the second-price security-bid auction is for agent i who receives signal $x_i = x$ to submit a security $\sigma(x;\alpha)$ such that $\mathbb{E}[\theta - s(\sigma(x;\alpha),\theta)|x;\alpha] = 0$. Furthermore, $\sigma(x;\alpha)$ is strictly increasing in x.*

Suppose that $\sigma(x;\alpha)$ is agent i's strategy in a symmetric equilibrium in the first-price security-bid auction, and $\sigma(x;\alpha)$ is differentiable and strictly increasing in x. Then,

$$x \in \arg\max_{x'} G(x'|\alpha)^{n-1}\mathbb{E}\left[\theta - s(\sigma(x';\alpha),\theta)|x;\alpha\right].$$

Hence, σ must satisfy

$$\sigma_x(x;\alpha) = \frac{(n-1)g(x|\alpha)\mathbb{E}[\theta - s(\sigma(x;\alpha),\theta)|x;\alpha]}{G(x|\alpha)\mathbb{E}[s_\sigma(\sigma(x;\alpha),\theta)|x;\alpha]}, \tag{2}$$

with the boundary condition that $\mathbb{E}[\theta - s(\sigma(0;\alpha),\theta)|0;\alpha] = 0$. Clearly, $\sigma_x(x;\alpha) > 0$. Suppose, in addition, that the ordered set of securities and the information structures satisfy the following assumption, the above first-order condition is also sufficient for optimality.

Assumption 2 *For all $\alpha \in \mathbb{A}$ and all (σ, x) such that $\mathbb{E}[\theta - s(\sigma,\theta)|x;\alpha] > 0$,*

$$\frac{\partial^2}{\partial x \partial \sigma} \log \mathbb{E}[\theta - s(\sigma,\theta)|x;\alpha] > 0.$$

This assumption is standard in the auction literature. For example, it is also used in Maskin and Riley (1984) and DeMarzo et al. (2005) to ensure the existence and uniqueness of the symmetric equilibrium in a first-price auction. Under Assumption 2, we have the following characterization of the symmetric equilibrium in the first-price security-bid auction:

Lemma 2 *Suppose Assumptions 1 and 2 hold. Given $\alpha_i = \alpha$ for all i, there exists a unique symmetric equilibrium in the first-price security-bid auction. It is the unique solution to the differentiale equation (2) with the boundary condition $\mathbb{E}[\theta - s(\sigma(0;\alpha),\theta)|0;\alpha] = 0$. Furthermore, $\sigma(x;\alpha)$ is strictly increasing and differentiable in x.*

3.2 Information acquisition

Consider now an agent's information acquisition problem prior to an auction when his opponents would choose accuracy α and play the symmetric equilibrium strategy as if all agents had accuracy α. The payoff of agent i from choosing accuracy η is

$$R(\eta;\alpha) := \int_0^1 \left[\max_\sigma \int_\Theta u(\theta_i,\sigma)\mathrm{d}G(\theta_i|x_i;\eta)\right]\mathrm{d}G(x_i|\eta),$$

where $u(\theta_i,\sigma)$ denote agent i's expected payoff when his true type is θ_i and he bids σ, which depends on the auction formats. In the second-price auction,

$$u(\theta_i,\sigma) = \int_0^{\sigma^{-1}(\sigma;\alpha)} [\theta_i - s(\sigma(z_i;\alpha),\theta_i)]\,\mathrm{d}G^{n-1}(z_i|\alpha),$$

where $\sigma(\cdot;\alpha)$ is given by Lemma 1 and $\sigma^{-1}(\cdot;\alpha)$ denotes its inverse function. In the first-price auction,

$$u(\theta_i,\sigma) = [\theta_i - s(\sigma,\theta_i)]\,G^{n-1}(\sigma^{-1}(\sigma;\alpha)|\alpha),$$

where $\sigma(\cdot;\alpha)$ is given by Lemma 2 and $\sigma^{-1}(\cdot;\alpha)$ denote its inverse function. Then, agent i's information acquisition problem is

$$\max_\eta R(\eta;\alpha) - C(\eta).$$

For later use, we define the marginal return from increasing accuracy when all agents have accuracy α as

$$MR(\alpha) := \frac{\partial}{\partial\eta}R(\eta;\alpha)\bigg|_{\eta=\alpha}.$$

3.2.1 Information order

Before proceeding, we first define the notion of informativeness used to rank the accuracy of different signals.

Definition 4 (Lehmann (1988)) $G(\cdot,\cdot;\alpha)$ *is more* accurate *than* $G(\cdot,\cdot;\eta)$ *if*

$$T_{\alpha,\eta}(x|\theta) := G^{-1}(G(x|\theta;\eta)|\theta;\alpha)$$

is non-decreasing in θ*, for every* x*.*

Accuracy, which weakens Blackwell's sufficiency condition (Blackwell et al. (1951)), was first proposed by Lehmann (1988). To better understand the notion of accuracy, note that if x is distributed according to $G(\cdot|\theta;\eta)$, then $T_{\alpha,\eta}(x|\theta)$ is distributed according to $G(\cdot|\theta;\alpha)$. That is, we can obtain a more accurate signal by subjecting the less accurate signal to the $T_{\alpha,\eta}(\cdot|\theta)$ transformation. Since $T_{\alpha,\eta}(x|\theta)$ is non-decreasing in θ, the new signal obtained via the transformation is higher (or lower) if θ is higher (or lower). In other words, the new signal is more correlated with v than the original one. Persico (2000) shows that all decision makers with single-crossing preferences prefer one signal $G(\cdot,\cdot;\alpha)$ over another $G(\cdot,\cdot;\eta)$ for all priors if and only if $G(\cdot,\cdot;\alpha)$ is more accurate than $G(\cdot,\cdot;\eta)$. Throughout Section 3, we assume that the information structures are ordered by accuracy:

Assumption 3 $\alpha' > \alpha$ *implies that* $G(\cdot,\cdot;\alpha')$ *is more accurate than* $G(\cdot,\cdot;\alpha)$*.*

3.3 Ranking security designs

We show that an agent's marginal return to information depend upon the steepness of the securities. To do so, we follow DeMarzo et al. (2005) and define the notion of steepness by how securities cross each other. A function H is said to be steeper than a function J if H crosses J from below only once. As in DeMarzo et al. (2005), we say one security s^1 is steeper than another security s^2 if the payment to the seller $s(\theta)$ is steeper under the first security, or equivalently, the payoff to the agent $\theta - s(\theta)$ is flatter under the first security. More formally,

Definition 5 (Karamardian and Schaible (1990)) *A function* $H(z)$ *is* quasi-monotone *if* $z' > z$ *and* $H(z) > 0$ *imply* $H(z') \geq 0$*.*

Definition 6 *An ordered set of securities* $\mathscr{S}^1$ *is steeper than an ordered set of securities* $\mathscr{S}^2$ *if for all* $s^1 \in \mathscr{S}^1$ *and* $s^1 \in \mathscr{S}^2$*,* $s^1 - s^2$ *is quasi-monotone.*

This definition of steepness is slightly different from that in DeMarzo et al. (2005). In DeMarzo et al. (2005), the information structure is exogenous. For fixed α, they say $\mathscr{S}^1$ is steeper than $\mathscr{S}^2$ if for all $s^1 \in \mathscr{S}^1$ and $s^2 \in \mathscr{S}^2$, $\mathbb{E}[s^1(\theta)|x, \alpha] = \mathbb{E}[s^2(\theta)|x; \alpha]$ implies that $\partial\mathbb{E}[s^1(\theta)|x; \alpha]/\partial x > \partial\mathbb{E}[s^2(\theta)|x; \alpha]/\partial x$. The following lemma shows that our definition of steepness is an adaption of DeMarzo et al. (2005) to the setting with endogenous information:

Lemma 3 *Suppose $s^1 - s^2$ is quasi-monotone and $x' > x$. Then, for all $\alpha \in \mathbb{A}$, $\mathbb{E}[s^1(\theta)|x; \alpha] = \mathbb{E}[s^2(\theta)|x; \alpha]$ implies that $\mathbb{E}[s^1(\theta)|x'; \alpha] \geq \mathbb{E}[s^2(\theta)|x'; \alpha]$.*

Why is steepness related to the marginal return to information? Consider first a second-price security-bid auction. Agent i's expected utility when his true type is θ_i and he observes x_i is

$$u(\theta_i, \sigma(x_i; \alpha)) = \int_0^{x_i} [\theta_i - s(\sigma(z_i; \alpha), \theta_i)] \, \mathrm{d}G^{n-1}(z_i|\alpha),$$

where σ is such that $\mathbb{E}[\theta - s(\sigma(x; \alpha), \theta)|x; \alpha] = 0$ by Lemma 1. If the agent knows θ_i, he would choose a security σ^* such that $s(\sigma^*, \theta_i) = \theta_i$ to maximize his utility. Recall that a more accurate signal can be obtained by subjecting the less accurate signal to the transformation $y_i = T_{\alpha,\eta}(x_i|\theta_i)$. Since $T_{\alpha,\eta}(x_i|\theta_i)$ is non-decreasing in θ_i, y_i would be larger (or smaller) than x_i if θ_i is high (or low). Thus, for each θ_i, the security $\sigma(y_i; \alpha)$ is closer than $\sigma(x_i; \alpha)$ to σ^*. That is, an increase in signal accuracy increases agent i's payoff. How much the payoff increases, however, depends on how steeply $u(\theta_i, \sigma(x_i; \alpha))$ changes as x_i moves towards θ_i, which is measured by $\partial u(\theta_i, \sigma(x_i; \alpha))/\partial x_i$. Suppose the ordered set of securities $\mathscr{S}^1$ is steeper than $\mathscr{S}^2$. Let $u^1(\theta_i, \sigma^1(x_i; \alpha))$ and $u^2(\theta_i, \sigma^2(x_i; \alpha))$ denote agent i's expected utilities from the second-price auctions using $\mathscr{S}^1$ and $\mathscr{S}^2$, respectively. Then,

$$\begin{aligned} &\frac{\partial}{\partial x_i} \left[u^2(\theta_i, \sigma^2(x_i; \alpha)) - u^1(\theta_i, \sigma^1(x_i; \alpha))\right] \\ &= \left[s^1(\sigma^1(x_i; \alpha), \theta_i) - s^2(\sigma^2(x_i; \alpha), \theta_i)\right] (n-1)G^{n-2}(x_i|\alpha)g(x_i|\alpha), \end{aligned}$$

which is quasi-monotone in θ_i since $\mathscr{S}^1$ is steeper than $\mathscr{S}^2$. Thus, $u^2(\theta_i, \sigma^2(x_i; \alpha))$ is more sensitive than $u^1(\theta_i, \sigma^1(x_i; \alpha))$ to changes in signal x_i. This leads to the following main result:

Proposition 1 *Suppose Assumptions 1 and 3 hold, and the ordered set of securities $\mathscr{S}^1$ is steeper than $\mathscr{S}^2$. For second-price security-bid auctions, an agent's marginal return to information in a symmetric equilibrium is lower using $\mathscr{S}^1$ than using $\mathscr{S}^2$.*

We now turn our attention to first-price security-bid auctions. In this case, an additional assumption is required to compare different sets of securities:

Definition 7 *An ordered set of securities $\mathscr{S}$ is convex if it is equal to its convex hull.*

Proposition 2 *Suppose Assumptions 1-3 hold, the ordered set of securities $\mathscr{S}^1$ and $\mathscr{S}^2$ are convex, and $\mathscr{S}^1$ is steeper than $\mathscr{S}^2$. For first-price security-bid auctions, an agent's marginal return to information in a symmetric equilibrium is lower using $\mathscr{S}^1$ than using $\mathscr{S}^2$.*

DeMarzo et al. (2005) show that when information available to agents is exogenous, security-bid auctions using steeper set of securities yield higher revenues. However, as we argued earlier, security-bid auctions using steeper set of securities provide less incentives for agents to acquire information. The accuracy of information affects the seller's revenue in two opposite ways. On the one hand, less accurate information reduces the efficiency of the auction and thus the seller's revenue. On the other hand, less accurate information reduces the information rent accrued to the agents which increases the seller's revenue. Thus, when the first effect dominates, the revenue ranking might be reversed when information is endogenous. This is illustrated by the following example.

Example 1 *Assume for simplicity that $\underline{\theta} = 0$. Consider an ordered set of securities $s(r, \theta)$, indexed by $r \in [0, 1]$, where $s(r, \theta) = r\theta + X$ for some $X > 0$. Consider the second-price security-bid auction in which each agent submits a share r; the agent submitting the highest r wins; and the winner pays according to the second highest r. Given $\alpha_i = \alpha$ for all i, in the unique weakly undomitated equilibrium, agent i submits $r(x_i) = 1 - X/\mathbb{E}[\theta|x_i; \alpha]$, which* *is strictly increasing in* x_i*. Let* $v_i := \mathbb{E}[\theta|x_i;\alpha_i]$ *denote agent* i*'s expected value and* $H(v_i;\alpha_i)$ *denote its distribution. Let* $v^i := \max_{j\neq i} v_j$ *denote the highest expected value among all agents except for agent* i*. Then, agent* i*'s information acquisition problem is*

$$\max_{\alpha_i} \int_\Theta \int_\Theta \max\left\{\frac{X}{v^i}v_i - X, 0\right\} \mathrm{d}H(v_i;\alpha_i)\mathrm{d}H^{n-1}(v^i;\alpha) - C(\alpha_i).$$

The seller's revenue from this auction is

$$R^S(\alpha) := \mathbb{E}\left[V_{(n)}\left(1 - \frac{X}{V_{(n-1)}}\right) + X \middle| \alpha_i = \alpha \ \forall i\right],$$

where $V_{(n)}$ *denotes the highest* v_i *and* $V_{(n-1)}$ *denotes the second highest* v_i*.*

Compare this with a second-price cash-bid auction, in which agent i *submits* v_i *in equilibrium. Then, agent* i*'s information acquisition problem is*

$$\max_{\alpha_i} \int_\Theta \int_\Theta \max\left\{v_i - v^i, 0\right\} \mathrm{d}H(v_i;\alpha_i)\mathrm{d}H^{n-1}(v^i;\alpha) - C(\alpha_i).$$

The seller's revenue from this auction is

$$\pi^C(\alpha) := \mathbb{E}\left[V_{(n-1)}\middle| \alpha_i = \alpha \ \forall i\right].$$

Given α*, for almost all realization of* $\boldsymbol{v}$*,*

$$V_{(n)}\left(1 - \frac{X}{V_{(n-1)}}\right) + X - V_{(n-1)} = (V_{(n)} - V_{(n-1)})\left(1 - \frac{X}{V_{(n-1)}}\right) > 0.$$

Hence, $\pi^S(\alpha) > \pi^C(\alpha)$*. However, since*

$$\max\left\{v_i - v^i, 0\right\} - \max\left\{\frac{X}{v^i}v_i - X, 0\right\} = \max\left\{\left(1 - \frac{X}{v^i}\right)(v_i - v^i), 0\right\}, \tag{3}$$

is non-decreasing in v_i *and strictly increasing in* v_i *when* $v_i > v^i$*, it follows from the argu-*

ments in Bergemann and Välimäki (2002) that $MR^C(\alpha) > MR^S(\alpha)$*. When* $X \to 0$, $\pi^S(\alpha)$ *converges to the full surplus, while the value of information to agent* i *goes to zero. Let* α^S *(or* α^C*) denote the information choice in a symmetric equilibrium in the security-bid auction (or the cash-bid auction). Then,* $\lim_{X\to 0} \pi^S(\alpha^S(X)) = \mathbb{E}[\theta_i]$*. If* $n > 3$ *and* $H(\cdot;\alpha)$ *is unimodal and symmetric,* $\pi^C(\alpha) = \mathbb{E}[V_{(n-1)}|\alpha] \geq H^{-1}((n-1)/n;\alpha) > \mathbb{E}[\theta_i]$ *for all* α *and* X*. Thus,* $\pi^S(\alpha^S) < \pi^C(\alpha^C)$ *for* $X > 0$ *sufficiently small (i.e., the revenue ranking between security-bid and cash-bid auctions are reversed).*

The same result holds for first-price auctions with a slightly more complex analysis.

4 Linear mechanisms

Security design affects the seller's revenue in two ways. First, as shown in DeMarzo et al. (2005), given the amount of information acquired by agents, security design affects the seller's revenue by affecting the competitiveness of the auction. Second, Section 3 shows that security design affects the seller's revenue by affecting the incentives for agents to acquire information. A natural question arises: what is the revenue-maximizing security deign with endogenous information? For tractability, we restrict our attention to mechanisms in which the payment is a linear security, i.e., cash, equity or a mixture of the two.

The private information of each agent i is two-dimensional, including the accuracy of his information $\alpha_i \in \mathbb{A}$ and the realized signal $x_i \in [0,1]$. A *linear contract* for agent i consists of a royalty rule $r_i : [0,1]^n \times \mathbb{A}^n \to [0,1]$ and a transfer rule $t_i : [0,1]^n \times \mathbb{A}^n \to \mathbb{R}$. A (direct) *linear mechanism* is a triple $(\boldsymbol{q}, \boldsymbol{r}, \boldsymbol{t})$, consisting of an allocation rule $q_i : [0,1]^n \times \mathbb{A}^n \to [0,1]$ and a linear contract (r_i, t_i) for each agent i. The ex post payoff of agent i with true type θ_i from such a linear mechanism is

$$q_i(\boldsymbol{x}, \boldsymbol{\alpha})[1 - r_i(\boldsymbol{x}, \boldsymbol{\alpha})]\theta_i - t_i(\boldsymbol{x}, \boldsymbol{\alpha}),$$

where $(\boldsymbol{x}, \boldsymbol{\alpha})$ is the profile of reported private information. The fact that $r_i(\boldsymbol{x}, \boldsymbol{\alpha}) \in [0,1]$

ensures that the payment is a feasible security satisfying the monotonicity assumption. A special case of the linear mechanism is the standard *cash* mechanism in which $r_i(\cdot) \equiv 0$.

Remember that the private information of an agent is two-dimensional, which suggests that the design problem is multi-dimensional and could potentially be very complicated. However, when the payments are linear, agent i's expected valuation of the asset, $v_i(x_i, \alpha_i) := \mathbb{E}[\theta_i | x_i; \alpha_i]$, completely captures the dependence of his payoff on the two-dimensional private information:

$$\mathbb{E}_{\theta_i}\left[q_i(\boldsymbol{x}, \boldsymbol{\alpha}) r_i(\boldsymbol{x}, \boldsymbol{\alpha}) \theta_i - t_i(\boldsymbol{x}, \boldsymbol{\alpha}) | x_i; \alpha_i\right] = q_i(\boldsymbol{x}, \boldsymbol{\alpha}) r_i(\boldsymbol{x}, \boldsymbol{\alpha}) v_i(x_i, \alpha_i) - t_i(\boldsymbol{x}, \boldsymbol{\alpha}).$$

Furthermore, the seller cannot screen the two pieces of information separately. Hence, without loss of generality, we can focus on linear mechanisms in which agents report their expected values directly.

For ease of notation, we use v_i to denote $v_i(x_i, \alpha_i)$ and $\boldsymbol{v} := (v_1, \ldots, v_n)$ to denote a vector of expected values. Then, a linear mechanism can be written as $\{q_i(\boldsymbol{v}), r_i(\boldsymbol{v}), t_i(\boldsymbol{v})\}_{i=1}^n$, where $q_i(\boldsymbol{v})$ is the probability of winning the asset for agent i when the vector of reports is $\boldsymbol{v}$, and $(r_i(\boldsymbol{v}), t_i(\boldsymbol{v}))$ specifies agent i's corresponding linear payment. For later use, let $H(v|\alpha)$ denote the cumulative distribution function of v_i, and $h(v|\alpha)$ denote the corresponding density function.

Given a mechanism $(\boldsymbol{q}, \boldsymbol{r}, \boldsymbol{t})$, let $\boldsymbol{\alpha}^* := (\alpha_1^*, \ldots, \alpha_n^*)$ denote the equilibrium vector information choices. Then, agent i's interim probability of winning the asset is

$$Q_i(v_i) := \mathbb{E}_{v_{-i}}[q_i(v_i, v_{-i}) | \alpha_{-i}^*], \tag{4}$$

where α_{-i}^* are his opponents' information choices. If agent i's true expected value is v_i and he reports $\hat{v}_i$, his interim payoff is

$$U_i(v_i, \hat{v}_i) := \mathbb{E}_{v_{-i}}\left[q_i(\hat{v}_i, v_{-i})\left[1 - r_i(\hat{v}_i, v_{-i})\right] v_i - t_i(\hat{v}_i, v_{-i}) | \, \alpha_{-i}^*\right].$$

Let $T_i(v_i) := \mathbb{E}_{v_{-i}}\left[t_i(v_i, v_{-i})\middle|\, \alpha^*_{-i}\right]$ denote agent i's interim cash payment. If $Q_i(v_i) \neq 0$, let $R_i(v_i) := \mathbb{E}_{v_{-i}}\left[q_i(v_i, v_{-i})r_i(v_i, v_{-i})\middle|\, \alpha^*_{-i}\right]/Q_i(v_i)$; otherwise let $R_i(v_i) := 0$. By construction, $R_i \in [0, 1]$. Note that if $Q_i(v_i) = 0$, then $q_i(v_i, v_{-i}) = 0$ for almost all v_{-i} and therefore $\mathbb{E}_{v_{-i}}\left[q_i(v_i, v_{-i})r_i(v_i, v_{-i})\middle|\, \alpha^*_{-i}\right] = 0$. Hence, $Q(v_i)R(v_i) = \mathbb{E}_{v_{-i}}\left[q_i(v_i, v_{-i})r_i(v_i, v_{-i})\middle|\, \alpha^*_{-i}\right]$ for all v_i. Hence, agent i's interim payoff is

$$U_i(v_i, \hat{v}_i) = Q_i(\hat{v}_i)\left[1 - R_i(\hat{v}_i)\right] v_i - T_i(\hat{v}_i).$$

Note that Q_i, T_i, R_i and U_i also depends on α^*_{-i}. Here, we suppress the dependence for ease of notation.

We focus on linear mechanisms satisfying the following properties. First, a mechanism is (interim) individually rational (IR) if

$$U_i(v_i) := U(v_i, v_i) \geq 0 \text{ for all } v_i. \tag{IR}$$

(IR) ensures that all agents are willing to participate in the mechanism. Second, a mechanism is Bayesian incentive compatible (IC) if

$$U_i(v_i) \geq U(v_i, \hat{v}_i) \text{ for all } v_i, \hat{v}_i. \tag{IC}$$

(IC) ensures that truth-telling is a Bayes-Nash equilibrium. Lastly, with costly information acquisition, a mechanism also needs to satisfy the information acquisition constraint (IA): no agent has an incentive to deviate from his equilibrium choice α^*_i:

$$\alpha^*_i \in \underset{\alpha_i}{\operatorname{argmax}}\, \mathbb{E}\left[U_i(v_i)\middle|\, \alpha_i, \alpha_j = \alpha^*_j\ \forall j \neq i\right] - C(\alpha_i). \tag{IA}$$

The seller's problem, denoted by ($\mathcal{P}$), is to choose a linear mechanism $(\boldsymbol{q}, \boldsymbol{r}, \boldsymbol{t})$ and a vector of recommendations of information choices $\boldsymbol{\alpha}^*$ to maximize her expected revenue:

$$\max_{\boldsymbol{\alpha}^*,(\boldsymbol{q},\boldsymbol{r},\boldsymbol{t})} \mathbb{E}_{\boldsymbol{v}}\left[\sum_i \left(v_i q_i(\boldsymbol{v}) r_i(\boldsymbol{v}) + t_i(\boldsymbol{v})\right)\middle|\, \alpha_i = \alpha_i^* \ \forall i\right], \tag{$\mathcal{P}$}$$

subject to (IC), (IR), (IA) and the feasibility constraint (F):

$$0 \le q_i(\boldsymbol{v}) \le 1, \ \sum_i q_i(\boldsymbol{v}) \le 1, \ \forall \boldsymbol{v} \in [\underline{\theta}, \overline{\theta}]^n. \tag{F}$$

For tractability, we restrict our attention to mechanisms that treat all agents symmetrically as well as symmetric equilibria in which all agents make the same information choice (i.e., $\alpha_i^* = \alpha^*$ for all i).[1] Note that when a mechanism is symmetric, the corresponding Q_i, R_i, T_i and U_i are independent of i. From here on, we drop the subscript i from Q, R, T, U, v and α whenever the meaning is clear.

The seller's problem is challenging because of the presence of then nonstandard constraint (IA), which prevents us from solving the problem directly. To overcome this difficulty, we focused on reduced-form auctions. Formally, $\boldsymbol{q}$ implements Q, and Q is the reduced form of $\boldsymbol{q}$ if $\boldsymbol{q}$ satisfies (4) and (F). Q is implementable if $\boldsymbol{q}$ exists implementing Q.

By the standard argument, (IC) holds if and only if

$$Q(v)\,[1 - R(v)] \text{ is non-decreasing,} \tag{MON}$$

U is absolutely continuous and satisfies the following envelope condition:

$$U(v) = U(v(0,\alpha)) + \int_{v(0,\alpha)}^{v} Q(\nu)\,[1 - R(\nu)]\,\mathrm{d}\nu.$$

Thus, (IR) holds if and only if $U(v(0,\alpha)) \ge 0$.

We now turn to the information acquisition problem. Since (IA) is difficult to work with directly, we follow the first-order approach and relax the seller's problem by replacing

[1] The formal definition of symmetric mechanisms can be found in appendix.

the (IA) constraint with a one-sided first-order necessary condition. As will become clear later, if we ignore (IA), then the optimal mechanism leaves agents no incentive to acquire information. Hence, we hypothesize that to ensure that (IA), it suffices to ensue that no agent has incentive to acquire less accurate signals than recommended. Suppose agent i chooses α_i and all the other agents choose α^*, then by the envelope condition, his expected payoff is

$$\mathbb{E}_{v_i}\left[U_i(v_i)|\alpha_i, \alpha_j = \alpha^*\right] - C(\alpha_i) = U_i(v(0,\alpha_i)) + \int_{v(0,\alpha_i)}^{v(1,\alpha_i)} [1 - H(v_i|\alpha_i)] Q(v_i)\left[1 - R(v_i)\right] \mathrm{d}v_i - C(\alpha_i).$$

If agent i does not gain by deviating to $\alpha_i < \alpha^*$, then α^* satisfies the following one-sided first-order necessary condition:

$$\int_{v(0,\alpha^*)}^{v(\overline{x},\alpha^*)} -H_{\alpha_i}(v_i|\alpha^*) Q(v_i)\left[1 - R(v_i)\right] \mathrm{d}v_i \geq C'(\alpha^*), . \tag{IA$'$}$$

Subsequently, we first relax the seller's problem by replacing (IA) with (IA$'$) and then show that (IA$'$) holds with equality when α^* is feasible. We follow the first-order approach and relax the seller's problem by replacing the (IA) constraint with (IA$'$). The first-order approach is valid if the second-order condition of the agents' optimization problem is satisfied. Later on, we provide sufficient conditions for the first-order approach to be valid.

One important prior result we use is the necessary and sufficient condition that characterizes the set of interim allocation rules implementable by symmetric mechanisms.[2] By Theorem 1 in Matthews (1984b), any implementable Q satisfies the following necessary condition:

$$Y(v) := \int_{v}^{\overline{\theta}} \left[H(z|\alpha^*)^{n-1} - Q(z)\right] h(z|\alpha^*)\mathrm{d}z \geq 0, \ \forall v \in [\underline{\theta}, \overline{\theta}]. \tag{F$'$}$$

The above condition says that the probability of assigning the object to an agent whose posterior mean is above w must not exceed the probability that an agent whose posterior

[2]See Maskin and Riley (1984), Matthews (1984b), Border (1991) and Che et al. (2013).

mean is above v exists. If Q is nondecreasing, Theorem 1 in Matthews (1984b) proves that this condition is also sufficient. Unlike the mechanism design problem with only cash payments, (IC) no longer ensures that Q is nondecreasing. In what follows, we relax the seller's problem even more by replacing (F) with (F′), and then show that the optimal Q is non-increasing. Note that in equilibrium, the support of posterior means is $V := [v(0,\alpha^*), v(1,\alpha^*)] \subset [\underline{\theta}, \overline{\theta}]$. Therefore, (F′) imposes no restriction on Q outside V.

Finally, using the envelope condition, the seller's expected revenue can be written as

$$\begin{aligned}
&\int_{v(0,\alpha)}^{v(1,\alpha)} [Q(v)R(v)v + T(v)]\,\mathrm{d}H(v|\alpha^*) \\
=&\int_{v(0,\alpha)}^{v(1,\alpha)} \left[v - \frac{1-H(v|\alpha^*)}{h(v|\alpha^*)}(1-R(v))\right] Q(v)\mathrm{d}H(v|\alpha^*) - U(v(0,\alpha)).
\end{aligned}$$

Clearly, it is optimal to set $U(v(0,\alpha)) = 0$.

Then, the seller's relaxed problem ($\mathcal{P}'$) can be written as the following reduced-form problem:

$$\max_{\alpha^*,Q,R} \int_{v(0,\alpha^*)}^{v(1,\alpha^*)} \left[v - \frac{1-H(v|\alpha^*)}{h(v|\alpha^*)}(1-R(v))\right] Q(v)\mathrm{d}H(v|\alpha^*), \tag{$\mathcal{P}'$}$$

subject to (MON), (IA′) and (F′).

Remark 1 *If $\alpha_i = \alpha^*$ is exogenous given, as Crémer (1987) points out, full surplus extraction can be achieved by letting $r_i(\cdot) \equiv 1$, $t_i(\cdot) \equiv 0$, and the allocation rule be ex post efficient: for all $\boldsymbol{v}$ and all i,*

$$q_i(\boldsymbol{v}) = \begin{cases} 1 & \text{if } v_i > \max_{j\neq i} v_j, \\ 0 & \text{otherwise.} \end{cases}$$

This mechanism is no longer optimal if agents need to acquire information at some cost since it leaves agents no incentive to acquire information.

4.1 Optimal linear mechanism for fixed information choice

As is customary, we focus on the seller's relaxed problem that implements a given information choice α^*, denoted by ($\mathcal{P}$-α^*):

$$\max_{Q,R} \int_{v(0,\alpha^*)}^{v(1,\alpha^*)} \left[v - \frac{1 - H(v|\alpha^*)}{h(v|\alpha^*)} (1 - R(v)) \right] Q(v) \mathrm{d}H(v|\alpha^*), \qquad (\mathcal{P}\text{-}\alpha^*)$$

subject to (MON), (IA′) and (F′). The solution to ($\mathcal{P}$-α^*) will provide us rich insights into the properties of optimal mechanisms. To ease our exposition, from here on, we impose the following regularity condition on the distribution of conditional expectations:

Assumption 4 (Monotone hazard rate)

$$\frac{1 - H(v|\alpha^*)}{h(v|\alpha^*)} \textit{ is non-increasing in } v \textit{ for all } \alpha^*.$$

We consider a different information order from that in Section 3.

Assumption 5 (Supermodularity) *The information structures are supermoular ordered, i.e., $v(\cdot, \cdot)$ is supermodular: for all $x, x' \in (0,1)$, $x > x'$ and $\alpha > \alpha'$,*

$$v(x, \alpha) - v(x', \alpha) \geq v(x, \alpha') - v(x', \alpha').$$

The notion of "supermodular precision" was introduced by Ganuza and Penalva (2010), and it orders different information structures based on their impacts on the distribution of conditional expectations. Roughly speaking, if an information structure is more supermodular precise than another, then it leads to a more disperse distribution of conditional expectations. By contrast, accuracy orders different information structures based on their value to decision makers with single-crossing preferences. Since agent i's payoff in a linear mechanism is completely determined by his conditional expectation $v_i(x_i, \alpha_i)$, it is natural to consider supermodular ordered information structures.

Ganuza and Penalva (2010) show that supermodular precision and accuracy are consistent, but neither is stronger than the other. Any two information structures ordered in terms of accuracy will be equally ordered in terms of supermodular precision if they can be ordered based on the latter notion. However, the order can be lost. Similarly, the order based on supermoduler precision can be lost, but not reversed, in terms of accuracy.

Shi (2012) and Li (2019) adopt the supermodular assumption for some of their results. By a similar argument to that of Lemma 1 in Li (2019), Assumption 5 holds if and only if

$$-\frac{H_\alpha(v|\alpha^*)}{h(v|\alpha^*)} \text{ is non-decreasing in } v \text{ for all } \alpha^*.$$

Let $\hat{v} := \inf\{v : -H_\alpha(v|\alpha^*) > 0\} \in (v(0,\alpha^*), v(1,\alpha^*))$. To ensure that α^* is feasible, assume

$$\int_{\hat{v}}^{v(1,\alpha^*)} -H_\alpha(v|\alpha^*)H(v|\alpha^*)^{n-1}\mathrm{d}v \leq C'(\alpha^*),$$

where the left-hand side is an agent's maximum marginal benefit from choosing α^* under any symmetric mechanism. To exclude trivialties, assume $C'(\alpha^*) > 0$.

We now provide an informal argument to derive the optimal solution. If we ignore (MON), we can use the following Lagrangian relaxation to get an intuition for the optimal solution:

$$\mathscr{L} := \int_{v(0,\alpha^*)}^{v(1,\alpha^*)} \left[v + \left(-\frac{1 - H(v|\alpha^*)}{h(v|\alpha^*)} - \lambda^* \frac{H_\alpha(v|\alpha^*)}{h(v|\alpha^*)} \right) (1 - R(v)) \right] Q(v) h(v|\alpha^*) \mathrm{d}v.$$

We can choose R and Q to maximize $\mathscr{L}$ pointwise: let $R(v) = 0$ if $-\frac{1-H(v|\alpha^*)}{h(v|\alpha^*)} - \lambda^* \frac{H_\alpha(v|\alpha^*)}{h(v|\alpha^*)} > 0$ and $R(v) = 1$ otherwise; let $Q(v) = H(v|\alpha^*)^{n-1}$ if $\max\left\{v, v - \frac{1-H(v|\alpha^*)}{h(v|\alpha^*)} - \lambda^* \frac{H_\alpha(v|\alpha^*)}{h(v|\alpha^*)}\right\} > 0$ and $Q(v) = 0$ otherwise. Under Assumptions 4 and 5, the corresponding $Q\,(1 - R)$ is non-decreasing and (MON) holds. In addition, Q is non-decreasing, and therefore (F′) is also a sufficient condition for Q to be implementable. This leads to the following result:

Proposition 3 *Suppose Assumptions 4 and 5 hold. The optimal linear mechanism that implements α^* satisfies*

1. *The allocation rule is ex post efficient:*

$$q_i(\boldsymbol{v}) = \begin{cases} 1 & \textit{if } v_i > \max_{j \neq i} v_j, \\ 0 & \textit{otherwise.} \end{cases}$$

2. *Agent i pays if and only if he wins, and there exists v^* such that the payment satisfies*

$$\begin{cases} r_i(\boldsymbol{v}) = 0,\ t_i(\boldsymbol{v}) = v^* & \textit{if } v_i > v^*, \\ r_i(\boldsymbol{v}) = 1,\ t_i(\boldsymbol{v}) = 0 & \textit{if } v_i < v^*. \end{cases}$$

In addition, (IA′) *holds with equality.*

In contrast to the optimal cash mechanism, the optimal linear mechanism is ex post efficient. Furthermore, the winner pays in cash when their expected values are high and pays in stock when their expected values are low. Intuitively, a high royalty rate reduces the information rent accrued to the agents, but it also reduces the incentive for the agents to acquire information. The optimal payment method needs to balance this trade-off. Roughly speaking, increasing the royalty rate for low type and high type have a similar impact on the agents' incentives to acquire information, yet any information rent received by low type must also be received by high type. Therefore, the optimal payment method specifies a high royalty rate for low type and a low royalty rate for high type.

Corollary 1 *Suppose Assumptions 4 and 5 hold. The optimal threshold v^* decreases as the marginal cost from increasing information precision $C'(\alpha^*)$ increases.*

Corollary 1 implies that as it becomes marginally less costly to acquire more information, the winner is less likely to pay in cash.

5 Empirical implications and evidence

As an application, our results provide a theoretical explanation for the choice of payment method in mergers and acquisitions based on information acquisition. Proposition 3 predicts that the use of stock bids associates negatively with synergy measures, such as takeover premiums, abnormal returns of sellers, and combined abnormal returns of buyers and sellers. Takeover premiums reflect synergy values of the takeover from the buyers' perspectives, while abnormal target returns and combined abnormal returns reflect synergy values from the market perspective. Thus, the first set of tests focuses on how merge synergies affect payment methods.

Implication 1 *Stock payments are more likely than cash payments when premiums in takeovers or abnormal returns surrounding takeover announcements are low.*

Our model emphasizes the role of costly information acquisition in merges and acquisitions. Particularly, Corollary 1 predicts that stock payments are more likely when it is less difficult for bidders to acquire information about potential synergies. Since bidders are well informed about the fundamentals of themselves, we condition the tests on how difficult it is for bidders to acquire information about the sellers. Specifically,

Implication 2 *Stock payments are more likely than cash payments when it is easier for the bidder to acquire information about the seller and its fundamental value.*

We examine this information based implication by exploring the information asymmetry between the two parties to the transaction. Specifically, we condition the tests on measures of how easily bidders can acquire information about the sellers. Below, we describe the data and summary statistics

5.1 Data and summary statistics

The merge and acquisition data is from the Securities Data Company (SDC) Platinum. We include deals for U.S. public targets by U.S. public acquirers from 1977 to 2020. We require that deal value above $1 million and that acquirers are non-financial firms. We also require that both parties are domestic firms and only include merge deals. After these filterings, we have 5467 deals, including both successful and unsuccessful deals. We further exclude 377 deals with unknown payment methods and 196 deals with discretionary payment methods. The final sample includes 4894 deals.

We then match the data set with stock data from the Center for Research in Security Price (CRSP) to compute buyers' and sellers' 3-day abnormal returns around announcement dates. We use a 200-day window with 120 minimum valid returns to compute abnormal returns with three different benchmarks. We can match 4051 deals with sellers' abnormal returns and 3750 deals with both sellers' and buyers' abnormal returns. We also match the SDC merge and acquisition data with firm fundamentals from Compustat.

Table 1 lists the variables and their definitions. We winsorize variables at 1% and 99% of their distributions, and Table 2 presents the summary statistics. Cash and stock are the two most popular payment methods. On average, stock accounts for 45% of the payments, and cash accounts for another 42%. Moreover, 36% and 38% deals in our sample have all-cash and all-stock payments, respectively. Hereafter, we classify merge deals into all-cash, all-stock, and mixed payments.

We use three proxies for merge synergies: Takeover Premium, Target CAR, and Combined CAR. We calculate the Takeover Premium as the offer to target stock price premium four weeks before the announcement. The average Takeover Premium is 54.35%. The average 3-day cumulative abnormal return for targets (Target CAR) is about 21%, and the combined 3-day cumulative abnormal return (Combined CAR), calculated as the weighted average of buyers' and sellers' abnormal returns by the market capitalizations, has a mean of 2%. We consider three benchmarks in constructing the returns: market portfolio, CAPM, and the Fama-French three-factor model. All three generate similar average abnormal returns.

Our empirical proxies for information asymmetry are Local Deal, Target Urban, and Target Recent SEO. The first two variables capture the geographic proximity and location.

Local Deal is a dummy variable indicating that the distance between the acquirer and target is less than 30 miles. Target Urban is a dummy variable indicating that the target is within 15 miles of the center of one of the ten largest metropolitan areas. Our results are robust to other cutoff distances. We also use recent seasoned equity offering (SEO) by the target to capture information disclosure before the merge negotiation. Target Recent SEO is a dummy variable indicating that the target made an equity offering during the 18 months preceding the announcement.

We choose these variables on information asymmetry following Eckbo et al. (2018), who use them to proxy for the information quality. We argue that these variables can reflect information quality precisely because they represent information acquisition costs. We further focus on how costly acquirers can acquire information about targets, instead of the targets' information about acquirers as in Eckbo et al. (2018). Nevertheless, because Eckbo et al. (2018) find that targets' information about acquirers matters for payment methods, we also add Acquirer Urban and Acquirer Recent SEO, two proxies for targets' information about acquirers, in our analysis.

We do not include Recent Acquirer and Industry Complementarity, the other two proxies for information asymmetry used in Eckbo et al. (2018) because we find them less appealing for our analysis. First, we find that the association between payment methods and Recent Acquirer depends on whether the previous merge's target is public or private. Moreover, much fewer targets acquired another firm before. Second, the effect of Industry Complementarity becomes insignificant once we add both industry and time fixed effects. We provide a more detailed discussion in Appendix C.

We add capital structure variables of both buyers and sellers from Compustat, including Total Assets, Leverage, Market-to-book Ratio, R&D, and Tangibility. We also include the Herfindahl-Hirschman Index (HHI) representing industrial competition and Competition from Private Buyers that reflects external pressure to pay in cash, following Eckbo et al. (2018).

5.2 Synergy values and payment methods

Implication 1 states that payments are more likely in stock when synergy values are low. As a first look, Figure 1 plots the histograms of synergy values for deals with different payment methods. The upper panel uses Takeover Premium as the synergy measure, and the lower one plots the distributions of Combined CAR. Both panels consistently show that deals paid entirely in stock have lower synergy values than deals paid entirely in cash. The pattern remains similar when we contrast deals paid entirely in stock against the other deals.

We formally test the prediction with the following empirical specification:

$$\text{Payment Method}_i = \beta_0 + \beta_1 \,\text{Synergy Value}_i + \beta_2 \,\text{Controls}_i + \varepsilon_i, \tag{5}$$

where we consider both the fraction of stock in takeover payments and the choice of payment method (all-stock, mixed, or all-cash) as the left-hand-side variable. We control for both bidders' and sellers' capital structures with one-year lag, including total assets, leverage, market-to-book ratio, R&D expense, and asset tangibility, as well as proxies for competition and external pressure to pay in cash. We also include year fixed effects and bidder and seller industry fixed effects to control unobserved firm characteristics and aggregate shocks.

Table 3 presents the regression results. Column 1 regresses the fraction of stock against the Takeover Premium. A one within-group standard deviation increase (48.2) in the Takeover Premium decreases the fraction of stock payments by 3.5 percentage points. The effect is statistically significant, and about 0.1 within-group standard variation of the fraction of stock payments (0.40). Columns 2 through 4 use targets' 3-day cumulative abnormal return around announcement as the independent variable. We calculate abnormal returns based on three benchmarks: the market portfolio, CAPM, or the Fama-French three-factor model. It is consistent across three specifications that a one within-group standard deviation increase (23.1) in the abnormal returns leads to a 7.0 percentage points decrease in stock payments. Next, we consider Combined CAR in columns 5 through 7. The effect of Combined CAR on payment methods is statistically significant and economically more substantial. One within-group standard deviation increase (7.3) in the combined abnormal returns decreases the fraction of stock payments by 9.5 percentage points.

Given that one-third of the sample is all-stock payments and another one-third is all-cash payments, one concern arises that the linear regression model we used in Table 3 may not be adequate to account for the discrete outcome. We conduct logit regressions and multinomial probit regressions with the same set of variables and fixed effects to alleviate this concern.

Columns 1 and 2 of Table 4 consider the payments entirely in stock versus other payments in logit regressions. We find significant effects of synergy values on payment methods that are consistent with our implication. We further confirm this result by exploiting the multinomial probit estimation of choice between all-cash, all-stock, and mixed payments in Columns 3 through 6. The Target CAR and Combined CAR coefficients remain significant and similar to those in the logit regressions. Furthermore, although we only show regressions with Target CAR and Combined CAR, both based on the Fama-French three-factor benchmark, as proxies for merge synergies, the results are robust to the other proxies.

Overall, we find strong evidence to support Implication 1. The effect of merge synergies on payment robust is robust to various regression models and different proxies of merge synergies. Meanwhile, the control variables' coefficients reveal that stock payments associate positively with targets' total assets but negatively with acquirers' total assets. Intuitively, it is difficult for smaller acquirers relative to targets to raise enough cash. Thus, they may have to rely on stock payments in the transaction. Moreover, all-stock payments are more likely for acquirers and targets with lower leverages and higher market-to-book ratios. It is likely because levered acquirers tend to use stocks to reduce excess leverage (Harford et al., 2009), and acquirers with higher market-to-book ratios tend to use over-valued stocks (Rhodes-Kropf et al., 2005). We also conjecture that both leverages and the market-to-book ratios of targets are related to the cost of acquiring information. Finally, we find positive effects of acquirer R&D expenses and tangible assets on stock payment, but no significant effect of target R&D expenses and tangible assets, HHI, nor Competition from Private Buyers.

5.3 Costly information acquisition and payment methods

This subsection investigates Implication 2 empirically by relating proxies for how easily acquirers can learn about targets to payment methods. We use the following empirical specification:

$$\text{Payment Method}_i = \beta_0 + \beta_1 \,\text{Information Cost}_i + \beta_2 \,\text{Controls}_i + \varepsilon_i, \tag{6}$$

where we consider both the fraction of stock in takeover payments and the choice of payment method (all-stock, mixed, or all-cash) as the left-hand-side variable. We use three proxies, Local Deal, Target Urban, and Target Recent SEO, for the information cost; the larger these proxies are, the lower the cost. We also add HHI and Competition from Private Buyers to control competition and external pressure to pay in cash. We also include bidders' and sellers' capital structure variables, year fixed effects, and industry fixed effects to control firm characteristics and aggregate shocks in some regressions.

Column 1 of Table 5 presents coefficients from a linear regression for a fraction of stock in payment. All three proxies associate positively with stock payments. A one standard deviation increase of the variables leads to 5.2, 1.7, and 2.2 percentage points increases in the fraction of stock in payments, respectively. Reverse causality is not likely to be a concern here. Although targets may indeed strategically issue stocks and increase investment before merger deals, whether the deal is local (Local Deal) and whether the targets' headquarters are near city centers (Target Urban) are not subject to any potential merger deals or the payment methods.

In column 2, we address the concern that firm characteristics may drive our results. We control for both bidders' and sellers' capital structures with one-year lag, including total assets, leverage, market-to-book ratio, R&D expense, and asset tangibility, as well as the year fixed effects and both bidder and seller industry fixed effects. Although Target Urban becomes insignificant, we find a similar result for the Local Deal and Target Recent SEO. The effects of capital structure variables are consistent with what we find in Section 5.2.

We then conduct multinomial probit regressions of choice between all-cash, all-stock, and mixed payments to account for the discrete outcomes. Columns 3 through 6 of Table 5 report the regression results with and without capital structure controls and year and industry fixed effects. The results are consistent with those from the linear regression models. In particular, when we add capital structure variables and various fixed effects in columns 5 and 6, we find the proxies for acquirers' information cost are significant for comparing all-stock payments against all-cash payments. Also, as we expected, the comparisons between mixed payments and all-cash payments are less salient.

Given that Eckbo et al. (2018) show that the targets' information matters for payment methods, would the targets' information quality of acquirers drive our results? To alleviate this concern, we include two proxies for how easily targets can learn about acquirers. Consistent with Eckbo et al. (2018), we find a positive and significant coefficient on Acquirer Recent SEO with our sample. Eckbo et al. (2018) show a non-significant effect of Acquirer Urban and argue that other proxies may absorb its effect. We also find it broadly not significant across specifications in Table 5 and significantly negative in some regressions.[3] Notably, the coefficients of interest, Local Deal, Target Urban, and Target Recent SEO, remain significant after adding these two proxies. Thus, it reveals that how easily acquirers can obtain information about targets is crucial for payment methods. Moreover, our results of some proxies, such as the Urban status, point to a more critical role of acquirers' information about targets than targets' information about acquirers.

Overall, as important as [XXXXXXXXX]

[3]Local Deal represents information acquisition cost in both directions, while Urban status and Recent SEO only represent information acquisition cost in one direction.

6 Conclusion

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Figures and Tables

Figure 1: Frequencies of Synergy Values

This figure presents frequencies of two synergy measures for different payment methods. The upper panel uses offer to target stock price premium four weeks prior to announcement as the synergy measure, and the lower panel uses combined 3-day cumulative abnormal return around announcement with the Fama-French three factors model as the benchmark. Combined abnormal returns are computed as the weighted average of buyer and seller returns, using the market capitalization as the weight. Grey bars represent frequencies of synergy values of all-cash payments. Unfilled bars represent frequencies of synergy values of all-stock payments. I restrict the premium to be less than 200% and the CAR to be within 1st-99th percentiles to remove outliers.

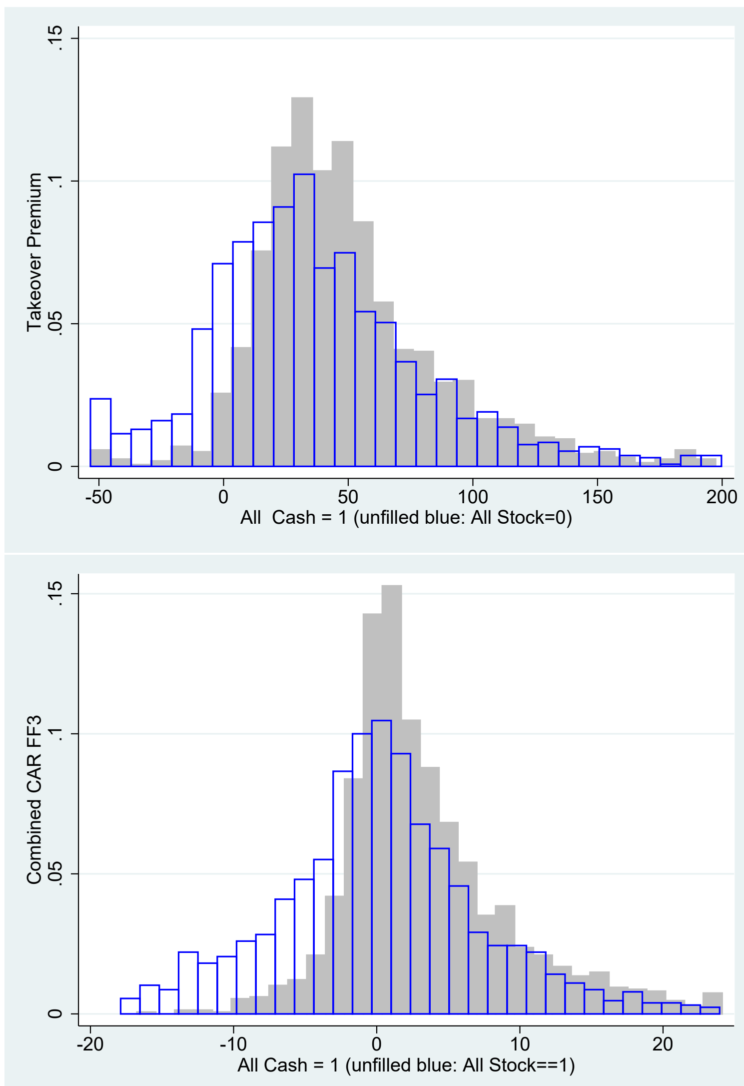

Table 1: Variable Definitions

Payment Methods	
All Stock	All-stock payment (consideration structure = shares), Securities Data Company (SDC).
All Cash	All-cash payment (consideration structure = casho), SDC.
Mixed	Consideration structure = hybrid or other, SDC.
Fraction of Stock	Fraction of stock in the payment, SDC.
Fraction of Cash	Fraction of cash in the payment, SDC.
Synergy	
Takeover Premium	Offer Price to Target Stock Price Premium 4 Week Prior to Announcement.
Target CAR(-1,+1)	Three-day target cumulative abnormal returns calculated using market portfolio, CAPM, or the Fama-French three factors as benchmarks, Center for Research in Security Prices (CRSP).
Combined CAR(-1,+1)	Three-day combined cumulative abnormal returns (average of buyer and seller returns, weighted by the market capitalizations) calculated using market portfolio, CAPM, or the Fama-French three factors as benchmarks, CRSP.
Information	
Local Deal	Dummy = 1 if the bidder and target are located within 30 miles. Firm location data are from the zip codes in SDC.
Urban Status	Dummy = 1 if a firm is located within 15 miles of one of the ten largest metropolitan areas (New York City, Los Angeles, California, Chicago, Illinois, Washington, DC, San Francisco, California, Philadelphia, Pennsylvania, Boston, Massachusetts, Detroit, Michigan, Dallas, Texas, and Houston, Texas).
Recent SEO	Dummy = 1 if a firm issued stock within 18 months prior to the bid, SDC.
Firm capital structure	
Total Assets	Natural log of total assets, Compustat.
Leverage	Total debt/total assets, Compustat.
M/B	Market-to-book equity ratio, Compustat.
R&D	Research and development expense/total assets, Compustat.
Asset Tangibility	Property, plant, and equipment/total assets, Compustat.
Competition, and external pressure to pay in cash	
HHI	Herfindahl–Hirschman Index of the bidder's FF49 industry and year, Compustat.
Competition from Private Buyers	Fraction of all merger bids in the target's Fama and French 49 (FF49) industry and year in which the bidder is private, SDC.

Table 2: Summary Statistics

This table presents summary statistics for the deal-level data used in the analysis. The sample consists of 4894 merger bids for U.S. public targets by U.S. public acquirers from 1977 to 2020. We require that the deal values are above $1 million and that acquirers are non-financial firms. All variables are defined in Table 1.

	count	mean	sd	p10	p25	p50	p75	p90
All Stock	4894	0.36	0.48	0.00	0.00	0.00	1.00	1.00
All Cash	4894	0.38	0.49	0.00	0.00	0.00	1.00	1.00
Mixed Payment	4894	0.25	0.43	0.00	0.00	0.00	1.00	1.00
Fraction of Stock	4894	0.45	0.46	0.00	0.00	0.32	1.00	1.00
Fraction of Cash	4894	0.42	0.46	0.00	0.00	0.07	1.00	1.00
Takeover Premium	3808	46.73	50.30	0.80	18.43	37.15	62.75	101.88
Target CAR	4051	21.73	24.41	-2.43	5.46	17.26	32.77	52.11
Target CAR CAPM	4051	21.60	24.50	-2.74	5.24	17.08	32.67	52.23
Target CAR FF3	4051	21.60	24.50	-2.75	5.22	17.15	32.73	52.08
Combined CAR	3750	2.06	7.60	-6.25	-1.73	1.34	5.53	11.64
Combined CAR CAPM	3750	1.91	7.59	-6.35	-1.93	1.25	5.36	11.30
Combined CAR FF3	3750	1.92	7.58	-6.23	-1.84	1.26	5.25	11.28
Local Deal	4894	0.16	0.37	0.00	0.00	0.00	0.00	1.00
Target Urban	4894	0.19	0.39	0.00	0.00	0.00	0.00	1.00
Target Recent SEO	4894	0.18	0.38	0.00	0.00	0.00	0.00	1.00
Acquirer Urban	4894	0.22	0.41	0.00	0.00	0.00	0.00	1.00
Acquirer Recent SEO	4894	0.27	0.44	0.00	0.00	0.00	1.00	1.00
Target Total Assets	3810	5.21	1.88	2.91	3.87	5.00	6.39	7.80
Target Leverage	3795	0.49	0.25	0.17	0.29	0.49	0.66	0.80
Target M/B	3729	2.78	3.88	0.66	1.12	1.84	3.17	5.87
Target R&D	4894	0.05	0.11	0.00	0.00	0.00	0.06	0.17
Target Tangibility	3801	0.27	0.24	0.04	0.08	0.19	0.41	0.66
Acquirer Total Assets	4368	6.85	2.26	3.85	5.31	6.95	8.38	9.82
Acquirer Leverage	4359	0.51	0.22	0.21	0.37	0.52	0.65	0.78
Acquirer M/B	4271	3.84	4.99	0.92	1.47	2.49	4.25	7.80
Acquirer R&D	4894	0.04	0.07	0.00	0.00	0.00	0.05	0.12
Acquirer Tangibility	4359	0.28	0.23	0.05	0.10	0.21	0.42	0.66
HHI	4894	0.07	0.06	0.02	0.03	0.05	0.09	0.14
Competition from Private Buyers	4894	0.19	0.14	0.00	0.09	0.17	0.27	0.39

Table 3: Synergy Values and Payment Methods

This table presents coefficients from regressions relating the fraction of stock in payment to takeover synergies. The specification in column 1 uses offer to target stock price premium four weeks before the announcement as the synergy measure. The specification in Columns 2-4 uses Targets' 3-day cumulative abnormal return around the announcement. The specification in Columns 5-7 uses the combined 3-day cumulative abnormal return around announcement as the synergy value. We calculate abnormal returns with three benchmarks: the market portfolio, CAPM, and the Fama-French three-factor model. Controls include bidder and seller capital structure variables, Competition from Private Buyers, and HHI. All variables are defined in Table 1. Industry dummies indicate the 2-digit SIC industry. Robust standard errors are in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	Fracton of stock in payment						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Takeover Premium	-0.001***						
	(0.000)						
Target CAR		-0.003***					
		(0.000)					
Target CAR CAPM			-0.003***				
			(0.000)				
Target CAR FF3				-0.003***			
				(0.000)			
Combined CAR					-0.012***		
					(0.001)		
Combined CAR CAPM						-0.013***	
						(0.001)	
Combined CAR FF3							-0.013***
							(0.001)
Competition from Private Buyers	-0.012	-0.053	-0.053	-0.054	-0.019	-0.019	-0.020
	(0.058)	(0.051)	(0.051)	(0.051)	(0.052)	(0.052)	(0.052)
HHI	0.173	0.258*	0.257*	0.258*	0.211	0.217	0.224
	(0.173)	(0.151)	(0.151)	(0.151)	(0.154)	(0.154)	(0.154)
Target Total Assets	0.051***	0.036***	0.037***	0.037***	0.052***	0.052***	0.052***
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
Target Leverage	-0.087**	-0.126***	-0.126***	-0.126***	-0.144***	-0.143***	-0.143***
	(0.037)	(0.034)	(0.034)	(0.034)	(0.036)	(0.036)	(0.036)
Target M/B	0.011***	0.009***	0.009***	0.009***	0.012***	0.012***	0.012***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Target R&D	0.070	0.107	0.109	0.110	0.080	0.084	0.082
	(0.092)	(0.087)	(0.087)	(0.087)	(0.086)	(0.086)	(0.086)
Target Tangibility	-0.053	-0.036	-0.037	-0.037	-0.011	-0.010	-0.011
	(0.060)	(0.053)	(0.053)	(0.053)	(0.054)	(0.054)	(0.054)
Acquirer Total Assets	-0.064***	-0.055***	-0.055***	-0.055***	-0.074***	-0.074***	-0.074***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Acquirer Leverage	-0.086*	-0.092**	-0.091**	-0.089**	-0.036	-0.035	-0.031
	(0.045)	(0.042)	(0.042)	(0.042)	(0.043)	(0.043)	(0.043)
Acquirer M/B	0.006***	0.005***	0.005***	0.005***	0.004**	0.004**	0.004**
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Acquirer R&D	0.599***	0.517***	0.518***	0.518***	0.519***	0.514***	0.520***
	(0.144)	(0.142)	(0.142)	(0.143)	(0.139)	(0.139)	(0.139)
Acquirer Tangibility	0.174***	0.203***	0.204***	0.205***	0.180***	0.179***	0.184***
	(0.062)	(0.054)	(0.054)	(0.054)	(0.056)	(0.056)	(0.055)
Observations	2813	3162	3162	3162	3049	3049	3049
Adjusted R^2	0.249	0.295	0.295	0.295	0.310	0.312	0.312
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Acquirer Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Target Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 4: Robustness: Synergy Values and Payment Methods

This table presents coefficients from regressions relating payment methods to takeover synergies. Columns 1-2 reports the coefficient estimates from logit regressions for the all-stock dummy. Columns 3-6 reports the multinomial probit regressions for the choice of payment method, where the outcomes are all-stock, mixed, and all-cash (baseline) deals. The controls include bidder and seller capital structures, Competition from Private Buyers, and HHI. All variables are defined in Table 1. Industry dummies indicate the 2-digit SIC industry. Robust standard errors are in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	Logit regression		Multinomial probit regression			
	All-stock		All-stock	Mixed	All-stock	Mixed
	(1)	(2)	(3)	(4)	(5)	(6)
Target CAR FF3	-0.016***		-0.017***	-0.012***		
	(0.002)		(0.002)	(0.002)		
Combined CAR FF3		-0.063***			-0.072***	-0.052***
		(0.007)			(0.006)	(0.006)
Competition from Private Buyers	-0.443	-0.192	-0.324	-0.253	-0.048	-0.120
	(0.370)	(0.380)	(0.339)	(0.330)	(0.346)	(0.334)
HHI	0.268	0.144	0.799	1.854**	0.659	1.560*
	(0.952)	(0.996)	(0.878)	(0.926)	(0.906)	(0.930)
Target Total Assets	0.078**	0.153***	0.230***	0.383***	0.325***	0.455***
	(0.037)	(0.039)	(0.035)	(0.037)	(0.036)	(0.039)
Target Leverage	-1.181***	-1.274***	-0.686***	0.307	-0.765***	0.280
	(0.242)	(0.251)	(0.202)	(0.198)	(0.217)	(0.212)
Target M/B	0.044***	0.062***	0.048***	0.041***	0.064***	0.048***
	(0.014)	(0.014)	(0.012)	(0.013)	(0.013)	(0.013)
Target R&D	0.397	0.257	0.391	0.132	0.184	0.050
	(0.526)	(0.535)	(0.485)	(0.522)	(0.489)	(0.535)
Target Tangibility	-0.210	-0.110	-0.307	-0.091	-0.188	0.081
	(0.344)	(0.359)	(0.318)	(0.325)	(0.328)	(0.339)
Acquirer Total Assets	-0.224***	-0.315***	-0.318***	-0.355***	-0.437***	-0.452***
	(0.032)	(0.034)	(0.029)	(0.032)	(0.031)	(0.034)
Acquirer Leverage	-0.737***	-0.594**	-0.445*	0.122	-0.168	0.369
	(0.261)	(0.283)	(0.235)	(0.238)	(0.255)	(0.255)
Acquirer M/B	0.042***	0.036***	0.024**	-0.016	0.017	-0.023*
	(0.011)	(0.012)	(0.010)	(0.011)	(0.011)	(0.012)
Acquirer R&D	1.870**	1.981**	3.295***	3.213***	3.955***	3.829***
	(0.865)	(0.899)	(0.891)	(0.940)	(0.862)	(0.907)
Acquirer Tangibility	0.613*	0.507	0.925***	0.648**	0.919***	0.718**
	(0.353)	(0.367)	(0.334)	(0.328)	(0.345)	(0.348)
Observations	3157	3029	2815		2711	
Pseudo R^2 / Log likelihood	0.197	0.205	-2279.139		-2163.026	
Year FE	Yes	Yes	Yes		Yes	
Acquirer Industry FE	Yes	Yes	Yes		Yes	
Target Industry FE	Yes	Yes	Yes		Yes	

Table 5: Information Acquisition and Payment Methods

This table presents coefficients from regressions relating payment methods to information acquisition. Columns 1-2 reports the coefficient estimates from linear regressions for the fraction of stock in takeover payments. Columns 3-6 reports the multinomial probit regressions for the choice of payment method, where the outcomes are all-stock, mixed, and all-cash (baseline) bids. The controls include bidder and seller capital structures, Competition from Private Buyers, and HHI. All variables are defined in Table 1. Industry dummies indicate the 2-digit SIC industry. Robust standard errors are in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	Linear regression		Multinomial probit regression			
	Fraction of stock		All-stock	Mixed	All-stock	Mixed
	(1)	(2)	(3)	(4)	(5)	(6)
Local Deal	0.141***	0.079***	0.513***	0.306***	0.345***	0.122
	(0.018)	(0.020)	(0.075)	(0.079)	(0.113)	(0.120)
Target Urban	0.043**	0.016	0.168**	0.237***	0.226**	0.134
	(0.017)	(0.019)	(0.072)	(0.074)	(0.111)	(0.112)
Target Recent SEO	0.059***	0.078***	0.200***	0.188**	0.392***	0.222**
	(0.017)	(0.017)	(0.072)	(0.075)	(0.105)	(0.109)
Acquirer Urban	-0.077***	-0.014	-0.235***	0.097	-0.157	-0.042
	(0.016)	(0.018)	(0.068)	(0.069)	(0.108)	(0.107)
Acquirer Recent SEO	0.110***	0.061***	0.467***	0.416***	0.340***	0.347***
	(0.015)	(0.016)	(0.062)	(0.065)	(0.092)	(0.095)
Competition from Private Buyers	-0.215***	-0.054	-0.851***	-0.321	-0.368	-0.234
	(0.044)	(0.051)	(0.188)	(0.202)	(0.329)	(0.320)
HHI	-0.157	0.230	0.119	-1.082**	0.481	1.662*
	(0.112)	(0.153)	(0.442)	(0.491)	(0.885)	(0.923)
Target Total Assets		0.042***			0.256***	0.422***
		(0.005)			(0.034)	(0.035)
Target Leverage		-0.112***			-0.584***	0.327*
		(0.033)			(0.196)	(0.188)
Target M/B		0.010***			0.054***	0.046***
		(0.002)			(0.012)	(0.012)
Target R&D		0.040			0.219	0.120
		(0.085)			(0.460)	(0.502)
Target Tangibility		-0.025			-0.297	-0.070
		(0.052)			(0.306)	(0.318)
Acquirer Total Assets		-0.062***			-0.339***	-0.365***
		(0.005)			(0.028)	(0.031)
Acquirer Leverage		-0.071*			-0.372*	0.082
		(0.040)			(0.225)	(0.228)
Acquirer M/B		0.004**			0.017*	-0.018*
		(0.002)			(0.009)	(0.010)
Acquirer R&D		0.514***			3.063***	3.031***
		(0.138)			(0.847)	(0.894)
Acquirer Tangibility		0.172***			0.653**	0.456
		(0.053)			(0.319)	(0.318)
Observations	4894	3321	4894		2963	
Adjusted R^2 / Log likelihood	0.020	0.295	-5202.988		-2433.014	
Year FE	No	Yes	No		Yes	
Acquirer Industry FE	No	Yes	No		Yes	
Target Industry FE	No	Yes	No		Yes	

A Omitted proofs in Section 3

Lemma 4 (**Milgrom and Weber (1982)**) *The following three conditions are equivalent:*

1. *The random variables θ and x are strictly affiliated.*
2. *(strict monotone likelihood ratio property) $\frac{g(x|\theta';\alpha)}{g(x|\theta;\alpha)}$ is strictly increasing in x if $\theta' > \theta$.*
3. *$\frac{g(\theta|x';\alpha)}{g(\theta|x;\alpha)}$ is strictly increasing in θ if $x' > x$.*

Lemma 5 *For every $\alpha \in \mathbb{A}$, the following two statements are true.*

1. *$\mathbb{E}[s(\theta)|x,\alpha]$ is strictly increasing in x if x and θ are strictly affiliated, and $s(\theta)$ is non-decreasing and non-constant.*
2. *$\mathbb{E}[\theta - s(\theta)|x,\alpha]$ is strictly increasing in x if x and θ are strictly affiliated, and $\theta - s(\theta)$ is non-decreasing and non-constant.*

Proof. For all $x' > x$,

$$[s(\theta)|x',\alpha] - \mathbb{E}[s(\theta)|x,\alpha] = \int_{\underline{\theta}}^{\overline{\theta}} s(\theta)\,[g(\theta|x';\alpha) - g(\theta|x;\alpha)]\,\mathrm{d}\theta > 0, \tag{7}$$

since $s(\theta)$ is non-decreasing and non-constant, $g(\theta|x';\alpha)/g(\theta|x;\alpha)$ is strictly increasing, and $\int_{\underline{\theta}}^{\overline{\theta}}(g(\theta|x';\alpha) - g(\theta|x;\alpha))\mathrm{d}\theta = 0$. Hence, $\mathbb{E}[s(\theta)|x,\alpha]$ is strictly increasing in x.

Similarly, $\mathbb{E}[\theta - s(\theta)|x,\alpha]$ is strictly increasing in x if x and θ are strictly affiliated, and $\theta - s(\theta)$ is non-decreasing and non-constant. ∎

Proof of Lemma 1. Fix α. For ease of notation, we use $\sigma(x)$ to denote $\sigma(x;\alpha)$ in this proof. By assumption there exists a unique $\sigma(x)$ such that

$$\mathbb{E}[\theta - s(\sigma(x),\theta)|x;\alpha] = 0. \tag{8}$$

Clearly, submitting $\sigma(x)$ when $x_i = x$ is the unique undominated strategy for agent i. Next, we argue that $\sigma(x)$ is strictly increasing in x. Let $x' > x$. Suppose to the contrary that $\sigma(x') \leq \sigma(x)$. Then, $\mathbb{E}[\theta - s(\sigma(x'), \theta)|x;\alpha] \geq \mathbb{E}[\theta - s(\sigma(x), \theta)|x;\alpha]$. Since $\chi \notin \mathscr{S}$, $\theta - s(\sigma(x'), \theta)$ and $\theta - s(\sigma(x), \theta)$ are non-constant. By Lemma 5,

$$\mathbb{E}[\theta - s(\sigma(x'), \theta)|x';\alpha] > \mathbb{E}[\theta - s(\sigma(x'), \theta)|x;\alpha] \geq \mathbb{E}[\theta - s(\sigma(x), \theta)|x;\alpha] = 0,$$

a contradiction to (8). Hence, $\sigma(x') > \sigma(x)$. ■

Proof of Lemma 2. Fix α. For ease of notation, we use $\sigma(x)$ to denote $\sigma(x;\alpha)$ in this proof. Suppose σ^* is a symmetric equilibrium, and it is strictly increasing and differentiable. Clearly, $\mathbb{E}[\theta - s(\sigma^*(0), \theta)|0;\alpha] = 0$. Let $U(x', x)$ denote the expected payoff to an agent who observes x and submits $\sigma^*(x')$:

$$U(x', x) \equiv G(x'|\alpha)^{n-1}\mathbb{E}[\theta - s(\sigma^*(x'), \theta)|x;\alpha].$$

Then,

$$\begin{aligned}
&\frac{\partial U(x', x)}{\partial x'} \\
=&(n-1)G(x'|\alpha)^{n-2}g(x'|\alpha)\mathbb{E}[\theta - s(\sigma^*(x'), \theta)|x;\alpha] - G(x'|\alpha)^{n-1}\mathbb{E}[s_\sigma(\sigma^*(x'), \theta)|x;\alpha]\sigma^{*\prime}(x') \\
=&G(x'|\alpha)^{n-1}\mathbb{E}[s_\sigma(\sigma^*(x'), \theta)|x;\alpha]\left\{\frac{(n-1)g(x'|\alpha)}{G(x'|\alpha)}\frac{\mathbb{E}[\theta - s(\sigma^*(x'), \theta)|x;\alpha]}{\mathbb{E}[s_\sigma(\sigma^*(x'), \theta)|x;\alpha]} - \sigma^{*\prime}(x')\right\}.
\end{aligned}$$

By Assumption 2, $\frac{\mathbb{E}[\theta - s(\sigma^*(x'), \theta)|x;\alpha]}{\mathbb{E}[s_\sigma(\sigma^*(x'), \theta)|x;\alpha]}$ is strictly increasing in x. If σ^* is the solution to the differential equation (2), then $\frac{\partial U(x', x)}{\partial x'}|_{x'=x} = 0$. Furthermore,

$$\frac{\partial U(x', x)}{\partial x'} \begin{cases} < 0 & \text{if } x' > x, \\ > 0 & \text{if } x' < x. \end{cases}$$

Hence, $x' = x$ is a global maximizer of $U(x', x)$. Finally, it is obviously not a profitable deviation for agents to bid $\sigma \notin [\sigma^*(0), \sigma^*(1)]$. This establishes the existence.

Suppose σ^* is a symmetric equilibrium. To prove uniqueness, we first show that there exists no σ such that $\mathbb{P}(\sigma^*(x) = \sigma) > 0$. Suppose not, and let $x' > x$ be two types such that $\sigma^*(x) = \sigma^*(x') = \sigma$. Then, either $\mathbb{E}[\theta - s(\sigma, \theta)|x'; \alpha] > 0$ or $\mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha] < 0$ since otherwise $\mathbb{E}[\theta - s(\sigma, \theta)|x'; \alpha] = \mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha] = 0$, which is impossible by Lemma 5 and the fact that $\chi \notin \mathscr{S}$. If $\mathbb{E}[\theta - s(\sigma, \theta)|x'; \alpha] > 0$, it is a profitable deviation for an agent with signal x' to bid $\sigma + \varepsilon$ for $\varepsilon > 0$ sufficiently small. If $\mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha] < 0$, it is a profitable deviation for an agent with signal x to bid σ_0. Hence, there exists no σ such that $\mathbb{P}(\sigma^*(x) = \sigma) > 0$.

Second, σ^* is strictly increasing. To see this, consider $x' > x$. Let $\sigma = \sigma^*(x)$, $\sigma' = \sigma^*(x')$, $p = \mathbb{P}(\sigma^*(\tilde{x}) \leq \sigma)$ and $p' = \mathbb{P}(\sigma^*(\tilde{x}) \leq \sigma')$. Suppose $p' < p$, which implies $\sigma' < \sigma$. By the optimality of σ^*, we have

$$p'\mathbb{E}[\theta - s(\sigma', \theta)|x'; \alpha] \geq p\mathbb{E}[\theta - s(\sigma, \theta)|x'; \alpha], \tag{9}$$

and

$$p\mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha] \geq p'\mathbb{E}[\theta - s(\sigma', \theta)|x; \alpha]. \tag{10}$$

If $p' = 0$, then $\mathbb{E}[\theta - s(\sigma, \theta)|x'; \alpha] = \mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha] = 0$ by (9) and (10), which is impossible by Lemma 5 and the fact that $\chi \notin \mathscr{S}$. Hence, $p' > 0$. Clearly, $\mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha] \geq 0$. If $\mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha] = 0$, then $\mathbb{E}[\theta - s(\sigma', \theta)|x; \alpha] \leq 0$ by inequality (10) and the fact that $p' > 0$, a contradiction to $\sigma' < \sigma$. Hence, $\mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha] > 0$. Dividing (9) by (10) yields

$$\frac{\mathbb{E}[\theta - s(\sigma', \theta)|x'; \alpha]}{\mathbb{E}[\theta - s(\sigma', \theta)|x; \alpha]} \geq \frac{\mathbb{E}[\theta - s(\sigma, \theta)|x'; \alpha]}{\mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha]}. \tag{11}$$

However, by Assumption 2,

$$\log \mathbb{E}[\theta - s(\sigma', \theta)|x'; \alpha] - \log \mathbb{E}[\theta - s(\sigma', \theta)|x; \alpha] < \log \mathbb{E}[\theta - s(\sigma, \theta)|x'; \alpha] - \log \mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha],$$

or equivalently,

$$\frac{\mathbb{E}[\theta - s(\sigma', \theta)|x'; \alpha]}{\mathbb{E}[\theta - s(\sigma', \theta)|x; \alpha]} < \frac{\mathbb{E}[\theta - s(\sigma, \theta)|x'; \alpha]}{\mathbb{E}[\theta - s(\sigma, \theta)|x; \alpha]},$$

which contradicts to (11). Hence $p' \geq p$, i.e., σ^* is non-decreasing. Combining this and the fact that there exists no σ such that $\mathbb{P}(\sigma^*(x) = \sigma) > 0$, we can conclude that σ^* is strictly increasing.

Lastly, we show that σ^* is differentiable. Clearly, σ^* is continuous. By the optimality of σ^*, we have

$$G(x|\alpha)^{n-1}\mathbb{E}[\theta - s(\sigma^*(x), \theta)|x; \alpha] \geq G(x'|\alpha)^{n-1}\mathbb{E}[\theta - s(\sigma^*(x'), \theta)|x; \alpha],$$

and

$$G(x'|\alpha)^{n-1}\mathbb{E}[\theta - s(\sigma^*(x'), \theta)|x'; \alpha] \geq G(x|\alpha)^{n-1}\mathbb{E}[\theta - s(\sigma^*(x), \theta)|x'; \alpha].$$

By the mean-value theorem, we have

$$\left[G(x|\alpha)^{n-1} - G(x'|\alpha)^{n-1}\right] \mathbb{E}[\theta - s(\sigma^*(x), \theta)|x; \alpha] \geq G(x'|\alpha)^{n-1}\mathbb{E}[s_\sigma(\sigma^{**}, \theta)|x; \alpha](\sigma^*(x) - \sigma^*(x')),$$

and

$$\left[G(x'|\alpha)^{n-1} - G(x|\alpha)^{n-1}\right] \mathbb{E}[\theta - s(\sigma^*(x'), \theta)|x'; \alpha] \geq G(x|\alpha)^{n-1}\mathbb{E}[s_\sigma(\sigma^\dagger, \theta)|x'; \alpha](\sigma^*(x') - \sigma^*(x)),$$

where σ^{**} and $\sigma^\dagger$ are between $\sigma^*(x)$ and $\sigma^*(x')$. Combining the above two inequalities

$$\begin{aligned} \frac{\left[G(x'|\alpha)^{n-1} - G(x|\alpha)^{n-1}\right] \mathbb{E}[\theta - s(\sigma^*(x'), \theta)|x'; \alpha]}{G(x|\alpha)^{n-1}\mathbb{E}[s_\sigma(\sigma^\dagger, \theta)|x'; \alpha](x' - x)} &\geq \frac{\sigma^*(x') - \sigma^*(x)}{x' - x} \\ &\geq \frac{\left[G(x'|\alpha)^{n-1} - G(x|\alpha)^{n-1}\right] \mathbb{E}[\theta - s(\sigma^*(x), \theta)|x; \alpha]}{G(x'|\alpha)^{n-1}\mathbb{E}[s_\sigma(\sigma^{**}, \theta)|x; \alpha](x' - x)}. \end{aligned}$$

Since σ^* is continuous, the left- and right-most terms of the above inequality converges to

$$\frac{(n-1)G(x|\alpha)^{n-2}g(x|\alpha)\mathbb{E}[\theta - s(\sigma^*(x),\theta)|x;\alpha]}{G(x|\alpha)^{n-1}\mathbb{E}[s_\sigma(\sigma^*(x),\theta)|x;\alpha]}$$

as $x' \to x$. Hence, σ^* is differentiable and satisfies (2) everywhere. ■

Proof of Lemma 3.

$$\mathbb{E}[s^1(\theta) - s^2(\theta)|x';\alpha] = \int_{\underline{\theta}}^{\overline{\theta}} (s^1(\theta) - s^2(\theta)) \left(\frac{g(\theta|x';\alpha)}{g(\theta|x;\alpha)} - 1 \right) g(\theta|x;\alpha)\mathrm{d}\theta. \tag{12}$$

Since $s^1 - s^2$ is quasi-monotone, $\mathbb{E}[s^1(\theta)|x;\alpha] = \mathbb{E}[s^2(\theta)|x;\alpha]$, and $\frac{g(\theta|x';\alpha)}{g(\theta|x;\alpha)}$ is strictly increasing, by Lemma 1 in Persico (2000), $\mathbb{E}[s^1(\theta) - s^2(\theta)|x';\alpha] \geq 0$. ■

Definition 8 *A function $u(\theta, x)$ has the* single crossing property *in $(\sigma; x)$ if for any pair $x' > x$, $u(\theta, x') - u(\theta, x)$ is quasi-monotone in θ.*

Definition 9 *Given two differentiable functions $u^1(\theta, x)$ and $u^2(\theta, x)$, we say that u^1 is* more risk-sensitive *than u^2 (and we write $u^1 \succeq u^2$) if $\partial \left[u^1(\theta, x) - u^2(\theta, x)\right]/\partial x$ is quasi-monotone in θ.*

This definition of risk-sensitivity is slightly different from that in Persico (2000), who says that $u^1 \succeq u^2$ if $u^1 - u^2$ has the single-crossing property in $(x; \theta)$. It is easy to see that if u^1 and u^2 are differentiable, then $u^1 - u^2$ has the single-crossing property in $(x; \theta)$ implies that $\partial[u^1(\theta, x) - u^2(\theta, x)]/\partial x$ is quasi-monotone in x. Theorem 2 in Persico (2000) shows that if $u^1(\theta, \sigma^1(x;\alpha)) \succeq u^2(\theta, \sigma^2(x;\alpha))$, then $MR^1(\alpha) \geq MR^2(\alpha)$.

Proof of Proposition 1. Fix $\alpha_i = \alpha$ for all i. Let $z_i \equiv \max_{j\neq i} x_j$ denote the highest signal among all agents except for i, then the marginal distribution of z_i is $G(z_i|\alpha)^{n-1}$. Let $\sigma^m(\cdot;\alpha)$ denote the symmetric equilibrium in the second-price auction using $\mathscr{S}^m$ $(m = 1, 2)$. Agent i's expected utility from the second-price auction using $\mathscr{S}^1$ when his true type is θ_i and he observes x_i is

$$u^1(\theta_i, \sigma^1(x_i;\alpha)) = \int_0^{x_i} \left[\theta_i - s^1(\sigma^1(z_i;\alpha), \theta_i)\right] \mathrm{d}G^{n-1}(z_i|\alpha),$$

where σ satisfies (8). A similar expression holds for the second-price auction using $\mathscr{S}^2$. Thus,

$$\begin{aligned} &\frac{\partial}{\partial x_i}\left[u^2(\theta_i, \sigma^2(x_i;\alpha)) - u^1(\theta_i, \sigma^1(x_i;\alpha))\right] \\ &= \left[s^1(\sigma^1(x_i;\alpha), \theta_i) - s^2(\sigma^2(x_i;\alpha), \theta_i)\right](n-1)G^{n-2}(x_i|\alpha)g(x_i|\alpha), \end{aligned}$$

which is quasi-monotone in θ_i since $\mathscr{S}^1$ is steeper than $\mathscr{S}^2$. Hence, $u^2 \succeq u^1$. The rest of the proof follows that of Theorem 2 in Persico (2000). ■

Proof of Proposition 2. Since $\mathscr{S}^m$ $(m = 1, 2)$ is convex, there exists a non-decreasing function $r^m : [\sigma_0, \sigma_1] \to [0, 1]$ such that $s^m(\sigma, \theta_i) = (1 - r^m(\sigma))s^m(\sigma_0, \theta_i) + r^m(\sigma)s^m(\sigma_1, \theta_i)$. Let $\sigma^m(\cdot;\alpha)$ denote the symmetric equilibrium in the first-price auction using $\mathscr{S}^m$ $(m = 1, 2)$. Hence, agent i's expected utility from the first-price auction using $\mathscr{S}^1$ when his true type is θ_i and he observes x_i is

$$u^1(\theta_i, \sigma^1(x_i;\alpha)) = G^{n-1}(x_i|\alpha)\left[\theta_i - s^1(\sigma^1(x_i;\alpha), \theta_i)\right].$$

Hence,

$$\begin{aligned} &\frac{\partial u^1(\theta_i, \sigma^1(x_i;\alpha))}{\partial x_i} \\ &= \left\{\theta_i - \left[\frac{G(x_i|\alpha)}{(n-1)g(x_i|\alpha)} r^{1\prime}(\sigma^1)\sigma^1_x + r^1(\sigma^1)\right]\left[s^1(\sigma_1, \theta_i) - s^1(\sigma_0, \theta_i)\right] - s^1(\sigma_0, \theta_i)\right\} \frac{(n-1)G^{n-2}(x_i|\alpha)}{g(x_i|\alpha)} \\ &= \left\{\theta_i - \frac{\mathbb{E}[\tilde{\theta}_i - s^1(\sigma_0, \tilde{\theta}_i)|x_i;\alpha]}{\mathbb{E}[s^1(\sigma_1, \tilde{\theta}_i) - s^1(\sigma_0, \tilde{\theta}_i)|x_i;\alpha]}\left[s^1(\sigma_1, \theta_i) - s^1(\sigma_0, \theta_i)\right] - s^1(\sigma_0, \theta_i)\right\} \frac{(n-1)G^{n-2}(x_i|\alpha)}{g(x_i|\alpha)}, \end{aligned}$$

where the last line holds by (2). A similar expression holds for the first-price auction using

$\mathscr{S}^2$. Since $s^m(\sigma_0, \theta_i) \leq \theta_i \leq s^m(\sigma_1, \theta_i)$ for all θ_i,

$$\frac{\mathbb{E}[\tilde{\theta}_i - s^m(\sigma_0, \tilde{\theta}_i)|z_i; \alpha]}{\mathbb{E}[s^m(\sigma_1, \tilde{\theta}_i) - s^m(\sigma_0, \tilde{\theta}_i)|x_i; \alpha]} \in [0, 1].$$

Therefore, there exists $\sigma^{m*}(x_i)$ such that for all $\theta_i \in \Theta$

$$\frac{\mathbb{E}[s_m(\sigma_1, \tilde{\theta}_i)|x_i; \alpha]}{\mathbb{E}[s_m(\sigma_1, \tilde{\theta}_i) - s_m(\sigma_0, \tilde{\theta}_i)|x_i; \alpha]} [s^m(\sigma_1, \theta_i) - s^m(\sigma_0, \theta_i)] + s^m(\sigma_0, \theta_i) = s^m(\sigma^{m*}(x_i), \theta_i).$$

Thus,

$$\begin{aligned} &\frac{\partial}{\partial x_i} \left[u^2(\theta_i, \sigma^2(x_i; \alpha)) - u^1(\theta_i, \sigma^1(x_i; \alpha))\right] \\ = &\left[s^1(\sigma^{1*}(x_i), \theta_i) - s^2(\sigma^{2*}(x_i), \theta_i)\right] (n-1)G^{n-2}(x_i|\alpha)g(x_i|\alpha), \end{aligned}$$

which is quasi-monotone in θ_i since $\mathscr{S}^1$ is steeper than $\mathscr{S}^2$. Hence, $u^2 \succeq u^1$. The rest of the proof follows that of Theorem 2 in Persico (2000). ■

B Omitted proofs in Section 4

Before proceeding, we first define symmetric mechanisms formally. Let $\sigma_{i,j} : V^n \to V^n$ denote the function that interchanges the ith and the jth coordinates, i.e.,

$$\sigma_{i,j}(v_1, \dots, v_n) = (v_1, \dots, v_{i-1}, v_j, v_{i+1}, \dots, v_{j-1}, v_i, v_{j+1}, \dots, v_n), \ \forall (v_1, \dots, v_n).$$

A function $\boldsymbol{\varphi} := (\varphi_1, \dots, \varphi_n)$ is symmetric if φ is such that $\varphi_1(\sigma_{i,j}(\boldsymbol{v}))$ for all $i, j \neq 1$ and all $\boldsymbol{v}$ and $\varphi_i(\boldsymbol{v}) = \varphi_1(\sigma_{1,i}(\boldsymbol{v}))$ for all $i \neq 1$ and all $\boldsymbol{v}$. A mechanism $(\boldsymbol{q}, \boldsymbol{r}, \boldsymbol{t})$ is *symmetric* if its allocation rule $\boldsymbol{q}$, royalty rule $\boldsymbol{r}$ and transfer rule $\boldsymbol{t}$ are all symmetric.

Proof of Proposition 3. We first solve the seller's relaxed problem by ignoring (MON), and then verify that the optimal solution satisfies (MON).

For brevity, denote $v(0, \alpha^*)$ by $\underline{v}$, $v(1, \alpha^*)$ by $\overline{v}$, $h(v|\alpha^*)$ by $h(v)$, $H(v|\alpha^*)$ by $H(v)$ and $H_\alpha(v|\alpha^*)$ by $H_\alpha(v)$. Let $X(v) := \int_{\underline{v}}^{v} H_\alpha(z)Q(z)\mathrm{d}z$ for all $v \in [\underline{v}, \overline{v}]$. Then, the seller's relaxed problem can be written as a control problem with state variables (X, Y), and control variables $(Q, R) \in [0, 1]^2$. The evolution of the state variables is governed by

$$X'(v) = -H_\alpha(v)Q(v)\left(1 - R(v)\right), \tag{13}$$

$$Y'(v) = -[H(v)^{n-1} - Q(v)]h(v). \tag{14}$$

To appeal to the optimal control theory, we restrict attention to Q and R that are piecewise continuous and piecewise continuously differentiable.

We now derive the necessary conditions that an optimal solution of ($\mathcal{P}$-α^*) must satisfy. The problem ($\mathcal{P}$-α^*) can be summarized as follows:

$$\max_{X,Y,Q,R} \int_{\underline{v}}^{\overline{v}} \left[z - \frac{1 - H(z)}{h(z)}\left(1 - R(z)\right)\right] Q(z)h(z)\mathrm{d}z,$$

subject to (13), (14),

$$X(\underline{v}) = 0,\ X(\overline{w}) \geq C'(\alpha^*), \tag{15}$$

$$Y(\underline{v}) \geq 0,\ Y(\overline{v}) = 0, \tag{16}$$

$$Y(z) \geq 0. \tag{17}$$

We say that some property holds virtually everywhere if the property is fulfilled at all z except for a countable number of z's. We use the following abbreviation for "virtually everywhere": $v.e.$ We define

$$\begin{aligned}\mathcal{H}(X, Y, Q, R, z) :=& \lambda_0 \left[z - \frac{1 - H(z)}{h(z)}(1 - R)\right] Qh(z) - \lambda_Y(z)[H(z)^{n-1} - Q]h(z) \\ & - \lambda_X(z)H_\alpha(z)(1 - R)Q \text{ for } z \in [\underline{v}, \overline{v}].\end{aligned}$$

By Theorem 4.3.2 in Seierstad and Sydsæter (1987), we have

Lemma 6 *Let* (X, Y, Q, R) *be an admissible pair that solves* $(\mathcal{P}\text{-}\alpha^*)$. *Then there exist a number* λ_0, *vector functions* $(\lambda_X, \lambda_Y, \lambda_Q)$, *and a nondecreasing function* η_Y, *all having one-sided limits everywhere, such that the following condition holds:*

$$\lambda_0 = 0 \text{ or } \lambda_0 = 1, \tag{18}$$

$$(\lambda_0, \lambda_X(z), \lambda_Y(z), \eta_Y(\overline{v}) - \eta_Y(\underline{v})) \neq 0, \ \forall z, \tag{19}$$

$$(Q(v), R(v)) \text{ maximizes } \mathcal{H}(X, Y, Q, R, v) \text{ for } (Q, R) \in [0,1]^2, v.e. \tag{20}$$

$$\eta_Y \text{ is constant in any interval where } Y > 0. \tag{21}$$

$$\eta_Y \text{ is continuous at all } v \text{ where } Y(v) = 0 \text{ and } Q \text{ is discontinuous.} \tag{22}$$

$$\lambda_X \text{ is continuous.} \tag{23}$$

$$\lambda'_X(z) = 0, \ \ v.e. \tag{24}$$

$$\lambda_Y(z) + \eta_Y(z) \text{ is continuous,} \tag{25}$$

$$\lambda'_Y(z) + \eta'_Y(z) = 0, \ \ v.e. \tag{26}$$

$$\lambda_X(\overline{v}) \geq 0 (= 0 \text{ if } X(\overline{v}) > C'(\alpha^*)), \tag{27}$$

$$\lambda_Y(\underline{v}) \leq 0 (= 0 \text{ if } Y(\underline{v}) > 0). \tag{28}$$

In what follows, we assume that (X, Y, Q, R) is an admissible pair that solves $(\mathcal{P}\text{-}\alpha^*)$ and that $(X, Y, Q, R, \lambda_0, \lambda_X, \lambda_Y, \eta_Y)$ satisfy the conditions in Lemma 6.

Since λ_X is continuous and $\lambda'_X(z) = 0$ virtually everywhere, $\lambda_X(z)$ is constant in $[\underline{v}, \overline{v}]$. We abuse the notation slightly and denote this constant by λ_X. Then, (27) is equivalent to

$$\lambda_X \geq 0 (= 0 \text{ if } X(\overline{v}) > C'(\alpha^*)).$$

Similarly, because $\lambda_Y + \eta_Y$ is continuous and $\lambda'_Y(z) + \eta'_Y(z) = 0$ virtually everywhere, $\lambda_Y(z) + \eta_Y(z)$ is constant in $[\underline{v}, \overline{v}]$. We can assume without loss of generality that $\lambda_Y(z) + \eta_Y(z) = 0$.

Then, $\eta_Y = -\lambda_Y$, and condition (21) is equivalent to

$$\lambda_Y(z) \text{ is constant in any interval where } Y(z) > 0. \tag{29}$$

Furthermore, η_Y is nondecreasing if and only if λ_Y is nonincreasing and $\lambda_Y(\underline{v}) \leq 0$.

Suppose $\lambda_0 = 1$. (20) holds if and only if

$$R(v) = \begin{cases} 1 & \text{if } -\frac{1-H(v)}{h(v)} - \lambda_X \frac{H_\alpha(v)}{h(v)} < 0 \\ 0 & \text{if } -\frac{1-H(v)}{h(v)} - \lambda_X \frac{H_\alpha(v)}{h(v)} > 0 \end{cases}, v.e. \tag{30}$$

and

$$Q(v) \begin{cases} = 1 & \text{if } \max\left\{v, v - \frac{1-H(v)}{h(v)} - \lambda_X \frac{H_\alpha(v)}{h(v)}\right\} + \lambda_Y(v) > 0 \\ = 0 & \text{if } \max\left\{v, v - \frac{1-H(v)}{h(v)} - \lambda_X \frac{H_\alpha(v)}{h(v)}\right\} + \lambda_Y(v) < 0 \\ \in [0,1] & \text{if } \max\left\{v, v - \frac{1-H(v)}{h(v)} - \lambda_X \frac{H_\alpha(v)}{h(v)}\right\} + \lambda_Y(v) = 0 \end{cases}, v.e. \tag{31}$$

We argue that $Y(v) = 0$ for all $v \in [\underline{v}, \overline{v}]$. Suppose, to the contrary, that $Y(v) > 0$ in an interval (v^1, v^2) with $Y(v^1) = Y(v^2) = 0$. Then, $\lambda_Y(v)$ is constant in (v^1, v^2). Since $\int_{v^1}^{v^2} [H(v)^{n-1} - Q(v)]h(v)\mathrm{d}v = 0$ and $\max\left\{v, v - \frac{1-H(v)}{h(v)} - \lambda_X \frac{H_\alpha(v)}{h(v)}\right\}$ is strictly increasing in v, there exists $v^\sharp \in (v^1, v^2)$ such that $Q(v) = 0$ for $v \in (v^1, v^\sharp)$ and $Q(v) = 1$ for $v \in (v^\sharp, v^2)$. However, this implies that for $v \in (v^\sharp, v^2)$,

$$Y(v) = Y(v^2) + \int_v^{v^2} \left[H(z)^{n-1} - 1\right] h(z)\mathrm{d}z < 0,$$

a contradiction. Hence, $Y(v) = 0$ for all $v \in [\underline{v}, \overline{v}]$. This implies that $Q(v) = H(v)^{n-1}$ for all $v \in [\underline{v}, \overline{v}]$. Then, by (31), $\lambda_Y(v) = -\max\left\{v, v - \frac{1-H(v)}{h(v)} - \lambda_X \frac{H_\alpha(v)}{h(v)}\right\}$ for all $v \in [\underline{v}, \overline{v}]$, which is strictly decreasing.

Suppose $\lambda_X = 0$. Then, $R = 1$ and therefore, $X(\overline{v}) = 0 < X'(\alpha^*)$, a contradiction.

Hence, $\lambda_X > 0$, which implies that $X(\overline{v}) = C'(\alpha^*)$. Let v^* be such that

$$-\frac{1 - H(v^*)}{h(v^*)} - \lambda_X \frac{H_\alpha(v^*)}{h(v^*)} = 0. \tag{32}$$

Then, $R(v) = 1$ if $v < v^*$ and $R(v) = 0$ if $v > v^*$. An agent's marginal benefit from increasing accuracy is

$$X(\overline{v}) = \int_{v^*}^{\overline{v}} -H_\alpha(v)H(v)^{n-1}\mathrm{d}v,$$

which is strictly decreasing in v^*. Since v^* is strictly decreasing in λ_X, there exists a unique λ_X such that $X(\overline{v}) = C'(\alpha^*)$.

Since $\mathcal{H}$ is independent of (X, Y) and Y is linear in Y, by Theorem 4.3.3 of Seierstad and Sydsæter (1987), the solution found above is optimal. Finally, $Q(v)[1 - R(v)]$ is nondecreasing.

Suppose $\lambda_0 = 0$. (20) holds if and only if

$$R(v) = \begin{cases} 1 & \text{if } -\lambda_X \frac{H_\alpha(v)}{h(v)} < 0 \\ 0 & \text{if } -\lambda_X \frac{H_\alpha(v)}{h(v)} > 0 \end{cases}, v.e. \tag{33}$$

and

$$Q(v) \begin{cases} = 1 & \text{if } \max\left\{0, -\lambda_X \frac{H_\alpha(v)}{h(v)}\right\} + \lambda_Y(v) > 0 \\ = 0 & \text{if } \max\left\{0, -\lambda_X \frac{H_\alpha(v)}{h(v)}\right\} + \lambda_Y(v) < 0 \\ \in [0,1] & \text{if } \max\left\{0, -\lambda_X \frac{H_\alpha(v)}{h(v)}\right\} + \lambda_Y(v) = 0 \end{cases}, v.e. \tag{34}$$

We argue that $\lambda_X > 0$. Suppose, to the contrary, that $\lambda_X = 0$. Then, (19) implies that $\lambda_Y(\overline{v}) < 0$ since otherwise $(\lambda_0, \lambda_X(z), \lambda_Y(z), \eta_Y(\overline{v}) - \eta_Y(\underline{v})) = 0$ for all z, a contradiction. Note that λ_Y is nonincreasing. Let $\hat{v} := \inf\{v : \lambda_Y(v) < 0\}$. Then, $\lambda_Y(v) < 0$ for all $v \in (\hat{v}, \overline{v})$. Then, $Q(v) = 0$ for virtually all $v \in (\hat{v}, \overline{v})$. Hence, $Y(\hat{v}) > 0$. However, this implies that there exists $\varepsilon > 0$ such that $Y(v) > 0$ for all $v \in (\hat{v} - \varepsilon, \hat{v})$, and therefore, $\lambda_Y(\hat{v} - \varepsilon) = \lambda_Y(\hat{v}) < 0$, a contradiction to the definition of $\hat{v}$. Hence, $\lambda_X > 0$.

Recall that $\hat{v} = \inf\{v : -H_\alpha(v) > 0\}$. Clearly, $v^* > \hat{v}$. By a similar argument to that in the case of $\lambda_0 = 1$, we can show that $Y(v) = 0$ for all $v \in (\hat{v}, \overline{v})$. This implies that $Q(v) = H(v)^{n-1}$ for all $v \in (\hat{v}, \overline{v})$. By (33), $R(v) = 1$ if $v < \hat{v}$ and $R(v) = 0$ if $v > \hat{v}$. Hence, the seller's revenue is

$$\begin{aligned}
&\int_{\underline{v}}^{\hat{v}} vQ(v)h(v)\mathrm{d}v + \int_{\hat{v}}^{\overline{v}} \left[v - \frac{1-H(v)}{h(v)}\right] H(v)^{n-1}h(v)\mathrm{d}v \\
< &\int_{\underline{v}}^{\overline{v}} vH(v)^{n-1}h(v)\mathrm{d}v + \int_{v^*}^{\overline{v}} -\frac{1-H(v)}{h(v)} H(v)^{n-1}h(v)\mathrm{d}v,
\end{aligned}$$

where the right-hand side of the inequality is the seller's revenue obtained when $\lambda_0 = 1$. Hence, the pair of (Q, R) found when $\lambda_0 = 0$ is not an optimal solution. ■

Proof of Corollary 1. In the proof of Proposition 3, we show that the optimal $v^* \geq \hat{v}$ satisfies that

$$\int_{v^*}^{v(1,\alpha^*)} -H_\alpha(v|\alpha^*)H(v|\alpha^*)^{n-1}\mathrm{d}v = C'(\alpha^*),$$

where the left-hand side decreases as v^* increases. Clearly, if the marginal cost $C'(\alpha^*)$ increases, then the optimal threshold v^* decreases. ■

C Additional empirical evidence and tests

In this section, we discuss the two sets of variables on information asymmetry used in Eckbo et al. (2018) but excluded in our analysis. First, Target Recent M&A is a dummy variable indicating that the target announced a merger deal as acquirer during the 18 months preceding the deal. Acquirer Recent M&A is a dummy variable indicating that the acquirer acquired another firm during the 18 months preceding the deal. Second, Industry Complementarity is a measure used in Fan and Lang (2000), which captures how the bidder industry and the target industry complement each other. Eckbo et al. (2018) also consider five alternative measures of industry complementarities or similarities, including Vertical Relatedness, Same Primary SIC, Overlapping Industries (normalized by either the number of bidder industries or target industries), and Return Correlation. Table C1 presents details of how we construct these variables.

Table C2 shows the summary statistics of these variable for our sample. There are two reasons why we do not use Target Recent M&A in the main analysis. First, different from Acquirer Recent M&A, only three percent targets acquired another firm during the 18 months preceding the announcement dates. This finding is quite intuitive as targets are relatively smaller and have lower market-to-book ratio than acquirers, yet it means that there is little variation we can explore. Second, on the contrary to Eckbo et al. (2018), in our sample Acquirer Recent M&A negatively associates with All Stock payments. We conjecture that this is because we only include public targets in our sample. We verify this conjecture by analysing an extended sample with both public and private targets.

We follow Eckbo et al. (2018) to construct this extended sample. Specifically, we include merger deals for U.S. targets by U.S. public acquirers from 1980 to 2014 in the SDC merge and acquisition data. We then require that deal size above $10 million and that acquirers are non-financial firms. This extended sample includes 10,454 deals. The sample size is larger than that in Eckbo et al. (2018), primarily because we do not match these deals to CRSP or Compustat, but it is sufficient for our purpose.

What emerges from this investigation is whether the previous deal's target is public or private matters for the correlation between Acquirer Recent M&A and payment methods. We construct two dummy variables to illustrate this point: Acquirer Recent M&A with Public Target and Acquirer Recent M&A with Non-public Target. By construction, the sum of these two dummies should equal to Acquirer Recent M&A. Then, we compare their means of the all-stock and the all-cash subsamples.

As shown in Table C3, we find that although Acquirer Recent M&A with Non-public Target associates positively with all-stock payments, Acquirer Recent M&A with Public Target associates negatively with all-stock payments. One possible explanation of this finding is that recent mergers may contain opposite effects on information asymmetry. On the one hand, as Eckbo et al. (2018) claim, acquirers may disclose more information in previous merger deals. On the other hand, previous mergers may also complicate acquirers' capital structure and make it more costly for targets to learn about them. Given the opposite effects, the correlation between Acquirer Recent M&A and payment methods is not clear a priori and depends on the sample chosen. Therefore, we exclude Acquirer Recent M&A in our analysis.

We exclude variables on industrial complementarity in our analysis because targets' capital structure variables and fixed effects absorb their effects on payment methods. Table C4 compares regressions relating the fraction of payment in stock to industrial complementarity variables with and without controls on targets' capital structure and target industrial and year fixed effects. In Panel A, we replicate the finding of Eckbo et al. (2018) using our sample with bidder capital structure variables, HHI, and Competition from Private Buyers as controls and acquirer industry fixed effects. We indeed find significant effects of industrial complementarities on payment methods. Once we further include target industry fixed effects, year fixed effects, and controls on targets' capital structure in Panel B, only Return Correlation remains significant. This comparison suggests that industrial complementarities may explain the across-group variation in payment methods but not the within-group variation.

Table C1: Variable Definitions

Information	
Target Recent M&A	Dummy = 1 if a target acquired another firm within 18 months prior to the sample bid, SDC.
Acquirer Recent M&A	Dummy = 1 if an acquirer acquired another firm within 18 months prior to the sample bid, SDC.
Industry Complementarity	Based on Fan and Lang (2000), the proxy captures how the bidder industry and the target industry complement to each other. Joseph Fan's website.
Vertical Relatedness	Based on Fan and Lang (2000), the proxy captures how much input and output of the bidder industry is bought from and sold to the target industry. Joseph Fan's website.
Same Primary SIC Dummy	Dummy = 1 if the bidder primary four-digit SIC is similar to target primary four-digit SIC, SDC.
Overlapping Ind./Bidder Ind.	Number of overlapping four-digit SIC codes between the bidder and target scaled by the number of bidder four-digit SIC codes, SDC.
Overlapping Ind./Target Ind.	Number of overlapping four-digit SIC codes between the bidder and target scaled by the number of target four-digit SIC codes, SDC.
Return Correlation	Daily stock return correlation between bidder and target. We use $[t-290:t-41]$ as the estimation window, which covers 250 trading days before the 40-day run-up period, $[t-40:t-1]$, CRSP.

Table C2: Summary Statistics for Variables in Table C1

This table presents summary statistics for our sample constructed in Section 5.1. It includes counts, means, and medians of the full sample (columns 1-3), all-stock deals (columns 4-6), and all-cash deals (Columns 7-9). Column 10 presents the difference in mean between all-stock and all-cash deals, and Column 11 shows the t-statistics of the difference. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	Full Sample			All-stock			All-cash				
	Count	Mean	Median	Count	Mean	Median	Count	Mean	Median	Mean Diff	t-stat
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Target Recent M&A	4894	0.03	0.00	1779	0.04	0.00	1875	0.03	0.00	0.01*	(1.98)
Acquirer Recent M&A	4894	0.17	0.00	1779	0.15	0.00	1875	0.18	0.00	-0.03**	(-2.60)
Industry Complementarity	3453	0.64	0.79	1166	0.66	1.00	1369	0.61	0.58	0.05**	(3.18)
Vertical Relateness	3515	0.05	0.01	1181	0.05	0.01	1385	0.04	0.01	0.01***	(4.63)
Same Prime SIC	4894	0.36	0.00	1779	0.36	0.00	1875	0.32	0.00	0.04**	(2.68)
Overlapping Ind. /Target Ind.	4894	0.19	0.14	1779	0.21	0.17	1875	0.16	0.11	0.05***	(6.80)
Overlapping Ind. /Acquirer Ind.	4894	0.15	0.13	1779	0.16	0.14	1875	0.13	0.10	0.03***	(5.81)
Return Correlation	3518	0.15	0.11	1223	0.14	0.11	1413	0.15	0.11	-0.00	(-0.45)

Table C3: Summary Statistics for All-stock and All-cash Subsamples

We construct two dummy variables: Acquirer Recent M&A with Public Target and Acquirer Recent M&A with Non-public Target, to show that the public status of the previous deal's target matters for the correlation between Acquirer Recent M&A and payment methods. This table presents their summary statistics for all-stock deals (Columns 1-3) and all-cash deals (Columns 4-6) in the extended sample constructed in Appendix C. Column 7 presents the difference in mean between all-stock and all-cash bids, and Column 8 shows the t-statistics of the difference. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	All-stock			All-cash				
	Count	Mean	Median	Count	Mean	Median	Mean Diff	t-stat
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Acquirer Recent M&A	3449	0.30	0.00	2335	0.29	0.00	0.02	(1.24)
Acquirer Recent M&A w. Public Targets	3449	0.14	0.00	2335	0.18	0.00	-0.04***	(-4.46)
Acquirer Recent M&A w. Non-pub. Targets	3449	0.16	0.00	2335	0.10	0.00	0.06***	(6.62)

Table C4: Industrial Complementarities and Payment Methods

This table presents coefficients from regressions relating the fraction of payment in stock to industrial complementarities using our sample constructed in Section 5.1. Both panels include bidder capital structure variables, HHI, and Competition from Private Buyers as controls and acquirer industry fixed effects. Panel B also includes target industry fixed effects, year fixed effects, and target capital structure variables in the regressions, while Panel A does not. All variables are defined in Table 1 or Table C1. Industry dummies indicate the 2-digit SIC industry. Robust standard errors are in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

Panel A: Without Target Industry FE, Year FE, or Target Capital Structure Variables

	Fracton of payment in stock					
Industry Complementarity	0.062***					
	(0.023)					
Vertical Relateness		0.414***				
		(0.148)				
Same Prime SIC			0.022			
			(0.014)			
Overlapping Ind./Acquirer Ind.				0.126***		
				(0.047)		
Overlapping Ind./Target Ind.					0.081**	
					(0.032)	
Return Correlation						0.070**
						(0.032)
Observations	3026	3080	4262	4262	4262	3295
Adjusted R^2	0.146	0.147	0.157	0.158	0.158	0.159

Panel B: With Target Industry FE, Year FE, and Target Capital Structure Variables

	Fracton of payment in stock					
Industry Complementarity	-0.010					
	(0.026)					
Vertical Relateness		0.055				
		(0.149)				
Same Prime SIC			-0.014			
			(0.016)			
Overlapping Ind./Acquirer Ind.				-0.021		
				(0.055)		
Overlapping Ind./Target Ind.					-0.011	
					(0.038)	
Return Correlation						0.108***
						(0.034)
Observations	2359	2396	3321	3321	3321	2901
Adjusted R^2	0.281	0.283	0.295	0.295	0.295	0.302