The Crisis of Expertise*

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Abstract

An expert advises a sequence of principals on their actions to match a hidden, randomly evolving state. The expert privately knows her competence. The principals learn about the state and the expert's competence from past advice and past action outcomes, both publicly observable. I find that the equilibrium can feature a "crisis of expertise," in which principals dismiss a competent expert's correct advice and rely only on public information. Notably, the crisis happens precisely when the quality of public information is low and thus when the competent expert's knowledge is much needed. I discuss policy implications for alleviating the crisis.

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1 Introduction

Experts give advice to guide decisions. Popular discussions suggest that technology can lead to a "crisis of expertise" by making information abundant and publicly accessible to decision makers (see, *e.g.*, Nichols, 2017; Gurri, 2018; Eyal, 2019). A common description of this crisis is that decision makers dismiss the genuine advice that informed experts offer and act solely on the basis of public information, for

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instance, from social media. Nichols (2017, p. 105) writes: "Ask any professional or expert about the death of expertise, and most of them will immediately blame the same culprit: the Internet." Similarly, in a 2019 interview with *CNBC*, a financial planner explains that this is the reason why people avoid obtaining professional financial advice: "There is a lot more information online these days, [...] so people feel like they can do it themselves."[1]

To understand how expertise is valued as information technology advances, important questions arise. Is the crisis of expertise due to the abundance of public information? And if so, is it simply a phenomenon where decision makers substitute high-quality public information for expert advice?

This paper addresses these questions in a stylized model of expert advice. In the model, decision makers take actions after assessing two relevant pieces of information—current expert advice and public information—the latter of which consists of past advice and past action outcomes. Public information has two faces. On the one hand, it gives decision makers access to resources that inform their actions and help them learn about the expert. On the other hand, it allows uninformed experts to learn and benefit by pretending to give informed advice, despite them knowing no more than the decision makers; the presence of these uninformed experts then causes skeptical decision makers to sometimes dismiss expert advice and act solely on the basis of public information. A striking prediction of my model is a complementarity between the quality of public information and decision makers' trust in expert advice: decision makers dismiss expert advice only when the quality of public information is low and thus precisely when informed experts' knowledge is much needed. Hence, the crisis of expertise is a consequence of the lack, as opposed to the availability, of high-quality public information. This observation leads to important policy implications regarding public information provision.

I describe my model in Section 2. An expert (she) faces a sequence of principals who arrive at random times. Each arriving principal (he) takes an action and wants his action to match a binary state. The principal faces uncertainty, as the state is hidden and evolves according to a Markov process. Before taking an action, the principal hires the expert by paying her upfront for advice, with the payment reflecting his perceived value of the advice. Once hired, the expert receives a private signal about the current state. The expert is privately informed about her own signal quality: she

[1] "99% of Americans don't use a financial advisor—here's why," *CNBC*, November 11, 2019.

is either an "informed" type who observes the state perfectly, or an "uninformed" type whose signal of the state is uninformative. The expert's advice and the state are publicly revealed after the principal takes an action. The goal of the expert is to maximize the payments that she receives from the principals.

A central feature of the model is that the principals have noisy, evolving beliefs about both the state and the expert's type given the public information, namely, the advice and what the state turned out to be at past principals' arrival times. The model is applicable to a variety of advising and consulting situations. To give a concrete example, suppose that each principal is a firm that chooses to raise capital to fund either a risky project or a safe project. A project succeeds only if enough investors invest in it. The state of the market varies over time and can be "fear" or "greed"; if the market is fearful (resp., greedy), then only the safe (resp., risky) project receives enough funds. The firms are unsure about the state and thus hire a financial expert. They evaluate the advice and then determine their decisions, based on the other firms' past capital-raising outcomes as well as the expert's past pieces of advice.

As the opening paragraphs indicate, the goal of my analysis is to elucidate how principals value genuine, informed advice and make decisions given their access to both current expert advice and public information. In Sections 3 and 4, I thus characterize equilibria in which the informed expert truthfully reports her private information relevant to each arriving principal's decision-making, namely her type and current private signal. In any such equilibrium, given each history of play at which a principal arrives, there is a region of public beliefs about the state and the expert's type that corresponds to a crisis of expertise: the principal acts independently of the expert's report if and only if he arrives with public beliefs in this "crisis" region. The main result of the analysis is to show that the crisis region excludes precise enough, inconclusive public state beliefs.

In each arrival, the uninformed type strives to appear informed, and thus it is impossible for an informed type's report to convey information about her type to the principal. Hence, the expert's report is effectively a report of the current state. The informed type reports her correct state observation truthfully. The uninformed type mixes between reporting either state. To see how the uninformed expert's mixture is determined, call the state that is most likely given the public information the *agreeing* state, and the other state is the *disagreeing* state. From the uninformed type's perspective, disagreeing is likely to be incorrect and hence to reveal her type. However, if it is correct, then her reputation, namely the principals' belief that she is informed, will increase and given the equilibrium strategies, so will the future payments. The uninformed type may thus "gamble" on the unlikely event that disagreeing is correct for her reputation. On the other hand, the principal matches his action with the expert's report if and only if he believes that the report is likely to be sent by an informed expert, because the principal understands that the uninformed expert knows nothing more about the state than he does.

I then turn to show the main result that the crisis region excludes precise enough, inconclusive beliefs that the current state is equal to the last state: if a principal arrives with such precise state beliefs and receives an agreeing report, then he matches his action with the report, as he believes that the report reinforces the high-quality public information about the state. In addition, given the high-quality public information about the state, the principal expects the uninformed type to find a disagreeing report likely to be incorrect, and thus to have little incentive to gamble. Hence, if the principal receives a disagreeing report instead of an agreeing report, then he also matches his action with the report, as he believes that the report is likely to be sent by an informed type.

My main insight that high-quality public information preempts the crisis of expertise is novel and contributes broadly to the literature on public information provision. Indeed, this insight contrasts with a key message of this literature, namely that the provision of public information crowds out economic agents' use of their socially valuable private information. See, for instance, Morris and Shin (2002) and also the social learning literature, starting with Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992). In my analysis, public information does not crowd out the informed expert's use of her socially valuable private information; instead, it crowds out decision makers' trust of the informed expert's private information. In addition, public information of sufficiently high quality sustains the trust, resulting in an efficient use of the informed expert's private information. The complementarity between the quality of public information and decision makers' trust in experts resonates with the recent findings in the survey "Trust and Mistrust in Americans' Views of Scientific Experts" by the *Pew Research Center*, which documents that confidence in scientific experts is higher among survey participants who possess higher science knowledge (Funk, Hefferon, Kennedy and Johnson, 2019). Importantly, the complementarity suggests the need for policies that promote the provision of high-quality public information.

This finding supports recent advocacy by political campaigns and consumer groups who determinedly point to the need to equip consumers with high-quality public information (see, *e.g.*, Howells, 2005).

From a methodological perspective, my analysis contributes to the literature on reputational cheap talk (see, *e.g.*, Scharfstein and Stein, 1990; Trueman, 1994; Ottaviani and Sørensen, 2006a,b,c) by developing a tractable dynamic framework that explicitly examines principals' learning and decision-making as well as expert's career concerns, both of which are absent in the existing models. As is well known, the construction of a tractable multi-period model of reputational cheap talk that captures learning about both the expert's ability as well as the state is challenging (see, for instance, the survey by Marinovic, Ottaviani and Sørensen (2013) for further discussion).[2] My model allows for a clean characterization of such learning, and in turn a clean characterization of the principals' and the expert's equilibrium behavior and payoffs.

Exploiting such tractability, my equilibrium analysis obtains new and richer predictions about expert advice than the existing models do; these predictions in turn shed light on both anecdotal and empirical evidence regarding expert advisors' behavior over their careers. As my analysis proceeds, I relate my characterization results to concerns in the financial advice industry that the presence of "fake" experts undermines the value of informed advisors. I argue that the results rationalize social conventions that reward successful "contrarians" in financial markets and entrepreneurship. Moreover, my model predicts that both expert types alternate between agreeing and disagreeing as their state beliefs evolve, and uninformed experts' tendency to disagree dissipates as their careers progress. I relate these predictions to recent empirical evidence on gambling by careerist financial analysts. Finally, I show that the crisis of expertise is a short-run phenomenon in the equilibrium, as public information allows informed experts to build their reputations. I then point to the importance of technologies that publicly account for experts' individual track records by discussing how additional economic factors that are absent in the model might limit principals' learning about the expert and thus affect the short-run nature of the crisis of expertise.

The broader literature on expert reputation examines models with different incen-

[2] The idea that experts strive to appear informed motivated a broad literature. See also the references in the survey by Marinovic et al. (2013), as well as more recent contributions such as Klein and Mylovanov (2017), Pavesi and Scotti (2019), Rüdiger and Vigier (2019), Smirnov and Starkov (2019), Backus and Little (2020), Silva (2020) and Camara and Dupuis (2021).

tive structures. One strand of the literature consists of models where experts take costly actions to appear competent, starting with the seminal work by Holmström (1999).[3] Another strand of the literature studies experts who communicate to bias principals' decisions yet strive to appear unbiased (see, *e.g.*, Sobel, 1985; Benabou and Laroque, 1992; Morris, 2001; Olszewski, 2004; Gentzkow and Shapiro, 2006). In contrast, as in existing models of reputational cheap talk, my model deliberately abstracts from such decision biases. This allows my analysis to starkly illustrate why rational decision makers might dismiss informed experts' advice even when they understand that the experts are not biased towards specific decisions of theirs and that the "good" experts act to maximize their interest. In this regard, my analysis also contrasts with bad reputation models (see, *e.g.*, Ely and Välimäki, 2003) in which "good" experts' desire to separate themselves from "bad" experts creates perverse incentives for them to act against the principals' interests.

Recently, Deb and Stewart (2018), Deb, Pai and Said (2018) and Deb, Mitchell and Pai (2020) examine dynamic mechanisms and contracts that screen experts' ability.[4] Different from these models, the expert in my model faces implicit incentives to appear informed, as her ability is evaluated by a "market," namely a sequence of short-lived principals, without commitment power.

2 Model

Setup. Time $t \geq 0$ is continuous, and the horizon is infinite. A long-lived expert faces a countable sequence of short-lived principals. The expert has a private type $\theta \in \Theta := \{\theta_I, \theta_U\}$. She is an informed type θ_I with probability $p_0 \in (0, 1)$, and is an uninformed type θ_U otherwise. The principals arrive at random times according to a Poisson process $(N_t)_{t\geq 0}$ with intensity normalized to one. A hidden state $(s_t)_{t\geq 0}$, taking values on $S := \{\underline{s}, \bar{s}\}$ with initial value $s_0 = \bar{s}$, evolves according to a Markov process with switching rate (or volatility) $\gamma > 0$ at each state.[5] The expert's type, the

[3]See, *e.g.*, Prendergast and Stole (1996), Dewatripont, Jewitt and Tirole (1999a,b), Majumdar and Mukand (2004), Bonatti and Hörner (2017), Cisternas (2018), and Halac and Kremer (2020).

[4]Relatedly, there is a literature on testing experts, starting with Foster and Vohra (1998), that examines the existence of a test that cannot be passed by an expert who does not know the true data-generating process but can be passed almost surely by an expert who knows the process (for a survey, see Olszewski, 2015).

[5]The analysis readily extends to general initial state distributions, as well as to asymmetric switching rates at the cost of extra notation. Section 5 discusses the extension to more states.

state process and the arrival process are independent.

Timing, actions and payoffs. When a principal arrives at time t, he pays the expert a wage $w_t \in \mathbb{R}_+$ upfront;[6] the wage is assumed to equal the principal's marginal gain from interacting with the expert, as I specify below in (4). The expert then sends a message $m_t \in M$ to the principal, where M is a finite set. The principal then takes an action $a_t \in S$. If the current state is $s_t = s$, then the principal obtains a benefit

$$b_t = \begin{cases} \bar{b}_s, & \text{if } a_t = s, \\ \underline{b}_s, & \text{if } a_t \neq s, \end{cases}$$

where $\underline{b}_s < \bar{b}_s$. Thus, the principal prefers to take an action that matches the current state. The principal's flow payoff equals his benefit minus the wage, namely $b_t - w_t$. The expert's flow payoff equals the wage w_t.

Observe that by assumption, even when an arriving principal does not expect the expert's message to be valuable, he still pays the expert a wage of zero and the expert still sends a message. This is plainly for simplicity, allowing my analysis to abstract from modeling principals' decisions of whether to hire the expert or not. As discussed in Section 5, the insights extend to a setting where the expert does not send a message whenever the arriving principal does not expect the expert's message to be valuable.

The flow payoffs are discounted at rate $r > 0$. Given a history of arrival times $(t_i)_{i=1}^{\infty}$ and wages $(w_{t_i})_{i=1}^{\infty}$ at those times, the expert's (normalized) realized payoff is

$$\sum_{i=1}^{\infty} r\, \mathrm{e}^{-rt_i}\, w_{t_i}.$$

Information. At each principal's arrival time t, before sending a message, the expert receives a private signal $y_t \in S$. If the expert is an informed type, then the signal is equal to the current state. If the expert is an uninformed type, then the signal is equal to either $\bar{s}$ or $\underline{s}$ with equal probabilities and is therefore pure noise. In addition, in each arrival, the message, the principal's action and his realized benefit are public; the current state is thus publicly revealed once the benefit is realized.

In fact, to show the main insight that principals dismiss the informed type's genuine

[6] Although output-contingent wages are theoretically attractive, I assume that wages are paid upfront, as is often the case in applications such as financial advice and business consulting.

advice only if public information about the state has low quality, the informed type's private signals need not reveal the current state perfectly. In the Online Appendix, I show that this insight carries over to an extension in which the informed expert is a commitment type who truthfully reports her signals which, despite always being more precise than the public information, are noisy. The assumption of perfect signals eases the notations and the exposition in the main text. Given this assumption, as will be clear, there is no need to assume that the informed expert is a truthful commitment type; the commitment assumption is thus not imposed in the main text.

Histories. A public history of length n is a list $h_n = (t_i, m_{t_i}, a_{t_i}, b_{t_i})_{i=1}^n$, collecting the times $(t_i)_{i=1}^n$ of the first n arrivals, and the message m_{t_i}, the action a_{t_i} and the benefit b_{t_i} in each such time-t_i arrival. A type-θ expert's private history of length n is a list $h_n^\theta = (h_n, (y_{t_i})_{i=1}^n)$, consisting of the length-n public history and her private signals in the first n arrivals. Let H_n be the set of length-n public histories, let $H := \cup_{n=0}^\infty H_n$, and let $h \in H$ denote a generic public history. Likewise, let H_n^θ be the set of type-θ expert's length-n private histories, let $H^\theta := \cup_{n=0}^\infty H_n^\theta$, and let $h^\theta \in H^\theta$ denote a generic type-θ expert's private history.

Strategies. Each type-θ expert's strategy is a collection $\sigma^\theta = \left(\sigma_t^\theta\right)_{t\geq 0}$, where each

$$\sigma_t^\theta : H^\theta \times S \to \Delta M$$

is a measurable mapping that takes her history h^θ and the current signal y_t to a distribution over messages at an arrival time t, subject to the restriction that all arrival times in the history h^θ are strictly smaller than t. Let $\sigma := \left(\sigma^I, \sigma^U\right)$.

Each principal's strategy is to choose an action given a public history h and a current message m at his arrival time t. Since the principal is short-lived, to maximize his payoff, he chooses the myopically optimal action and obtains a benefit that I denote by $b_t^*(h, m)$. Moreover, if a principal is indifferent between the two actions, I assume that he matches his action with the last revealed state (hereafter, the last state); this is without loss because the principal's action does not affect the information of the expert, the information of future principals, as well as future wages. Finally, when specifying an equilibrium below, I omit the principals' strategies to ease the notation.

Beliefs. At each time t, the uninformed type forms a state belief that the current state equals the last state. Since state transitions are Markov, her state belief at time t is a function of the time-lapse l_t since the last state was revealed and is given by

$$\mu_{l_t} := \frac{1}{2}\left(1 + \mathrm{e}^{-2\gamma l_t}\right). \tag{1}$$

This belief is depicted in Figure 1. Observe that $\mu_l > \frac{1}{2}$ for each $l \geq 0$. Thus, between the two possible states, the uninformed type believes that the current state is more likely to match the last state than to mismatch it. The belief falls as the time-lapse l grows, and it falls at a faster rate with l the higher is the state volatility γ.

Likewise, at each time t, the informed type's state belief that the current state equals the last state is a function of the time-lapse l_t since the last state was revealed. I denote this belief by $\mu^I_{l_t}$. Absent a principal's arrival at time t, the belief $\mu^I_{l_t}$ coincides with the uninformed type's state belief μ_{l_t}. If a principal arrives at time t, then the belief $\mu^I_{l_t}$ is either 1 or 0, depending on the private signal that she receives.

Finally, at each time t, an arriving principal forms a belief on the expert's type and the current state. This belief is a pair (p_t, μ_{l_t}), where p_t denotes the principal's belief that the expert is informed and represents the expert's reputation, and μ_{l_t} is the principal's belief that the state is equal to the last state given a time-lapse l_t. Observe that the principal's state belief is equal to the uninformed expert's state belief, because the uninformed expert's private signals about the state are pure noise.

Throughout, I refer to a generic pair (p, μ) as the public type and state beliefs, or simply the public beliefs. At each time, a higher public state belief captures higher quality of public information about the state. I also say that an action, a state, a signal or a report is *agreeing* with respect to the public state belief if it matches the last state, and is *disagreeing* otherwise. Finally, I say that an action, a signal or a report is *correct* if it matches the current state, and is *incorrect* otherwise.

Wages. As mentioned above, in each arrival, the wage that a principal pays the expert equals his expected marginal benefit from receiving the expert's message. To see how the wage is formulated, given a last state s, define

$$\Delta_s := \frac{\bar{b}_{\neg s} - \underline{b}_{\neg s}}{\bar{b}_s - \underline{b}_s} \tag{2}$$

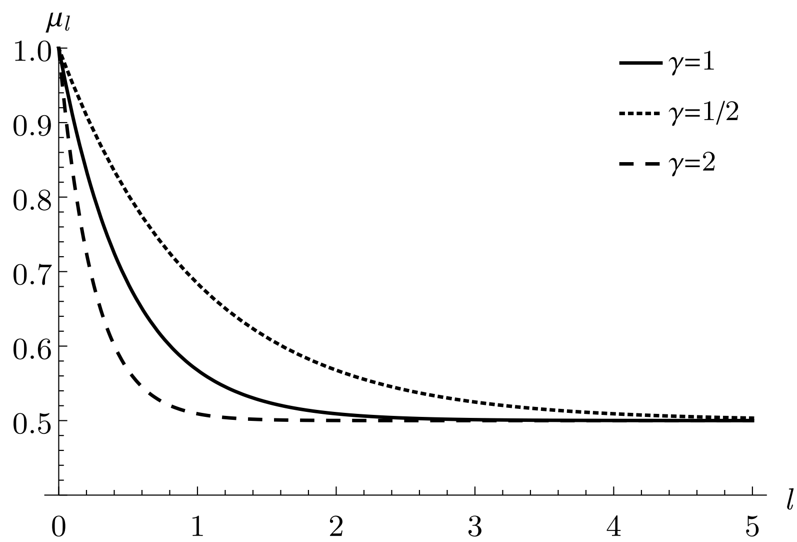


Figure 1: Public state belief μ_l

as the payoff stakes at the disagreeing state $\neg s$ relative to those at the agreeing state s. Given a last state s, if a principal had to optimally choose an action without the expert's advice, then he would choose a disagreeing action if the state belief is $\mu < \frac{\Delta_s}{1+\Delta_s}$ and would choose an agreeing action otherwise. His benefit is then

$$b_*(\mu, s) := \begin{cases} (1-\mu)\bar{b}_{\neg s} + \mu \underline{b}_s, & \text{if } \mu < \frac{\Delta_s}{1+\Delta_s}, \\ \mu \bar{b}_s + (1-\mu)\underline{b}_{\neg s}, & \text{otherwise.} \end{cases} \tag{3}$$

Hence, given a strategy σ and a time t at which a principal arrives with associated public history h, state belief μ and last state s, the wage is

$$w_t := \mathbf{E}\left[b_t^*\left(h, m\right) | h\right] - b_*\left(\mu, s\right), \tag{4}$$

where the expectation $\mathbf{E}$ is taken over messages m that the expert sends according to the strategy σ, conditional on the public history h.

Equilibrium. The solution concept that I use is perfect Bayesian equilibrium. In such an equilibrium, denoted by (σ, φ), each type-θ expert's strategy σ^θ is optimal given the beliefs $\varphi := (p_t, \mu_{l_t}, \mu_{l_t}^I)_{t \geq 0}$ and the beliefs are consistent with the strategy profile σ wherever possible. The message set is an ingredient of the solution concept.

In the spirit of Myerson (1986), I assume that $M = \Theta \times S$, allowing each type to disclose all her relevant private information, namely her type and her current private signal, in each arrival.

I focus on perfect Bayesian equilibria that satisfy Properties 1 and 2 below, henceforth "equilibria." Property 1 concerns the informed type's behavior on path:

Property 1. *On path, in each arrival, the informed type reports her private information, namely her type and her current private signal, truthfully.*

Given Property 1, it is without loss to assume that an uninformed type never truthfully reports her type, as such report leads to zero future wages. Accordingly, I simplify the structure of the message set and assume throughout that

$$M = S.$$

A message is interpreted as a report on the current state. By Property 1, on path, the informed type sends a correct report in each arrival. Thus, the property rules out equilibria in which the informed type babbles or mixes between truthfully reporting and misreporting in some arrival on path. This enables my focus on how principals value an informed expert who provides correct advice given their access to public information. On the other hand, without loss, I take the uninformed expert's strategy to be a probability of sending an agreeing report.

In equilibrium, beliefs are updated by Bayes' rule not only on path but also off path. Because Bayes' rule does not apply when beliefs are degenerate, Property 2 below, which is commonly assumed in the literature, pins down what happens following degenerate beliefs off path by identifying continuation games with degenerate reputations as complete-information games.

Property 2. *Degenerate reputations are absorbing, i.e., if at any history of play, the reputation is $p = 0$ (resp., $p = 1$), then at any concatenation of the history, the reputation is also $p = 0$ (resp., $p = 1$).*[7] *In addition, in each arrival with reputation $p = 0$, the principal's state belief follows* (1) *irrespective of the report he receives.*

[7] More precisely, a history of play at time t, denoted by η_t, consists of the realized type, the associated public history, the realized path of the state and the expert's history of private signals at time t. A concatenation of the history of play η_t followed by another history of play $\eta_{t'}$ is a history of play, denoted by $\eta_t\eta_{t'}$, at time $t + t'$. A concatenation of η_t is a history of play $\eta_t\eta_{t'}$ for some $\eta_{t'}$.

In addition, as will be evident in the analysis below, in the probability-zero event that a principal arrives with state belief $\mu = 1$, both types send an agreeing report and the principal chooses an agreeing action on path in any equilibrium. Without loss, I assume that in such arrival, if the principal receives a disagreeing report, then the expert's reputation becomes zero and the principal chooses an agreeing action.

Before turning to the equilibrium analysis, it is instructive to consider the constrained efficient expert's strategy that is consistent with Properties 1 and 2, given that the set of messages is $M = S$. I say that a strategy σ is *constrained efficient* if in each arrival with beliefs (p, μ) and last state s, the informed type reports the current state truthfully, while the uninformed type sends an agreeing report with probability

$$\alpha^{\text{ce}}_{\mu,s} := \begin{cases} 0, & \text{if } \mu < \frac{\Delta_s}{1+\Delta_s}, \\ 1, & \text{otherwise.} \end{cases} \tag{5}$$

Given a constrained efficient strategy, in each arrival, the informed type's report induces a correct action by the principal; the uninformed expert's report induces an action that the principal would prefer to take if he knew that the expert is uninformed. (Recall the discussion following (2) that given a state belief μ and a last state s, the principal optimally chooses the disagreeing action if $\mu < \frac{\Delta_s}{1+\Delta_s}$ and an agreeing action otherwise absent expert advice.) Thus, if the expert's strategy is constrained efficient, then each principal optimally matches his action with the expert's report.

3 Equilibrium

In this section and in Section 4, I present the main analysis. I begin by deriving the basic structure of any equilibrium and showing that an equilibrium exists. I then show that all equilibria exhibit qualitatively identical properties. Using these properties, I address the crisis of expertise and present the main result in Section 4. Proofs are in the appendix.

3.1 Basic Structure and Existence

Proposition 1 below derives the basic properties that both types' payoffs and strategies must satisfy in any equilibrium.

Proposition 1. *In any equilibrium (σ, φ), the following holds.*

1. *Payoff: at the end of each arrival when the principal's benefit is realized, if all past reports are correct, then the uninformed type's payoff is a function $V^U(p, s; \sigma, \varphi)$ of her reputation p and the revealed state s, and is strictly increasing in p; if at least one past report is incorrect, then both types' payoffs are zero.*
2. *Behavior: in each arrival, if all past reports are correct, then the uninformed type's strategy is Markov in the public beliefs (p, μ) and the last state s.*

In any equilibrium, on path, the principals value (only) the informed type's reports. The uninformed type thus benefits from a higher reputation. In addition, the uninformed type's payoff also depends on the revealed state, as the revealed state affects future principals' marginal benefits of receiving advice and thus future wages.

The Markovian structure of the uninformed type's strategy is due to the following observation. When choosing which report to send, the uninformed type takes into account the public state belief since it determines the chance that her report is correct, as well as the reputation and the last state since they characterize the resulting reputation and the revealed state when the principal's benefit is realized.

Proposition 1 also says that both types' payoffs are zero if at least one past report is incorrect. This follows immediately if an expert reports incorrectly when her reputation is short of unity, as the principals then infer that such a report is sent by an uninformed type. In contrast, if an expert reports incorrectly when her reputation is one, then her reputation remains at one by Property 2 and hence, *a priori*, her payoff need not jump down to zero; it jumps down to zero only if the equilibrium strategy prescribes that the informed type no longer sends reports that are valuable to the principals after she deviated to report incorrectly at reputation one. Proposition 1 shows that this must be the case, for otherwise the uninformed type's payoff exhibits a discontinuous jump at reputation one, precluding equilibrium existence.[8] Thus, in any equilibrium, in each arrival with at least one incorrect past report, it is without loss to assume that each type reports both states with equal probabilities irrespective of her current signal, as her payoff is then independent of her current report.

[8] Briefly, the proof shows that if the uninformed type's payoff exhibits a discontinuous jump at reputation one, then there are histories at which a principal arrives, the uninformed type prefers to send an agreeing report with probability that is arbitrarily close to one but not one. Indeed, while Property 2 is common in the literature, it is not void of any problems. See Madrigal, Tan and da Costa Werlang (1987) for an example illustrating how the property might conflict with equilibrium existence, and Nöldeke and van Damme (1990) for further discussion concerning its shortcomings.

Finally, on path, the informed type has no profitable deviation to report incorrectly, as such a report leads to the lowest possible payoff, namely zero. The above observations simplify the task of characterizing equilibria:

Corollary 1. *Any equilibrium is characterized by some agreeing function*

$$\alpha : [0,1] \times \left(\tfrac{1}{2}, 1\right] \times S \to [0,1]$$

that determines the uninformed type's agreeing probability $\alpha_{p,\mu,s}$ in each arrival with public beliefs (p, μ) and last state s, given that all past reports are correct. Conversely, for every agreeing function α, fixing the uninformed type's behavior as given by α, there exists an equilibrium (among other players); moreover, in every such equilibrium, at the end of each arrival when the principal's benefit is realized, if all past reports are correct, then the uninformed type's payoff is a function $V^U(p, s; \alpha)$ of the reputation p and the revealed state s.

By Corollary 1, to prove that an equilibrium exists, it suffices to show that there exists an agreeing function α that maximizes the uninformed type's initial payoff $V^U(p_0, \bar{s}; \alpha)$, and then to verify that when the principals expect the uninformed type's play follows such agreeing function, the uninformed type has no profitable deviation. This yields:

Lemma 1. *An equilibrium exists.*

In general, there are multiple equilibria, each characterized by an agreeing function that constitutes the uninformed type's best response given the informed type's behavior and the principals' expectations. In the remainder of the analysis, I show that all equilibria nonetheless exhibit qualitatively identical properties. Specifically, I fix an arbitrary agreeing function $\bar{\alpha}$ that characterizes an equilibrium. Since the function $\bar{\alpha}$ is arbitrarily chosen, hereafter, I refer to the equilibrium that it characterizes as "the equilibrium." I begin by considering the uninformed type's incentives and then turn to the principals'. I relegate the discussion of the informed type's equilibrium payoff structure to Appendix A, as it is not central to examining the crisis of expertise.

3.2 The Uninformed Expert

In this section, I examine in detail the uninformed type's behavior in each arrival with public beliefs (p, μ) and last state s given that all past reports are correct in the equilibrium. In such arrival, the reputation p must be positive. The uninformed type's agreeing probability $\bar{\alpha}_{p,\mu,s}$ is uniquely determined by her incentive constraint

$$\mu V^U\left(\frac{p}{p+(1-p)\bar{\alpha}_{p,\mu,s}}, s; \bar{\alpha}\right) - (1-\mu)V^U\left(\frac{p}{p+(1-p)(1-\bar{\alpha}_{p,\mu,s})}, \neg s; \bar{\alpha}\right) \begin{cases} \geq 0, & \text{if } \bar{\alpha}_{p,\mu,s} = 1, \\ = 0, & \text{if } \bar{\alpha}_{p,\mu,s} \in (0,1), \\ \leq 0, & \text{if } \bar{\alpha}_{p,\mu,s} = 0. \end{cases} \tag{6}$$

The left side of (6) is the difference between the uninformed type's payoff by agreeing and that by disagreeing. From her perspective, an agreeing report is correct with probability μ, and a correct such report leads to reputation

$$\frac{p}{p+(1-p)\bar{\alpha}_{p,\mu,s}} \tag{7}$$

and revealed state s. Likewise, her payoff by disagreeing follows by noting that such report is correct with probability $1-\mu$ and a correct such report leads to reputation

$$\frac{p}{p+(1-p)(1-\bar{\alpha}_{p,\mu,s})} \tag{8}$$

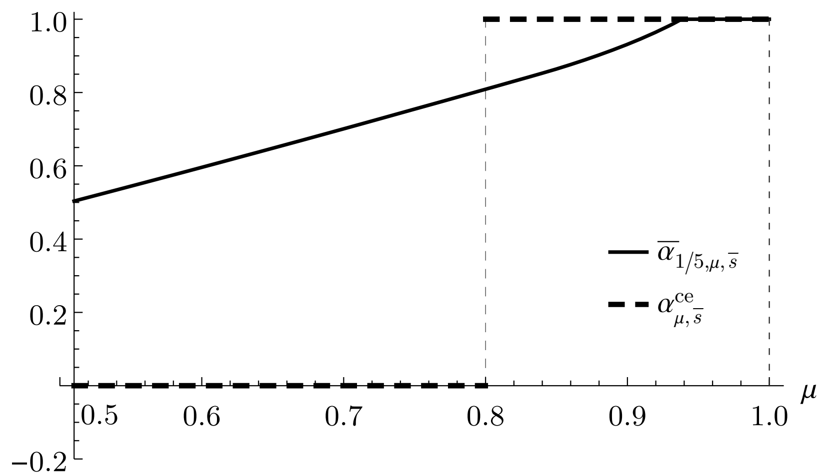
and revealed state $\neg s$. An agreeing probability that is too high (resp., low) violates the incentive constraint; this is because the reputation reward by sending a correct, disagreeing (resp., agreeing) report would then be too high, as the principals expect that such report is more likely to be sent by an informed type, triggering a profitable deviation to disagree (resp., agree) by the uninformed type.

Thus, at times, the uninformed type "gambles for her reputation" by disagreeing with positive probability. From her perspective, the agreeing report is more likely to be correct. Nonetheless, she might disagree to bet on the less likely event that such report is correct, because a correct such report yields a large reputation gain.

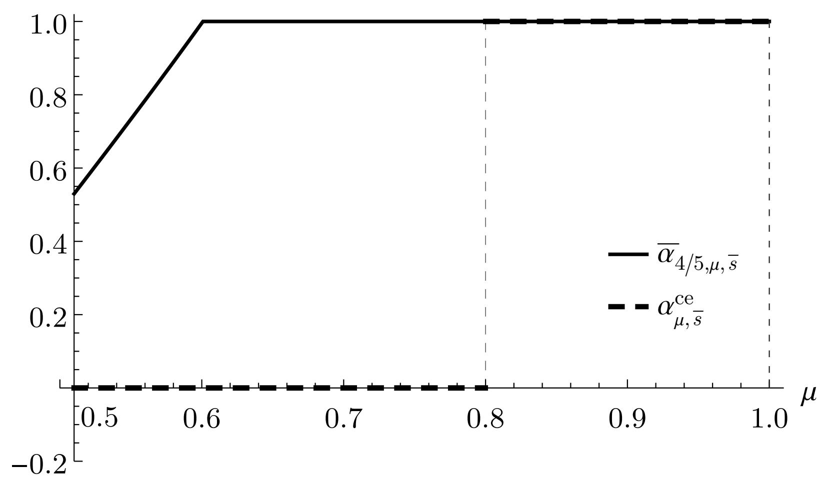
Proposition 2 below describes the structure of the agreeing probability $\bar{\alpha}_{p,\mu,s}$.

Proposition 2. *In the equilibrium, the following holds.*

1. *Higher state belief, more agreeing: for every $p > 0$ and $s \in S$, there is $\mu^* \equiv \mu^*(p,s)$ such that $\bar{\alpha}_{p,\mu,s}$ strictly increases in μ on $\left(\frac{1}{2}, \mu^*\right]$ and is 1 if $\mu \in (\mu^*, 1]$.*
2. *Higher reputation, more agreeing: for every $\mu \in (\frac{1}{2}, 1]$ and $s \in S$, $\bar{\alpha}_{p,\mu,s}$ increases in p on $(0,1]$; further, $\mu^*(p,s)$ strictly decreases in p on $(0,1]$.*



(a) $p = 1/5$



(b) $p = 4/5$

Figure 2: Agreeing probabilities $\bar{\alpha}_{p,\mu,\bar{s}}$

Parameters: $(\bar{b}_{\bar{s}}, \underline{b}_{\bar{s}}, \bar{b}_{\underline{s}}, \underline{b}_{\underline{s}}, \gamma, r) = (1, 0, 2, -2, 1, 1)$

Part 1 of Proposition 2 says that high-quality public information about the state mitigates uninformed gambling. Given a higher state belief, the uninformed type expects that an agreeing report is more likely to be correct and is therefore more likely to agree. In particular, state beliefs above the cutoff μ^* "preempt" the uninformed type's gambling. To illustrate, Figure 2a numerically plots the agreeing probability $\bar{\alpha}_{1/5,\mu,\bar{s}}$ against the state belief μ in a setting with parameters $(\bar{b}_{\bar{s}}, \underline{b}_{\bar{s}}, \bar{b}_{\underline{s}}, \underline{b}_{\underline{s}}, \gamma, r) = (1, 0, 2, -2, 1, 1)$. The figure also plots the constrained efficient agreeing probability $\alpha^{\text{ce}}_{\mu,\bar{s}}$, as defined in (5). Here, relative to the constrained efficient level, the uninformed type "agrees (resp., disagrees) too much" when the state belief is low (resp., high).

Part 2 says that given a higher reputation, the uninformed type agrees with a higher probability. This is because the reputation gain by correctly disagreeing is then smaller. To illustrate, Figure 2b numerically plots the agreeing probability $\bar{\alpha}_{4/5,\mu,\bar{s}}$ against the state belief μ, taking the parameter values as given in Figure 2a.

The uninformed type gambles because of the payoff reward upon correctly disagreeing relative to correctly agreeing that endogenously arises given the equilibrium strategies. In reality, such "contrarian-rewarding" conventions are common in financial markets and entrepreneurship (see, *e.g.*, Baddeley, 2018; Bozanic, Chen and Jung, 2019), and often appear in the form of implicit incentives as in the model. In addition, part 2 of the proposition sheds light on recent empirical evidence by Bozanic et al. (2019) who document that financial analysts who work at lower-tier brokerage houses are more likely to make calls contrary to the public beliefs for their career advancement.

Importantly, as Figure 2 illustrates, the uninformed type's behavior driven by her gambling incentives misaligns with what constrained efficiency (5) calls for. This misalignment, as I show next, causes principals to dismiss expert advice at times.

3.3 The Principals

In this section, I examine the principals' behavior in the equilibrium. In each arrival, if at least one past report is incorrect, then the principal expects the expert's report to contain no information about the state. Thus, as discussed in Section 2, if the state belief is μ and the last state is s, then the principal chooses an agreeing action if $\mu \geq \frac{\Delta_s}{1+\Delta_s}$, and chooses a disagreeing action otherwise. Proposition 3 below describes the principals' behavior in the arrival if all past reports are correct.

Proposition 3. *In the equilibrium, in each arrival with public beliefs (p, μ) and last state s, given that all past reports are correct, there exist $\kappa^A_{p,\mu,s} \in \mathbb{R}$ and $\kappa^D_{p,\mu,s} \in \mathbb{R}$ such that:*

1. *If $\mu < \frac{\Delta_s}{1+\Delta_s}$ and $\bar{\alpha}_{p,\mu,s} > \kappa^A_{p,\mu,s}$, then the principal chooses a disagreeing action irrespective of the expert's report.*

2. *If $\mu \geq \frac{\Delta_s}{1+\Delta_s}$ and $\bar{\alpha}_{p,\mu,s} \leq \kappa^D_{p,\mu,s}$, then the principal chooses an agreeing action irrespective of the expert's report.*

3. *Otherwise, the principal matches his action with the expert's report.*

When the state belief is low (resp., high), namely when $\mu < \frac{\Delta_s}{1+\Delta_s}$ (resp., $\mu \geq \frac{\Delta_s}{1+\Delta_s}$), the principal prefers to choose a disagreeing (resp., agreeing) action absent expert advice. Hence, upon receiving a disagreeing (resp., agreeing) report, he matches his action with the report because the report reinforces his (prior) belief that a disagreeing (resp., agreeing) action is optimal. In contrast, upon receiving an agreeing (resp., disagreeing) report, he dismisses the report and chooses a disagreeing (resp., agreeing) action whenever he believes that the report is sufficiently likely to be sent by an uninformed type, namely whenever $\bar{\alpha}_{p,\mu,s} > \kappa^A_{p,\mu,s}$ (resp., $\bar{\alpha}_{p,\mu,s} \leq \kappa^D_{p,\mu,s}$).

I next use the above results to examine the crisis of expertise and show my main insight that high-quality public information preempts the crisis.

4 The Crisis of Expertise

To set the stage, I define a correspondence $C : (0,1] \times S \rightrightarrows \left(\frac{1}{2}, 1\right]$ such that for each reputation $p > 0$ and for each state $s \in S$,

$$C(p,s) := \left\{\mu \in (\tfrac{1}{2}, 1] : \bar{\alpha}_{p,\mu,s} > \kappa^A_{p,\mu,s} \text{ if } \mu < \tfrac{\Delta_s}{1+\Delta_s} \text{ and } \bar{\alpha}_{p,\mu,s} \leq \kappa^D_{p,\mu,s} \text{ if } \mu \geq \tfrac{\Delta_s}{1+\Delta_s}\right\}.$$

The set contains state beliefs given which a principal acts independently of the expert's report given that all past reports are correct in the equilibrium.[9] I refer to a history of play at which the associated public beliefs (p, μ) and the associated last state s satisfy the condition $\mu \in C(p,s)$ conditional on an informed expert as a *crisis of expertise*. Thus, in a crisis, an arriving principal dismisses the informed expert's correct advice.

Proposition 4 below shows that in the equilibrium, in each arrival on path, given that all past reports are correct, the principal acts independently of the expert's report only if the quality of public information about the state is low (except at the probability-zero event that the state belief is precisely one).

Proposition 4. *The following holds.*

1. *C excludes high enough, inconclusive state beliefs: for every $p > 0$ and $s \in S$, there exists $\bar{\mu} \equiv \bar{\mu}(p, \Delta_s) < 1$ such that $\mu \notin C(p,s)$ if $\mu \in [\bar{\mu}, 1)$. Moreover, $1 \in C(p,s)$.*

[9] Observe that the definition of the correspondence C omits the case when the reputation p is zero. In the equilibrium, on path, the reputation is zero only after the expert has sent an incorrect report, upon which all arriving principals choose actions independently of the expert's reports.

2. *C is a singleton given high enough reputations: for every $s \in S$, there is $\bar{p} \in [0, 1)$ such that $C(p, s) = \{1\}$ if $p \geq \bar{p}$.*

3. *Wage is zero in a crisis: in the equilibrium, in each arrival with public beliefs (p, μ) and last state s, given that all past reports are correct, the wage is zero if and only if $\mu \in C(p, s)$.*

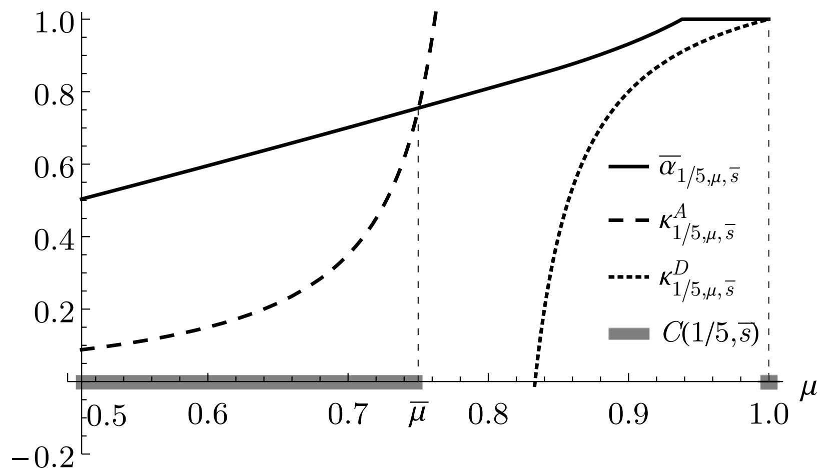
Part 1 of Proposition 4 says that in any arrival where the reputation p is positive, the set $C(p, s)$ excludes high enough interior state beliefs. Thus, unless public information about the state is conclusive, an arriving principal acts on the basis of public information irrespective of the expert's report only if the quality of public information about the state is low. When the state belief is sufficiently high, the principal prefers to choose the agreeing action absent expert advice. Moreover, by Proposition 2, the principal expects that such state belief disciplines the uninformed type's gambling behavior. Thus, the principal matches his action with the expert's report. Finally, the set $C(p, s)$ contains the unit state belief. In the probability-zero event in which a principal arrives with a unit state belief, he chooses the agreeing action for sure.

To illustrate, Figures 3a and 3b highlight the sets $C(1/5, \bar{s})$ and $C(1/5, \underline{s})$, taking the parameter values as given in Figure 2a. Because the payoff stakes at state $\bar{s}$ are smaller than those at state $\underline{s}$, the set $C(1/5, \bar{s})$ (resp., $C(1/5, \underline{s})$) contains interior state beliefs given which an arriving principal chooses the disagreeing action $\underline{s}$ (resp., agreeing action $\underline{s}$) independently of expert advice.[10]

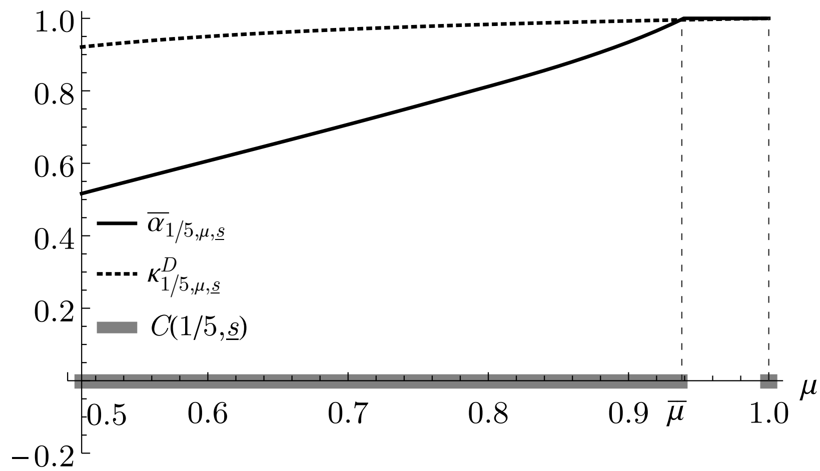
Part 2 says that the correspondence C contains only the unit state belief when the reputation is sufficiently high. Given a high enough reputation, an arriving principal expects that the report is likely to be sent by an informed type and therefore be correct. To illustrate, Figures 4a and 4b highlight the sets $C(4/5, \bar{s})$ and $C(4/5, \underline{s})$.[11]

[10] In Figures 3a and 3b, a crisis of expertise arises for all $\mu < \bar{\mu}$. Moreover, given an interior state belief and a last state $\bar{s}$ (resp., last state $\underline{s}$), a principal who chooses an action independently of the expert's report only chooses a disagreeing (resp., agreeing) action. In general, as Propositions 3 and 4 describe, a crisis can arise only if $\mu < \bar{\mu}$ but may not necessarily arise for all $\mu < \bar{\mu}$. In addition, given any $p > 0$ and $s \in S$, the set $C(p, s)$ may contain interior state beliefs given which an arriving principal chooses an agreeing action independently of the expert's report as well as those given which an arriving principal chooses a disagreeing action independently of the expert's report.

[11] More generally, it is ambiguous whether for each state s, $C(p, s)$ is decreasing (in the sense of set inclusion) in the reputation p. Given a higher reputation, an arriving principal expects that an agreeing report is more likely to be sent by an informed type, because the uninformed type has less gambling incentive. Thus, when the state belief is low, namely when $\mu < \frac{\Delta_s}{1+\Delta_s}$, so that an arriving principal prefers to choose a disagreeing action absent expert advice, the principal is more reluctant to match his action with the expert's report. In contrast, when the state belief is high, namely when



(a) $s = \bar{s}$



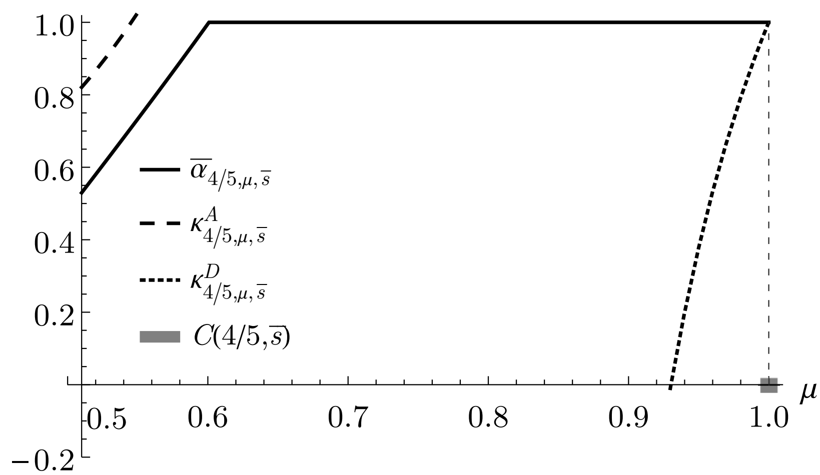
(b) $s = \underline{s}$

Figure 3: The crisis of expertise, $p = 1/5$

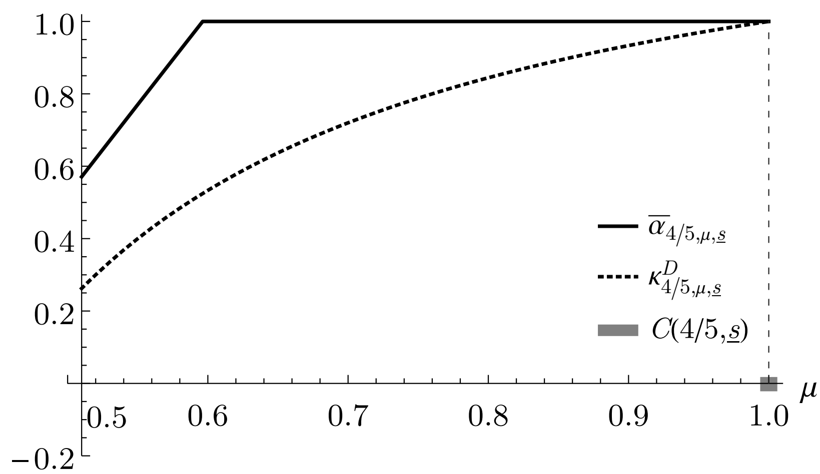
Part 3 says that wages reflect the crises. In the equilibrium, each principal values the expert's report and pays a positive wage if and only if he anticipates to match his action with the report. To illustrate, Figure 5 plots the wage, denoted by $\bar{w}_{1/5,\mu,\bar{s}}$, in an arrival given that all past reports are correct with reputation 1/5 and last state $\bar{s}$ against the state belief μ, taking the parameter values as given in Figure 2a.

Finally, Proposition 5 below turns to the long-run prediction about the crisis of expertise and shows that the crisis is a short-run phenomenon. Let $\mathbf{P}$ denote the probability measure over the set of outcomes of the game induced in the equilibrium.

$\mu \geq \frac{\Delta_s}{1+\Delta_s}$, so that an arriving principal prefers to choose an agreeing action absent expert advice, the principal is more willing to match his action with the report.



(a) $s = \bar{s}$



(b) $s = \underline{s}$

Figure 4: The crisis of expertise, $p = 4/5$

Proposition 5. *In the equilibrium, conditional on the expert being informed, on path, $(p_t)_{t\geq 0}$ is increasing; moreover, for every $\varepsilon > 0$, there exist $T, T' > 0$ such that*

$$\mathbf{P}\left[p_t \geq 1-\varepsilon, \forall t \geq T | \theta = \theta_I\right] \geq 1 - \varepsilon, \tag{9}$$

$$\mathbf{P}\left[\mu_{l_t} \notin C\left(p_t, s_{t-l_t}\right), \forall t \geq T' | \theta = \theta_I\right] \geq 1 - \varepsilon. \tag{10}$$

By (9), the principals eventually learn the informed expert's type; by (10), the crisis of expertise eventually disappears.[12] The proposition follows because the principals

[12]In Section 5, I discuss additional economic forces in practice that might weaken this result, hinting at the importance of policies that mitigate those forces.

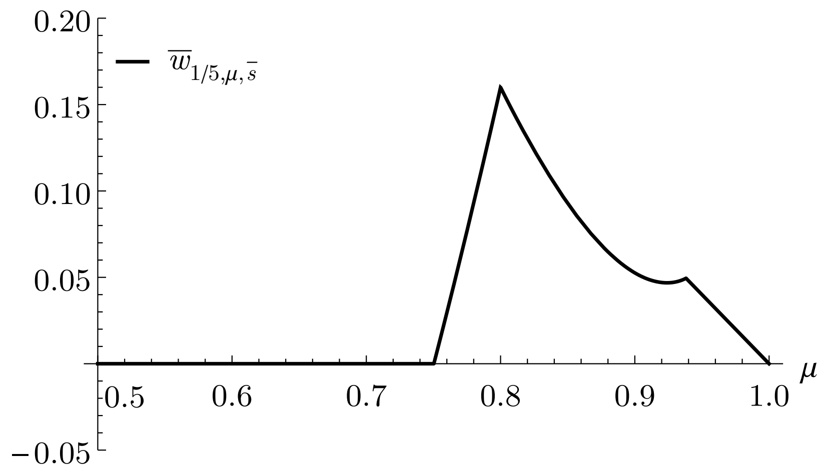


Figure 5: Wages

learn about the informed expert's type given her correct reports over time. Once the informed type's reputation becomes sufficiently high, part 2 of Proposition 4 implies that the crisis of expertise happens with zero probability.

Thus, conditional on the expert being informed, in the long run, the principals match their actions with the expert's report and wages fully reflect the value of the informed expert's advice.[13] These observations shed light on why, despite concerns about the crisis of expertise, reputable experts who have consistently issued correct forecasts are competitively sought after in financial markets (see, *e.g.*, Hong, Kubik and Solomon, 2000; Hong and Kubik, 2003), business consulting (see, *e.g.*, Bourgoin and Harvey, 2018), or entrepreneurship (see, *e.g.*, Baddeley, 2018) in practice.

In the remainder of this section, I discuss further implications of the analysis.

[13] Thus, if the informed expert is arbitrarily patient, her equilibrium payoff tends to the equilibrium payoff that she would have obtained if her type is commonly known at the outset. An analogous learning result can be established for the uninformed type. In the equilibrium, on path, the uninformed type successfully pools with the informed type so long as her reports are correct. Eventually, the uninformed type sends an incorrect report, and the principals learn her type. Formally, conditional on the expert being uninformed, on path, $(p_t)_{t\geq 0}$ is increasing until it jumps down to zero and stays at zero thereafter; moreover, for every $\varepsilon > 0$, there exists $T'' > 0$ such that

$$\mathbf{P}[p_t = 0, \forall t \geq T'' | \theta = \theta_U] \geq 1 - \varepsilon. \tag{11}$$

See Appendix B.8 for a proof.

4.1 The Two Faces of Public Information

The analysis speaks to the two faces of public information for decision-making. On the one hand, public information benefits principals' decision-making by facilitating their learning about both the state and the expert's type.

On the other hand, as Propositions 3 and 4 show, public information about the state allows the uninformed expert to learn how to gamble for her reputation at the expense of the principals, enabling the crisis of expertise. This observation speaks to recent concerns in the financial advice industry. Take, for example, the case of Patricia Russell, who appeared in 2019 on *LinkedIn* as a certified financial planner and a graduate from prestigious universities. Her financial advice was quoted in major media outlets, allowing Patricia to benefit from an undeserving reputation as a financial expert before she was discovered to be fake.[14] The discovery of Russell and other fake financial advisors attracted considerable concerns that the presence of these fake experts diminishes the value of informed financial advisors and their genuine advice.[15] The present results shed light on why these fake experts spring to life on the Internet. Moreover, Proposition 2 and Footnote 13 suggest that these fake experts tend to attract attention by initially making contrarian statements, and gradually tend to herd with public beliefs until their identities are exposed. Although their presence undermines the value of informed experts' genuine advice, the damage that it causes eventually dissipates in view of Proposition 5.

Importantly, as I discuss next, the results suggest why policies that improve the quality of public information can be effective measures for alleviating the crisis of expertise, despite the two faces of public information.

4.2 Complementarity: The Case for Public Information

In recent decades, political campaigns and consumer groups have often unambiguously advocated the need to equip client advisees with high-quality public information (see, *e.g.*, Howells, 2005). The idea is to protect the clients from low-quality expert advice.

My analysis supports such advocacy for a different reason. Specifically, my results identify a novel complementarity between the quality of public information and decision

[14] See "How This Fake Financial Expert Tricked Outlets Into Publishing Her Advice", *Huffpost*, August 1, 2019.

[15] See "Fake 'Expert' Diminishes the Value of Genuine Financial Help", *The Seattle Times*, August 24, 2019.

makers' trust in expert advice. As the discussion following (5) makes clear, when the quality of public information about the state is sufficiently high, the quality of expert advice is high. To see this, recall that constrained efficiency calls for an informed type's report to induce an action that matches her correct report and an uninformed type's report to induce an agreeing action in each arrival. Proposition 4 shows that given sufficiently high quality of public information about the state, arriving principals indeed match their actions with an informed type's correct reports. Moreover, the uninformed expert is less likely to gamble and is thus more likely to induce agreeing actions. In addition, Proposition 5 shows that public information allows principals to learn about the expert's type; such learning about the expert further mitigates the uninformed type's gambling in view of Proposition 2.

My results therefore suggest that policies such as auditing of experts or motivating the production of public information, complemented by high standards of expert certifications, can be effective measures for alleviating the crisis of expertise and improving the principals' decision-making. Consider first frequent auditing. Specifically, consider increasing the principals' arrival intensity from unity to some larger number. The interpretation is that some of these arriving principals are identical to those in the baseline model who care about matching their actions with the states, while the others are inspectors who audit the expert. Given a high enough arrival rate, principals on average are likely to arrive with sufficiently high state beliefs, alleviating the crisis.

Similarly, policies that motivate frequent production of public information about the state can alleviate the crisis. Suppose that public news arrives as an independent Poisson process such that upon each news arrival, the current state is publicly revealed. In this setting, the agreeing function $\bar{\alpha}$ continues to characterize the equilibrium. This is because news arrival is exogenous and does not affect the expert's incentives in each arrival, conditional on the public beliefs and the last state. As in the case of frequent auditing, if news arrives sufficiently frequently, then principals on average arrive with sufficiently high state beliefs, alleviating the crisis.

Finally, higher certification standards that improve experts' reputation can complement the above policies by mitigating uninformed gambling.

To be sure, in practice, there are factors outside of the model that prevents the principals from accessing the complete track records of experts and thus learning about the informed expert's ability, disrupting the short-run nature of the crisis of expertise. Likewise, decision makers might perceive an expert's competence based on the collective reputation of the profession that the expert belongs to, and the collective reputation might never be sufficiently high to preempt the crisis of expertise if decision makers are sufficiently worried about the "bad apples" in the profession. The present results thus point to the importance of policies or information technologies that make experts' individual track records publicly accessible.

5 Concluding Remarks

In this paper, I have developed a stylized model of expert advice to examine the crisis of expertise. The crisis has attracted considerable attention amid recent technological advancement that makes information abundant and publicly accessible. In my model, the crisis is an equilibrium phenomenon where decision makers dismiss the informed expert's correct advice and act solely on the basis of their noisy public information. I find that decision makers ignore valuable expert advice only when the quality of the public information is low. My analysis points to policies that improve the quality of public information as effective measures for alleviating the crisis.

I close the paper with a brief discussion of some of the assumptions that I made in my analysis. Some assumptions are plainly for convenience. For example, I have assumed that whenever the principals do not value expert advice, they still pay the expert a wage of zero and receive advice. One can alternatively assume that whenever the principals do not value expert advice, the expert cannot send messages in their arrivals and the principals choose actions absent expert advice. In such setting, in each arrival where the principal does not find expert advice to be valuable and hence the expert does not send a message, the principal would choose his action on the basis of the public state belief and the last state as in the baseline model where the principal would have chosen an action after receiving the report. In addition, because there exists a positive measure of state beliefs given which an arriving principal matches his action with the expert's report for each positive reputation and last state, the long-run predictions of the baseline model carry over to this alternative setting. Nonetheless, it would take "longer" for the principals to learn the expert's type and for the public beliefs to permanently exit the crisis region (for each state) in this alternative setting than in the baseline model.

I have also assumed that the informed expert only receives a signal about the state whenever a principal arrives. Alternatively, one can assume that the informed expert observes the state online, that is, she receives a signal that reveals the state at each time. The present analysis carries over as each arriving principal is only concerned about the current state for his decision-making, and the informed expert truthfully reports the correct current signal in each arrival in the equilibrium. Moreover, I have assumed that the public information available to the principals and the expert consists only of past advice and past action outcomes. In reality, there may be public shocks, *e.g.*, a pandemic, that arrive at random times and produce information about the state. Suppose that high shocks and low shocks arrive as independent Poisson processes such that upon each arrival of a high (resp., low) shock, the state publicly jumps to $\bar{s}$ (resp., $\underline{s}$). In this setting, as in the public-news setting discussed in Section 4.2, the agreeing function $\bar{\alpha}$ continues to characterize the equilibrium.

On the contrary, an important assumption that drives my results, which is standard in the literature on reputational cheap talk, is that the principals can verify the quality of the expert's past reports via the revealed states. For instance, the principals might learn the states directly by observing past principals' actions and their action outcomes, or indirectly from an evaluator who monitors the expert's interactions with past principals. An evaluator is often explicitly described in existing reputational cheap talk models, and the competitive labor market is usually thought of as one example of such evaluator. In contrast, my analysis does not speak to settings where public information about past outcomes is limited and principals can only assess the quality of the expert's past reports via certain indicators that are subject to the expert's strategic manipulation. Some recent work in the literature explores settings where the state might not be known even to "good" experts (see, *e.g.*, Backus and Little, 2020). Some other recent work in the reputational cheap talk literature considers such settings where the expert influences the feedback channels through which the principals learn her ability. For example, in Rüdiger and Vigier (2019), principals learn about the expert only through trade orders that are influenced by the expert's messages in a financial market; in Camara and Dupuis (2021), principals' ability to learn about the expert is assumed to depend on their interim beliefs about the state that are influenced by the expert's messages.

Finally, I have assumed that the state is binary. If the set of states is richer, for example, if the state is a real number and if each principal has a quadratic loss function such that he strictly prefers an action that is closer to the state, then the fundamental notion of agreeing and disagreeing, and more importantly the notion of a crisis of expertise, do not carry over. The nature of the problem is then different and is beyond the scope of this paper. I view this as a promising venue for future research.

Appendices

A Informed Type's Equilibrium Payoffs

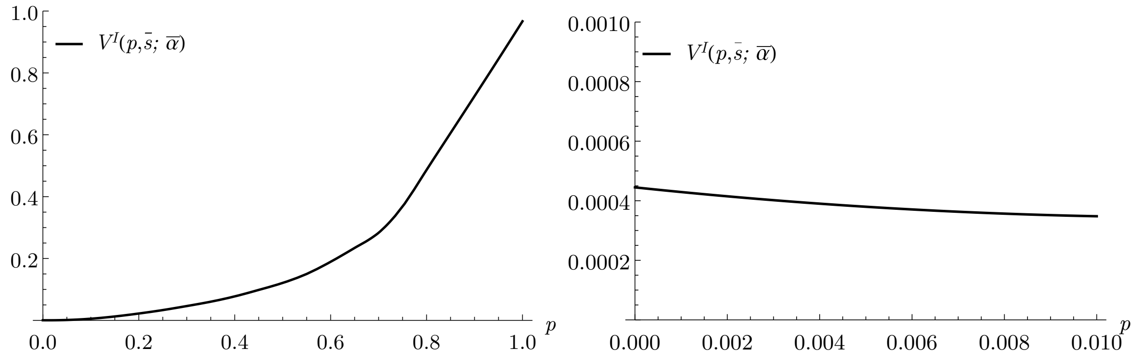
Proposition 6 describes the informed type's payoff in the equilibrium characterized by $\bar{\alpha}$. Different from the uninformed type, the informed type unambiguously benefits from a higher reputation only when the reputation is sufficiently high:

Proposition 6. *In the equilibrium, the informed type's payoff at the end of each arrival when the principal's benefit is realized is a function $V^I(p, s; \bar{\alpha})$ of her reputation p and the revealed state s. Moreover, there exists $p^\dagger \in [0, 1)$ such that for each state s, $V^I(p, s; \bar{\alpha})$ is strictly increasing in p on $[p^\dagger, 1]$.*

A higher reputation has two implications for each arriving principal's willingness to pay the expert. First, the principal believes that the report is more likely to be sent by an informed type. This has a positive effect on the wage. Second, the uninformed type agrees with a higher probability. This has a positive (resp., negative) effect on the wage when the state belief is high (resp., low) so that the principal would prefer to choose an agreeing (resp., disagreeing) action if he knew that the expert is uninformed.

When the reputation is high, the first, unambiguously positive effect dominates the second effect, and a higher reputation results in a higher payoff for the informed type. In contrast, when the reputation is low, the net effect of a higher reputation on the informed expert's payoff is ambiguous. Figure 6 illustrates that the negative effect might dominate the positive effect. Figure 6a plots the informed type's payoff $V^I(p, \bar{s}; \bar{\alpha})$ over reputations $p \in [0, 1]$, showing that the payoff strictly increases in p over large reputations. Figure 6b plots the same function over a subset of small reputations, showing that the payoff strictly decreases in p over these reputations.

B Proofs



(a) all reputations $p \in (0,1]$ **(b)** low reputations $p \in (0, 0.01]$

Figure 6: Informed type's payoff $V^I(p, \bar{s}; \bar{\alpha})$

Parameters: $(\bar{b}_{\bar{s}}, \underline{b}_{\bar{s}}, \bar{b}_{\underline{s}}, \underline{b}_{\underline{s}}, \gamma, r) = (1, -1, 6, -6, 8, 1)$

B.1 Proof of Proposition 1

Fix an equilibrium (σ, φ). Let $\mathcal{V}^U(h^U; \sigma, \varphi)$ denote the uninformed type's payoff at history h^U where an arrival ends and all past reports are correct. To prove the first part, I begin by showing that $\mathcal{V}^U(h^U; \sigma, \varphi)$ strictly increases in the reputation p associated with h^U. To emphasize the reputation p and the revealed state s associated with h^U, I write $\mathcal{V}^U(h^U; \sigma, \varphi)$ as $\mathcal{V}^U(h^U; p, s; \sigma, \varphi)$. Fix a continuation strategy at h^U by the uninformed type, and denote it by $\sigma^U|_{h^U}$. At h^U, the expected discounted wage in the next arrival is $\int_0^\infty \mathrm{e}^{-(r+1)l}\, w_{p,\mu_l,s}(\alpha_l)\,\mathrm{d}l$, where

$$\begin{aligned} w_{p,\mu_l,s}(\alpha_l) := \max\Big[0, p(\mu_l \bar{b}_s + (1-\mu_l)\bar{b}_{\neg s}) \\ + (1-p)\Big(\mu(\alpha_l \bar{b}_s + (1-\alpha_l)\underline{b}_s) + (1-\mu_l)(\alpha_l \underline{b}_{\neg s} + (1-\alpha_l)\bar{b}_{\neg s})\Big) - b_*(\mu_l, s)\Big] \end{aligned} \tag{12}$$

represents the wage that the next principal pays when he expects that an uninformed type agrees with probability α_l (as induced by the strategy σ) given a time-lapse l. Suppose that the associated reputation p at history h^U increases to some $p' > p$ and consider the continuation game. By choosing the continuation strategy $\sigma^U|_{h^U}$ fixed above (which is feasible), the uninformed expert's payoff is equal to some $\hat{\mathcal{V}}^U(h^U; p', s; \sigma, \varphi) > \mathcal{V}^U(h^U; p, s; \sigma, \varphi)$, as $\int_0^\infty \mathrm{e}^{-(r+1)l}\, w_{p',\mu_l,s}(\alpha_l)\,\mathrm{d}l > \int_0^\infty \mathrm{e}^{-(r+1)l}\, w_{p,\mu_l,s}(\alpha_l)\,\mathrm{d}l$ and the probability that her report is correct in each future arrival remains the same. Since the equilibrium continuation payoff $\mathcal{V}^U(h^U; p', s; \sigma, \varphi)$ must be at least $\hat{\mathcal{V}}^U(h^U; p', s; \sigma, \varphi)$, it then follows that the uninformed type's payoff strictly increases in the reputation.

In turn, given any two histories $h^U, \tilde{h}^U$ at which an arrival ends with an identical reputation p and revealed state s, $\mathcal{V}^U(h^U; \sigma, \varphi) = \mathcal{V}^U(\tilde{h}^U; \sigma, \varphi) =: V^U(p, s; \sigma, \varphi)$ for some function $V^U : [0,1] \times S \to \mathbb{R}_+$, for otherwise the uninformed expert has a profitable deviation at either h^U or $\tilde{h}^U$.

Next, it is clear that in any equilibrium, both types' payoffs upon sending an incorrect report are zero, if the incorrect report is sent when the expert's reputation is short of unity, as discussed in the main text. Before showing that this is also the case when the expert's reputation is one, I first turn to the second part of the proposition and show that in each arrival with beliefs (p, μ) and last state s where $p < 1$, if all past reports are correct, then the uninformed type's strategy is Markov in (p, μ, s). In such arrival, where the uninformed type's history is some h^U, the uninformed expert's agreeing probability, denoted by α_{h^U}, must satisfy

$$\begin{aligned} \alpha_{h^U} \in \arg\max_{\hat{\alpha}_{h^U} \in [0,1]} \Bigg[& \mu \hat{\alpha}_{h^U} V^U\left(\frac{p}{p + (1-p)\alpha_{h^U}}, s; \sigma, \varphi\right) \\ & + (1-\mu)(1-\hat{\alpha}_{h^U}) V^U\left(\frac{p}{p + (1-p)(1-\alpha_{h^U})}, \neg s; \sigma, \varphi\right)\Bigg], \end{aligned} \tag{13}$$

where the reputations upon correctly agreeing and correctly disagreeing in the objective are computed based on the principals' equilibrium expectation that the uninformed type agrees with probability α_{h^U}. Since all past reports are correct, the reputation p is positive. Moreover, since $V^U(p, s; \sigma, \varphi)$ is strictly increasing in p for each state s, α_{h^U} must be such that for some cutoff $\mu^*(p, s) \in \left[\frac{1}{2}, 1\right)$,

$$\alpha_{h^U} = \begin{cases} 1, & \text{if } \mu \geq \mu^*(p, s), \\ \alpha^*_{p,\mu,s}, & \text{otherwise,} \end{cases}$$

where $\alpha^*_{p,\mu,s}$ uniquely solves

$$\mu V^U\left(\frac{p}{p + (1-p)\alpha^*_{p,\mu,s}}, s; \sigma, \varphi\right) = (1-\mu) V^U\left(\frac{p}{p + (1-p)(1-\alpha^*_{p,\mu,s})}, \neg s; \sigma, \varphi\right).$$

Thus, the uninformed expert's equilibrium strategy in the arrival is Markov in the public beliefs (p, μ) and the last state s, given that all past reports are correct. In turn, by (12), the equilibrium wage in the arrival is plainly a function of (p, μ, s).

I now return to show that in any equilibrium, both types' payoffs upon sending an incorrect report when the reputation is one are zero. In the following, I show that this is the case for the uninformed type, which immediately implies that this must also be the case for the informed type. Suppose, towards a contradiction, that there exists an equilibrium (σ, φ) in which in some arrival with beliefs (p, μ) and last state s, where $p < 1$, the following holds: if the uninformed type deviates from her equilibrium prescribed behavior and sends a correct report, then her reputation jumps up to one and at reputation one, there is a positive measure of time-lapses such that in the next arrival with one such time-lapse, if the uninformed type sends an incorrect report, then her payoff remains positive. The uninformed expert's payoff $V^U(\cdot, s; \sigma, \varphi)$ must then be discontinuous at $p = 1$. To see the discontinuity, note that if $p \in [0, 1)$, then

$$\begin{aligned} V^U(p, s; \sigma, \varphi) = \int_0^\infty \mathrm{e}^{-(r+1)l} \Bigg[& r w^{\sigma,\varphi}_{p,\mu_l,s} + \mu_l \alpha_{p,\mu_l,s} V^U\left(\frac{p}{p + (1-p)\alpha_{p,\mu_l,s}}, s; \sigma, \varphi\right) \\ & + (1-\mu_l)(1-\alpha_{p,\mu_l,s}) V^U\left(\frac{p}{p + (1-p)(1-\alpha_{p,\mu_l,s})}, \neg s; \sigma, \varphi\right)\Bigg] \mathrm{d}l, \end{aligned}$$

where $w^{\sigma,\varphi}_{p,\mu,s}$ denote the equilibrium wage in an arrival with beliefs (p, μ) and last state s given that all past reports are correct, while if $p = 1$, then

$$\begin{aligned} V^U(1, s; \sigma, \varphi) = \int_0^\infty \mathrm{e}^{-(r+1)l} \Bigg[& r w^{\sigma,\varphi}_{1,\mu_l,s} + \mu_l \Big(\alpha_{1,\mu_l,s} V^U(1, s; \sigma, \varphi) + (1-\alpha_{1,\mu_l,s}) \underline{V}^U(1, \neg s; \sigma, \varphi)\Big) \\ & + (1-\mu_l)\Big((1-\alpha_{1,\mu_l,s}) V^U(1, \neg s; \sigma, \varphi) + \alpha_{1,\mu_l,s} \underline{V}^U(1, s; \sigma, \varphi)\Big)\Bigg] \mathrm{d}l, \end{aligned}$$

where $\underline{V}^U(1, s; \alpha)$ denotes the uninformed expert's payoff after sending one incorrect report at reputation one and the state is revealed to be s, and by assumption,

$$\int_0^\infty \mathrm{e}^{-(r+1)l} \left[\mu_l (1-\alpha_{1,\mu_l,s}) \underline{V}^U(1, \neg s; \sigma, \varphi) + (1-\mu_l)\alpha_{1,\mu_l,s} \underline{V}^U(1, s; \sigma, \varphi)\right] \mathrm{d}l > 0.$$

This shows the discontinuity.

To yield the desired contradiction, in the following, I show that in any equilibrium (σ, φ), the payoff $V^U(p, s; \sigma, \varphi)$ must be continuous on $p \in [p_0, 1]$ for each state s. If at some state s, the payoff $V^U(p, s; \sigma, \varphi)$ is discontinuous at some $p^\dagger \in [p_0, 1]$, then, because the uninformed expert's equilibrium payoff is monotone in p, for each

$p \in \left[p_0, p^\dagger\right]$, there exists $\hat{\alpha}^\dagger \in [0,1]$ such that $\frac{p}{p+(1-p)\hat{\alpha}^\dagger} = p^\dagger$ and

$$\lim_{\hat{\alpha}\uparrow\hat{\alpha}^\dagger} V^U\left(\frac{p}{p+(1-p)\hat{\alpha}}, s; \sigma, \varphi\right) > \lim_{\hat{\alpha}\downarrow\hat{\alpha}^\dagger} V^U\left(\frac{p}{p+(1-p)\hat{\alpha}}, s; \sigma, \varphi\right). \tag{14}$$

Observe also that given any $p < 1$, there exists a state $\hat{s}$ such that for a set of state beliefs μ with positive Lebesgue measure,

$$\mu V^U(p, \hat{s}; \sigma, \varphi) < (1-\mu) V^U(1, \neg\hat{s}; \sigma, \varphi). \tag{15}$$

Since the state s is arbitrarily chosen, assume without loss that $\hat{s} = s$. Consider then the arrival with reputation $p \in [p_0, 1)$ and last state s. By (15) and by monotonicity of $V^U(p, \cdot; \sigma, \varphi)$ in p, there is a positive measure of state beliefs μ such that

$$\begin{aligned}
\lim_{\hat{\alpha}\uparrow\hat{\alpha}^\dagger} \mu V^U\left(\frac{p}{p+(1-p)\hat{\alpha}}, s; \sigma, \varphi\right) &> \lim_{\hat{\alpha}\uparrow\hat{\alpha}^\dagger} (1-\mu)\left(\frac{p}{p+(1-p)(1-\hat{\alpha})}, \neg s; \sigma, \varphi\right), \\
\lim_{\hat{\alpha}\downarrow\hat{\alpha}^\dagger} \mu V^U\left(\frac{p}{p+(1-p)\hat{\alpha}}, s; \sigma, \varphi\right) &< \lim_{\hat{\alpha}\downarrow\hat{\alpha}^\dagger} (1-\mu)\left(\frac{p}{p+(1-p)(1-\hat{\alpha})}, \neg s; \sigma, \varphi\right).
\end{aligned}$$

Because $V^U(p, s; \sigma, \varphi)$ is monotone in p, there is no agreeing probability $\hat{\alpha} \in [0,1]$ given which the uninformed type's incentive constraint (6) holds. Thus, because the reputation p in each arrival given that all past reports are correct must be at least p_0, $V^U(p, s; \sigma, \varphi)$ must be continuous in $p \in [p_0, 1]$; this shows that both types' payoffs must be zero upon reporting incorrectly at reputation one.

Finally, to complete the proof, it suffices to show that the uninformed expert's strategy at a history h^U, where a principal arrives with public beliefs (p, μ) and last state s where the reputation is $p = 1$ and all past reports are correct, is also Markovian in (p, μ, s). This must be the case since an incorrect report in such arrival leads to zero payoff for the expert, and therefore the uninformed expert's agreeing probability in such arrival is also characterized by the solution to (13) as above.

B.2 Proof of Corollary 1

The proof is omitted, as the claim is clear from the main text.

B.3 Proof of Lemma 1

Let $L^\infty((0,1] \times \left(\frac{1}{2},1\right] \times S;\mathbb{R})$ be the set of Lebesgue measurable functions that maps from $(0,1]\times\left(\frac{1}{2},1\right]\times S$ to $\mathbb{R}_+$, and let $L_B = \left\{\alpha \in L^\infty\left((0,1] \times \left(\frac{1}{2},1\right] \times S;\mathbb{R}\right) : \|\alpha\|_\infty \leq 1\right\}$ denote the unit ball. The uninformed expert's payoff in an equilibrium that is best for her is the value of the following optimization problem, maximizing the uninformed expert's payoff over agreeing functions that the uninformed expert plays and the principals expect the uninformed expert to play:

$$\sup_{\alpha \in L_B} V^U(p_0, \bar{s}; \alpha). \tag{$*$}$$

The maximum in $(*)$ is attained, because the objective is weak-* continuous in α, and L_B is weak-* compact.[16] Let $\bar{\alpha}$ denote a solution to $(*)$. It remains to show that $\bar{\alpha}$ is well-defined as the uninformed type's best response when the principals expect the uninformed type to play $\bar{\alpha}$. Specifically, I show that there exists $\bar{\alpha}_{p,\mu,s} \in [0,1]$ that satisfies (6) for each (p,μ,s) with $p > 0$. Observe that the agreeing function $\bar{\alpha}$ that the principals expect the uninformed type to play must be continuous in (p,μ): as the proof of Proposition 1 shows, if $\bar{\alpha}$ characterizes an equilibrium, then the function $V^U(p,s;\bar{\alpha})$ must be continuous in $p \in [p_0, 1]$. In addition, since the reputation in each arrival given that all past reports are correct satisfies $p \in [p_0,1]$, it is without loss to assume that $V^U(p,s;\bar{\alpha})$ is continuous in $p \in (0,1]$. In turn, the probability $\bar{\alpha}_{p,\mu,s}$ that uniquely solves (6) must also be continuous in $(p,\mu) \in [p_0,1] \times \left(\frac{1}{2},1\right]$.

Because $V^U(p,s;\bar{\alpha})$ is strictly increasing in p, to show that there exists $\bar{\alpha}_{p,\mu,s} \in [0,1]$ that satisfies (6) for each (p,μ,s) with $p \in (0,1]$, it suffices to show that $V^U(p,s;\bar{\alpha})$ is indeed continuous in $p \in (0,1]$. Suppose that the principals expect that the uninformed expert's behavior is described by the agreeing function $\bar{\alpha}$. I show that for each $p \in (0,1]$, the payoff $V^U(p,s;\bar{\alpha})$ is the unique fixed point of the contraction T^U that maps from the set of continuous functions $f : (0,1] \times S \to \mathbb{R}_+$ to itself:

$$\begin{aligned} T^U f(p,s) = \max_{\hat{\alpha}:\left(\frac{1}{2},1\right]\to[0,1]} \int_0^\infty \mathrm{e}^{-(r+1)l} \Bigg[& r\bar{w}_{p,\mu_l,s} + \mu_l \hat{\alpha}_{\mu_l} f\left(\frac{p}{p+(1-p)\bar{\alpha}_{p,\mu,s}}, s\right) \\ & + (1-\mu_l)(1-\hat{\alpha}_{\mu_l}) f\left(\frac{p}{p+(1-p)(1-\bar{\alpha}_{p,\mu_l,s})}, \neg s\right)\Bigg] \mathrm{d}l, \end{aligned}$$

[16] Compactness follows as $L^\infty\left((0,1] \times (\frac{1}{2},1] \times S;\mathbb{R}\right)$ is the dual of $L^1\left((0,1] \times (\frac{1}{2},1] \times S;\mathbb{R}\right)$ and L_B is weak-* compact by Alaoglu's theorem (*e.g.*, Aliprantis and Border, 2006, Theorem 6.21).

where $\bar{w}_{p,\mu_l,s} := w_{p,\mu_l,s}(\bar{\alpha}_{p,\mu_l,s})$, namely the wage (12) in which the agreeing probability is induced by $\bar{\alpha}$ in an arrival with beliefs (p, μ_l) and last state s given that all past reports are correct. Because $\bar{\alpha}_{p,\mu,s}$ is continuous in $(p,\mu) \in (0,1] \times \left(\frac{1}{2}, 1\right]$, so is $\bar{w}_{p,\mu_l,s}$. Now, the objective on the right side is weak-* continuous in $\hat{\alpha}$ and continuous in p, and again the set of functions $\hat{\alpha} : \left(\frac{1}{2}, 1\right] \to [0,1]$ is weak-* compact. Moreover, the solution to this objective is single-valued. By the theorem of the maximum (*e.g.*, Stokey, Lucas and Prescott, 1989, Theorem 3.6, p. 62), $V^U(p, s; \bar{\alpha})$ is continuous in $p \in (0,1]$.

B.4 Proof of Proposition 2

Part 1 follows from the incentive constraint (6) as well as Proposition 1 that $V^U(\cdot, s; \bar{\alpha})$ is strictly increasing for each s. Similarly, in part 2, that $\bar{\alpha}_{p,\mu,s}$ increases in p on $(0,1]$ follows from the incentive constraint (6) as well as Proposition 1 that $V^U(p, \cdot; \bar{\alpha})$ is strictly increasing in p. Finally, to see that $\mu^*(p,s)$ is strictly decreasing in p on $(0,1]$, it suffices to observe that $\mu^*(p,s) = \min\left\{\mu \in [\frac{1}{2}, 1] : \mu V^U(p, s; \bar{\alpha}) \geq (1-\mu) V^U(1, \neg s; \bar{\alpha})\right\}$, and V^U is monotone in the reputation p.

B.5 Proof of Proposition 3

In each arrival with beliefs (p, μ) and last state s, if all past reports are correct, then the principal matches his action with an agreeing report if and only if his payoff by choosing an agreeing action exceeds that by choosing a disagreeing action:

$$\begin{aligned} &\frac{p\mu}{p\mu + (1-p)\bar{\alpha}_{p,\mu,s}} \times \bar{b}_s + \left(1 - \frac{p\mu}{p\mu + (1-p)\bar{\alpha}_{p,\mu,s}}\right) \times \left(\mu \bar{b}_s + (1-\mu)\underline{b}_{\neg s}\right) \\ &\quad \geq \frac{p\mu}{p\mu + (1-p)\bar{\alpha}_{p,\mu,s}} \times \underline{b}_s + \left(1 - \frac{p\mu}{p\mu + (1-p)\bar{\alpha}_{p,\mu,s}}\right) \times \left(\mu \underline{b}_s + (1-\mu)\bar{b}_{\neg s}\right). \end{aligned} \tag{16}$$

The constraint is violated if and only if

$$\mu < \frac{\Delta_s}{1 + \Delta_s} \quad \text{and} \quad \bar{\alpha}_{p,\mu,s} > \kappa^A_{p,\mu,s} := \frac{\mu p}{(1-p)((1-\mu)\Delta_s - \mu)}. \tag{17}$$

The principal matches his action with a disagreeing report if and only if $\mu < 1$ (by assumption, see Section 2) and his payoff by choosing a disagreeing action exceeds that by choosing an agreeing action:

$$\frac{p(1-\mu)}{p(1-\mu)+(1-p)(1-\bar{\alpha}_{p,\mu,s})}\times\bar{b}_{\neg s}+\left(1-\frac{p(1-\mu)}{p(1-\mu)+(1-p)(1-\bar{\alpha}_{p,\mu,s})}\right)(\mu\underline{b}_s+(1-\mu)\bar{b}_{\neg s})$$
$$>\frac{p(1-\mu)}{p(1-\mu)+(1-p)(1-\bar{\alpha}_{p,\mu,s})}\times\underline{b}_{\neg s}+\left(1-\frac{p(1-\mu)}{p(1-\mu)+(1-p)(1-\bar{\alpha}_{p,\mu,s})}\right)(\mu\bar{b}_s+(1-\mu)\underline{b}_{\neg s}). \tag{18}$$

These two conditions are violated if and only if

$$\mu\geq\frac{\Delta_s}{1+\Delta_s}\quad\text{and}\quad\bar{\alpha}_{p,\mu,s}\leq\kappa^D_{p,\mu,s}:=\frac{\mu(1-p)-\Delta_s(1-\mu)}{(1-p)(\mu-\Delta_s(1-\mu))}. \tag{19}$$

B.6 Proof of Proposition 4

Consider Part 1. By the definitions of $\bar{\alpha}$, (17) and (19), for every (p,s), there exists $\bar{\mu}\in[\mu^*,1)$ such that for every $\mu\in[\bar{\mu},1)$, $\mu\geq\frac{\Delta_s}{1+\Delta_s}$, $\bar{\alpha}_{p,\mu,s}=1$ and $\kappa^D_{p,\mu,s}<1$, and therefore $\mu\notin C(p,s)$. Finally, that for every (p,s) with $p>0$, $1\in C(p,s)$, follows by definition of C, (17), and (19). Consider next Part 2. Given each state s, there exists $p'<1$ such that for every $p\in[p',1]$, $\bar{\alpha}_{p,\mu,s}\leq\kappa^A_{p,\mu,s}$ for every $\mu\in\left(\frac{1}{2},\frac{\Delta_s}{1+\Delta_s}\right]$, and there exists $p''<1$ such that for every $p\in[p'',1]$, $\bar{\alpha}_{p,\mu,s}>\kappa^D_{p,\mu,s}$ for every $\mu\in\left(\frac{\Delta_s}{1+\Delta_s},1\right]$. Setting $\bar{p}=\max(p',p'')$ completes the proof of Part 2. Consider finally Part 3. In each arrival with beliefs (p,μ) and last state s, given that all past reports are correct, $\bar{w}_{p,\mu,s}$ is given by (12) in which the uninformed type's agreeing probabilities are induced by $\bar{\alpha}$. It is then straightforward to verify the claim.

B.7 Proof of Proposition 5

Let $\mathbf{P}^I$ denote the probability measure over the set of outcomes of the game induced in the equilibrium by conditioning on an informed type. Conditional on an informed type, given any history of play at which a principal arrives with beliefs (p,μ) and last state s, the reputation once the state is revealed is either (7) or (8), where at least one of them is strictly larger than p if $p<1$. Thus, $(p_t)_{t\geq 0}$ is increasing. Next, to show (9), fix an arbitrary history of play and a sequence of arrival times $(t_i)_{i=0}^{\infty}$. Given any arrival time t_i, for every $\xi>0$, there exists another arrival time $t_{i'}$, where $i'-i$ is sufficiently large, such that, $\mathbf{P}^I[p_{t_i}=p_{t_{i'}}]<\xi$. This is because for $p_{t_i}=p_{t_{i'}}$, it must hold that in all arrivals in between t_i and $t_{i'}$, the agreeing states must be correct and the time-lapses are sufficiently small such that the uninformed type agrees with probability one. This event happens with negligible probability if $i' - i$ is large. Hence, for every $\varepsilon > 0$, there exists $T > 0$ such that there is a large enough n given which

$$\mathbf{P}\left[N_t < n, \forall t \geq T\right] = 0, \quad \mathbf{P}^I\left[\{p_t \geq 1-\varepsilon, \forall t \geq T\} \cap \{N_t \geq n, \text{ for some } t \geq T\}\right] \geq 1-\varepsilon.$$

Thus,

$$\begin{aligned}\mathbf{P}^I\left[p_t \geq 1-\varepsilon, \forall t \geq T\right] &= \mathbf{P}^I\left[\{p_t \geq 1-\varepsilon, \forall t \geq T\} \cap \{N_t < n, \forall t \geq T\}\right] \\ &\quad + \mathbf{P}^I\left[\{p_t \geq 1-\varepsilon, \forall t \geq T\} \cap \{N_t \geq n, \text{ for some } t \geq T\}\right] \\ &= 0 + \mathbf{P}^I\left[\{p_t \geq 1-\varepsilon, \forall t \geq T\} \cap \{N_t \geq n, \text{ for some } t \geq T\}\right] \geq 1-\varepsilon.\end{aligned}$$

Analogously, to show (10), fix $\bar{p}$ such that for each $p \geq \bar{p}$, $C(p,s) = \{1\}$ for each state s. For every $\varepsilon > 0$, there exists $T' > 0$ such that there is a large enough n' given which

$$\mathbf{P}\left[N_t < n', \forall t \geq T'\right] = 0, \quad \mathbf{P}^I\left[\{p_t \geq \bar{p}, \forall t \geq T'\} \cap \{N_t \geq n', \text{ for some } t \geq T'\}\right] \geq 1-\varepsilon.$$

Thus,

$$\begin{aligned}\mathbf{P}^I\left[\mu_{l_t} \notin C(p_t, s_{t-l_t}), \forall t \geq T'\right] &\geq \mathbf{P}^I\left[p_t \geq \bar{p}, \forall t \geq T'\right] \\ &= \mathbf{P}^I\left[\{p_t \geq \bar{p}, \forall t \geq T'\} \cap \{N_t < n', \forall t \geq T'\}\right] \\ &\quad + \mathbf{P}^I\left[\{p_t \geq \bar{p}, \forall t \geq T'\} \cap \{N_t \geq n', \text{ for some } t \geq T'\}\right] \\ &= 0 + \mathbf{P}^I\left[\{p_t \geq \bar{p}, \forall t \geq T'\} \cap \{N_t \geq n', \text{ for some } t \geq T'\}\right] \\ &\geq 1-\varepsilon.\end{aligned}$$

B.8 Proof of Claim in Footnote 13

Let $\mathbf{P}^U$ denote the probability measure over the set of outcomes of the game induced in the equilibrium by conditioning on an uninformed type. Because the uninformed type sends an incorrect message in each arrival with positive probability (except in the zero-probability event that the state belief is one in the arrival), for every $\varepsilon > 0$, there exists $T'' > 0$ such that there is n'' given which

$$\mathbf{P}\left[N_t < n'', \forall t \geq T''\right] = 0, \quad \mathbf{P}^U\left[\{p_t = 0, \forall t \geq T''\} \cap \{N_t \geq n'', \text{ for some } t \geq T''\}\right] \geq 1-\varepsilon.$$

Then

$$\begin{aligned}\mathbf{P}^U\left[p_t=0,\forall t\geq T''\right] &= \mathbf{P}^U\left[\{p_t=0,\forall t\geq T''\}\cap\{N_t<n'',\forall t\geq T''\}\right]\\ &\quad+\mathbf{P}^U\left[\{p_t=0,\forall t\geq T''\}\cap\{N_t\geq n'',\text{ for some } t\geq T''\}\right]\\ &=0+\mathbf{P}^U\left[\{p_t=0,\forall t\geq T''\}\cap\{N_t\geq n'',\text{ for some } t\geq T''\}\right]\geq 1-\varepsilon.\end{aligned}$$

B.9 Proof of Proposition 6

In equilibrium, given that the uninformed expert's strategy is Markov in the public beliefs in each arrival, the informed expert's payoff at the end of an arrival with reputation p and revealed state s is characterized by the Bellman equation

$$\begin{aligned}V^I(p,s;\bar{\alpha}) = \int_0^\infty \mathrm{e}^{-(r+1)l}\Bigg\{&r\bar{w}_{p,\mu_l,s}+\mu_l V^I\left(\frac{p}{p+(1-p)\bar{\alpha}_{p,\mu_l,s}},s;\bar{\alpha}\right)\\ &+(1-\mu_l)V^I\left(\frac{p}{p+(1-p)(1-\bar{\alpha}_{p,\mu_l,s})},\neg s;\bar{\alpha}\right)\Bigg\}\,\mathrm{d}l.\end{aligned} \tag{20}$$

Finally, I show that there exists $p^\dagger\in[0,1)$ such that $V^I(\cdot,s)$ is strictly increasing for every s. By a standard property of contraction mapping (see, *e.g.*, Stokey et al., 1989, p. 52, Corollary 1), it suffices to verify that there exists $p^\dagger\in[0,1)$ such that for every $p\in[p^\dagger,1]$, irrespective of the state s, $\int_0^\infty \mathrm{e}^{-(r+1)l}\,\bar{w}_{p,\mu_l,s}\,\mathrm{d}l$ is strictly increasing in p. The latter follows from the properties of $\bar{\alpha}$ in Proposition 2 and that $\bar{\alpha}$ is continuous in p as argued in the proof of Lemma 1.

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