Free-Riding Under Joint Liability*

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Abstract

We introduce a stochastic stopping game to analyze the free-riding problem in jointly liable partnerships. In a joint project that requires collective contributions, a player enjoys free-riding benefits from ceasing to contribute, which also incurs negative externalities to his partners. The unique equilibrium features a "curse of productivity," namely, a more lucrative project may backfire by stimulating players' free-riding behavior. Also, a large joint liability ensures coordination by suppressing the free-riding benefits. Finally, we show that a group's ability to sustain coordination is non-monotonic in its size, as that depends on the group's vulnerability to a Domino effect identified in the paper.

Keywords: Dynamic Moral Hazard, Free-riding, Stochastic Partnership, Joint Liability, Group Size, Stopping Games.

JEL Codes: C73, D62, L22.

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1 Introduction

Joint liability is embedded in many real-world partnerships. It makes a party responsible when their partners fail to fulfill some responsibility. A notable example concerns group-based lending programs, which require borrowers to form groups and be jointly liable for the repayment of all group members. These programs, widely adopted by microfinance institutions in developing countries and personal loan companies in the United States, have proved successful in expanding credit access to a broader population.[1] However, they face a free-riding problem: Even when a borrower is able to fully repay his loan, he may deliberately default and count on other group members to pay for him (Giné, Jakiela, Karlan, & Morduch, 2010). Although this problem has been alleviated by the usage of social sanctions (De Quidt, Fetzer, & Ghatak, 2016; Giné & Karlan, 2014), it is still considered a reason for the recent shift of many microfinance institutions, including the Nobel-Prize-winning Grameen Bank of Bangladesh, from group lending towards individual lending.[2]

Besides group lending, joint liability is also a common practice in many different contexts, such as venture capital syndicates,[3] joint ventures, international organizations,[4] and, more broadly speaking, group assignments for students and parenting obligations within a family. All these contexts are subject to free-riding problems in one way or another.

This paper develops a dynamic framework to study the following questions: When does free-riding occur in a jointly liable partnership? How does the size of a group affect its members' free-riding intentions? How can we possibly mitigate this problem? In the baseline model, two players continuously contribute to a joint project whose output changes stochastically over time. Players can choose to stop contributing at any time:[5] The person who stops first is called the first

[1] See Karlan and Morduch (2010) for a survey on the achievements of group lending in developing countries. On the theoretical side, the existing literature (e.g., Ghatak (1999)) has profound analysis in the advantage of group lending *vis-à-vis* individual lending. For instance, group lending better elicits borrowers' information through peer selection and mitigates the moral hazard problem via peer monitoring.

[2] In particular, Grameen Bank has shifted to a new lending strategy described as "Grameen II," which no longer implements joint liability among borrowers. Other examples include BancoSol of Bolivia and Bandhan of India. See De Quidt et al. (2016) for more details.

[3] See Nanda and Rhodes-Kropf (2018). If one participating VC discontinues its investment in a funding round for a startup, other VCs in the syndicate must make additional investments to reach the pre-set financing goal.

[4] For example, following Brexit, Germany's projected annual contribution to the European Union's budget surged by 16%. Source: https://www.dw.com/en/germanys-eu-bill-to-rise-by-16-percent-post-brexit-report/a-41325116.

[5] The decision to stop contribution is completely irreversible in the baseline, and completely reversible in the discussion in Section 6.1. The example of group lending features complete irreversibility, while many other real-world cases lie between these two extremes with different level of irreversibility (or different cost of reversal).

mover while his partner is the second mover.[6] After the first mover stops, the entire responsibility of supporting the project falls on the shoulder of the second mover. If the second mover continues to contribute to the project, the first mover still receives some revenue from the project while avoiding the contribution cost. Otherwise, the project shuts down.

We focus on the Markov Perfect Equilibrium (MPE henceforth) of this stochastic stopping game. As shown in Section 3, this game features a "first mover advantage" and thus its MPE must be either a "coordinative equilibrium" or a "pre-emptive equilibrium," depending on whether pre-emption occurs in the presence of first mover advantage. Theorem 1 characterizes the unique Pareto-optimal MPE and highlights two findings. First, players may suffer from a "curse of productivity." A high-level output of the joint project is a double-edged sword: While it generates high revenue, it also enhances a second mover's willingness to run the project solo and thus stimulates free-riding in the first place. This finding provides a novel channel through which high productivity can be detrimental.[7] Second, the equilibrium also features a "blessing of joint liability." Although a large joint liability seems to exacerbate the second mover's burden, it may undermine the benefit from free-riding and ensure the existence of a coordinative equilibrium.

Then we generalize the model to more than two players and analyze how the group size affects the sustainability of a coordinative equilibrium. Compared with the baseline model, the novel feature of this generalization is the presence of a "Domino effect:" A player's stopping may trigger a second player to stop, which further triggers a third player to stop, and so on. Theorem 2 delineates the extent of this effect by characterizing the set of group sizes that can sustain a coordinative equilibrium. We show that the coordination-sustainability of a group size is not a matter of the magnitude per se (i.e., being sufficiently large or small), but rather, it depends on whether players' free-riding intentions can be properly deterred after they foresee the subsequent Domino effect. This finding stands in contrast to the classic results in a static setting where a large group size usually exacerbates free-riding (Olson, 1965). Additionally, we find that players' ability to renegotiate undermines the remaining contributors' commitment to punish a free-rider and make

[6] The identities of the first and second movers are endogenously determined by the players' strategies in the baseline. An alternative setting with pre-designated identities is analyzed in Section 5.

[7] To our knowledge, the most relevant studies on the possibility of detrimental productivity is the literature on ratchet effect (see Freixas, Guesnerie, and Tirole (1985)). We complement their discussion through a perspective of miscoordination instead of information friction. Curello (2020) also features detrimental productivity due to miscoordination, but in a different setting.

coordination more difficult.

Section 5 studies whether and how the free-riding problem is mitigated if a player can commit not to defect first. We analyze an alternative setting where the committed player is designated as the second mover and her partner is a designated first mover. They play a Stackelberg game where they sequentially choose when to stop. Theorem 3 shows that the no-first-defect commitment can lead to a Pareto improvement upon the baseline. For the designated second mover in particular, the cost of being inferior in a partnership may be outweighed by the benefit of immunization against pre-emption. Besides, we also find that the designated first mover's free-riding decision is non-monotonic: He chooses to free ride only when the project's productivity is intermediate (Proposition 4). High productivity makes free-riding not tempting, while low productivity renders it unlikely to succeed.

In Section 6, we discuss several modeling choices in the baseline model. First, we find that the irreversibility of defections is necessary for free-riding behavior to occur. If defections are reversible, we can punish a free-rider by loading him with more responsibility in the future, but that is physically impossible if defections are irreversible. In fact, we can always implement the first-best outcome in the case of reversible defections if we properly design the grim trigger strategy (Proposition 5). Second, we consider an alternative setting where a designer can flexibly distribute the project's overall operation cost among all the remaining contributors. By properly exploiting the Domino effect, a large group always outperforms a small one in deterring free-riding behavior (Proposition 6). Third, we argue that some insight from the baseline model is robust when players run separate projects. For instance, a more lucrative project can still backfire by making the project owner more committed to cover his partners' unfulfilled liability.

The paper proceeds as follows. Section 2 describes the baseline model. Section 3 solves the equilibrium. Section 4 analyzes how the group size affects coordination. Section 5 examines the usage of the no-first-defect commitment. Section 6 discusses modeling assumptions. Section 7 concludes.

1.1 Related Literature

First and foremost, this paper contributes to the studies on dynamic moral hazard in teams (Holmstrom, 1982).[8] In particular, a closely related literature is dynamic contribution games in which players build up a common stock of public goods through voluntary contributions (Marx & Matthews, 2000).[9] In that literature, players' free-riding intentions are mitigated by an "encouragement effect," namely, a player's short-run contribution encourages others to contribute more in the long run.[10] The crucial counterforce against free-riding in this paper is strategically similar: We identify a "discouragement effect" that a player's cessation to contribute will trigger others to opt out as well. Unlike that literature, the persistent effect of one's contribution in our paper does not stem from building up a common stock, but is a result of irreversible defections instead. As a consequence, we reach different conclusions in various aspects, especially regarding the effect of group size (see Section 4).

A second strand of related literature is stochastic stopping games (Dutta & Rustichini, 1993; Grenadier, 1996; Weeds, 2002).[11] For instance, in the context of strategic experimentations, Rosenberg, Solan, and Vieille (2007) and Murto and Välimäki (2011) consider the situation where experimenters can irreversibly stop their own experiments.[12] Their models feature information externalities among players, namely, an experimenter can learn from others' stopping decisions. In contrast, our paper mainly discusses direct payoff externalities among players.[13] Another related paper is Chassang (2010), where players keep receiving noisy signals about a payoff-relevant state and choose when to opt out of the partnership. That paper mainly discusses how noisy information impacts players' coordination behavior in the absence of learning, and thus the payoff-relevant state is assumed independent across time. We do not consider a noisy information structure but

[8] Our major deviations from the benchmark of this literature include stochastic payoffs and irreversible defections.

[9] Recent papers in this literature include Battaglini, Nunnari, and Palfrey (2014), Georgiadis (2015), Ramos and Sadzik (2019), Cetemen, Hwang, and Kaya (2020), etc.

[10] The discussion of "encouragement effect" also appears in the literature of strategic experimentation, but in a setting with information externalities instead. Section 2 of Hörner and Skrzypacz (2016) includes a detailed discussion.

[11] Depending on the setting, a stochastic stopping game is also referred to as a stochastic timing game or a real options game. For stopping games in a deterministic setting, see Fudenberg and Tirole (1985) and Simon (1987).

[12] Recent papers that analyze stopping games include Rosenberg, Salomon, and Vieille (2013), Anderson, Smith, and Park (2017), Guo and Roesler (2018), Margaria (2020), Kirpalani and Madsen (2020), Awaya and Krishna (2021), Cetemen, Urgun, and Yariv (2021), etc.

[13] There are also papers considering experimentation with payoff externalities but without the stopping game feature. See Strulovici (2010), Halac, Kartik, and Liu (2016), Cripps and Thomas (2019), and Thomas (2021) for examples.

allow the payoff-relevant state to admit some degree of persistence over time (i.e., following a Brownian motion). Hence, our state variable reflects not only the current payoff but also the future prospect of the partnership, which introduces dynamic strategic concerns.

Third, this paper is also related to stochastic partnership games.[14] McAdams (2011) considers a dynamic game of prisoners' dilemma where players can choose to work, shirk, or irreversibly quit the partnership, while the payoff-relevant state is stochastically evolving. That paper also exhibits payoff externalities, which result from the shirking action instead of the irreversible quitting action. Our paper differs in two ways. First, from a modeling point of view, the externality-generating action in ours is the same as the irreversible one. Second, our paper features two-way externalities in the sense that a player who quits will bring some externality to the remaining players, but will also suffer from the externalities generated by the remaining players' future quitting. Angeletos, Hellwig, and Pavan (2007) is another paper that models payoff externalities in a stochastic environment. They analyze a dynamic global game and focus on observational learning among players. While their stage game admits multiple symmetric equilibria à la the classic coordination game under some parameters, ours admits multiple asymmetric equilibria à la the *Game of Chicken*. This will drive our main results on pre-emption.

Finally, in terms of applications, this paper speaks to a very large literature on joint liability which is best understood in the context of group lending scheme adopted by microfinance institutions (surveyed by Karlan and Morduch (2010)). As to the potential problems of group lending, one early theoretical study is Besley and Coate (1995) who discuss the possibility of a "default contagion:" A borrower may default as failing to cover his partners' unfulfilled repayment. Our paper takes one step back to consider the initial default endogenously and focus on the free-riding problem: A borrower may deliberately default as counting on his partners to cover. Besides, Bond and Rai (2008) also mention the free-riding problem in group lending. We differ from theirs by adopting a dynamic framework and emphasizing the possibility of pre-emption.

[14] For partnership games in a deterministic setting, see Fujiwara and Okuno (2009) for an example.

2 Model Setup

2.1 Payoff

We consider a continuous-time model with an infinite horizon. Time is indexed by $t \in [0, \infty)$ and the discount rate for all the players is $r > 0$. Two players ($i = 1, 2$) form a partnership to run a joint project. Player i's realized lifetime utility is the exponentially discounted sum of his flow payoff, $\Pi_i = \int_0^\infty e^{-rt}\pi_{it}dt$.

	Contribute	Defect
Contribute	$\theta_t - c$, $\theta_t - c$	$\theta_t - \beta c$, $\alpha\theta_t$
Defect	$\alpha\theta_t$, $\theta_t - \beta c$	0 , 0

Table 1: Flow payoff at time t in the baseline model

The flow payoff at time t, (π_{1t}, π_{2t}), is depicted by Table 1. At each instant, each player is expected to contribute to the project by paying a flow cost $c > 0$. If both players contribute to the project, it returns a flow revenue $\theta_t > 0$ to each player. We interpret $\theta_t \in \Theta = \mathbb{R}^+$ as the project's output or productivity. We assume that it is observable to both players and follows a geometric Brownian motion, $\frac{d\theta_t}{\theta_t} = \mu dt + \sigma dZ_t$, where $\mu < r$, $\sigma > 0$, and Z_t is a standard Wiener process.

It is possible, however, for players to defect by withholding contribution. Suppose Player i defects while Player j runs the entire project solo, Player j's flow cost rises to βc, where $\beta > 1$ due to the joint liability embedded in the partnership.[15] In this situation, although Player i can still benefit from the ongoing project, his flow revenue is proportionally discounted to $\alpha\theta_t$ with $\alpha \in (0, 1)$.[16] Suppose both players defect, the project ceases operation and returns no revenue.

We call α the *free-riding parameter* as it captures how much a player benefits from having his partner undertake all the responsibility. We refer to β as the *joint liability parameter* since it reflects the amount of additional liability one needs to undertake in case his partner runs away. It can also be interpreted as how much one relies on his partner's contribution.

[15] Although it is more natural to let $\beta = 2$, we allow for some generality here. $\beta < 2$ can be justified by the economy of scale, while $\beta > 2$ can be explained by the additional frictional cost when the liability is shifted.

[16] This discount, which is essentially a punishment to Player i's defection, is best understood in real-world contexts. In group lending, for instance, a defaulter is usually subject to some social sanctions even after his group members covered his unfulfilled liability (De Quidt et al., 2016; Giné & Karlan, 2014).

2.2 Timeline

In the baseline model, we focus on the situation where players' defections are irreversible. That leads us to construct a canonical stochastic stopping game where players choose when to stop contributing. Players' past actions are publicly and perfectly observable. We follow the tradition of literature to formalize the timeline as the following two-stage dynamic game.

At Stage 1, each player chooses when to stop, given that no one has stopped yet. Player i's strategy is a $\mathcal{H}_{1t}$-adapted stopping time τ^i, where $\mathcal{H}_{1t}$ is the set of Stage-1 public history up to time t, with H_{1t} being a representative element. Since both players keep contributing during $[0, t)$ at this stage, H_{1t} only contains the information about the path of $(\theta_{\tilde{t}})_{\tilde{t}\in[0,t]}$. Stage 1 ends at $\tau := \min\{\tau^1, \tau^2\}$. It is possible, however, that both players attempt to stop at the same time (i.e., $\tau^1 = \tau^2$ for some H_{1t}). We make the following tie-breaking assumption in case that happens: Only one player (selected at random through a coin flip or other fair randomization devices) can successfully stop.[17] Under this assumption, there is only one player stopping at Stage 1 regardless of whether a coin flip is necessary, and we will call him the first mover ("he").

After the first mover stopped, the game immediately proceeds to Stage 2 where the remaining player, who we refer to as the second mover ("she"), chooses when to stop. Her strategy is a $\mathcal{H}_{2t}$-adapted stopping time $\tau^s \geq \tau$, where the set of public history $\mathcal{H}_{2t}$ contains not only the path of the payoff-relevant state, but also the players' history of actions at Stage 1. It is possible for the second mover to immediately follow suit after the first mover stopped (i.e., $\tau^s = \tau$), which we refer to as a "de facto joint stop." Therefore, a coin flip loser (if any) at Stage 1 is essentially given a chance to withdraw her stopping decision after losing the coin flip, while in case she does not withdraw, her stopping is still regarded as happening at Stage 2 for consistency.

[17] Such a tie-breaking assumption is common in the literature of stochastic timing games. We will discuss this assumption in Appendix B.1.

3 Equilibrium

3.1 Value Functions for the Stopping Game

We use backward induction to solve the equilibrium. At Stage 2, the second mover is facing an optimal stopping problem: She gets a flow payoff $\theta_t - \beta c$ until she stops and collects a zero lump-sum payoff. As is standard for a time-homogeneous problem of this sort, it is without loss of generality to consider Markov strategies. Hence, we let the second mover adopt a stopping set $\Theta^s \subseteq \Theta$ that specifies the value of θ_t under which she would stop. Denote $\gamma = \frac{\sigma^2 - 2\mu - \sqrt{(\sigma^2-2\mu)^2 + 8r\sigma^2}}{2\sigma^2}$, the negative root of the polynomial equation $\Gamma(x) = \mu x + \frac{\sigma^2}{2}x(x-1) - r = 0$.

Claim 1. *The second mover's optimal strategy at Stage 2 is to adopt the stopping set* $\Theta^s = (0, \theta^*)$ *where* $\theta^* := \frac{r-\mu}{r}\frac{\gamma}{\gamma-1}\beta c$. *Her corresponding value function at time* t *is*

$$S(\theta_t) = \begin{cases} -\frac{\beta c}{r}\left[1 - \left(\frac{\theta_t}{\theta^*}\right)^{\gamma}\right] + \frac{\theta_t}{r-\mu}\left[1 - \left(\frac{\theta_t}{\theta^*}\right)^{\gamma-1}\right] & \text{, when } \theta_t > \theta^* \\ 0 & \text{, when } \theta_t \leq \theta^*. \end{cases}$$

Proof. See Appendix A.1. □

Knowing the second mover's response at Stage 2, we can derive the first mover's lump-sum stopping payoff. After stopping, the first mover keeps receiving a flow payoff $\alpha\theta_t$ until the next time θ_t hits θ^*.

Claim 2. *If the first mover stops contribution at time* t*, his lump-sum stopping payoff is*

$$F(\theta_t) = \begin{cases} \frac{\alpha\theta_t}{r-\mu}\left[1 - \left(\frac{\theta_t}{\theta^*}\right)^{\gamma-1}\right] & \text{, when } \theta_t > \theta^* \\ 0 & \text{, when } \theta_t \leq \theta^*. \end{cases} \tag{1}$$

Proof. See Appendix A.2. □

Lemma 1. *There exists a unique* $\tilde{\theta} \in (\theta^*, \infty)$ *such that*

$$\begin{aligned}
F(\theta_t) &= S(\theta_t) && \text{for } \theta_t \in (0, \theta^*] \\
F(\theta_t) &> S(\theta_t) && \text{for } \theta_t \in (\theta^*, \tilde{\theta}) \\
F(\theta_t) &= S(\theta_t) && \text{for } \theta_t = \tilde{\theta} \\
F(\theta_t) &< S(\theta_t) && \text{for } \theta_t \in (\tilde{\theta}, \infty).
\end{aligned}$$

Proof. See Appendix A.3. □

Lemma 1 is important for later analysis. It indicates that a "first mover advantage" (i.e., $F(\theta_t) > S(\theta_t)$) exists in and only in one connected set of θ_t, namely, $(\theta^*, \tilde{\theta})$. Figure 1 illustrates this result.

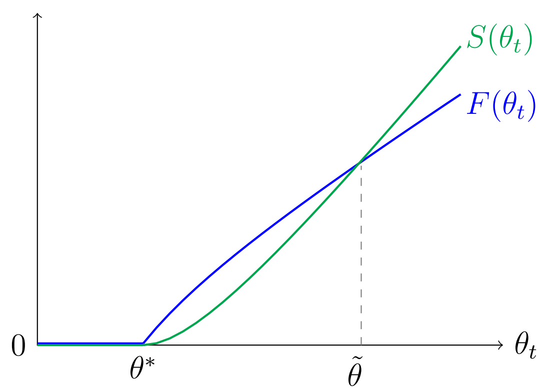


Figure 1: Value functions for the stopping game

3.2 Two Types of MPE

Since the second mover's optimal strategy at Stage 2 is unique (up to the indeterminacy at the threshold θ^*), we can induce backwards to Stage 1 where the players are facing the following stopping game: They each receive a flow payoff of $\theta_t - c$ until one of them stops and collects a lump-sum payoff $F(\theta_t)$ as the first mover, while the remaining player receives a lump-sum payoff $S(\theta_t)$ as the second mover.

We restrict attention to MPE, i.e., each player's equilibrium strategy is based on the current state θ_t alone and does not vary with time. Hence, Player i's strategy at Stage 1 can be represented by a reduced-form stopping set $\Theta^i \subseteq \Theta$ that identifies the value of θ_t under which Player i stops making contribution.

Our analysis starts with the following argument: Since $(\theta^*, \tilde{\theta})$ is a connected set of θ_t that features the first mover advantage, in this entire set both players either always contribute or always stop.[18] This argument is due to the nature of pre-emption: Once a player intends to stop in the presence of first mover advantage, his partner will react by stopping slightly earlier than him, and such pre-emption exercise will diffuse to a connected set until the first mover advantage vanishes.[19] With this argument, we can distinguish between two types of equilibrium.

Definition 1.

*(a) A **coordinative equilibrium** is one in which both players contribute at Stage 1 when $\theta_t \in (\theta^*, \tilde{\theta})$.*

*(b) A **pre-emptive equilibrium** is one where both stop at Stage 1 when $\theta_t \in (\theta^*, \tilde{\theta})$.*[20]

3.2.1 Coordinative Equilibrium

We first analyze coordinative equilibrium. Notice that it is dominant for players to contribute to the joint project when $\theta_t \geq \tilde{\theta}$.[21] Hence, in any coordinative equilibrium, players' stopping sets at Stage 1 are both subsets of $(0, \theta^*]$. This implies that both players will de facto jointly stop on the equilibrium path, since the second mover always immediately follows the first mover's stopping when $\theta_t \leq \theta^*$.

This observation enables us to determine the Pareto-dominant equilibrium within the set of coordinative equilibria (if existing). When players must jointly stop, the best possible outcome for them is to adopt the stopping set $(0, \theta^{**})$ where $\theta^{**} := \frac{r-\mu}{r}\frac{\gamma}{\gamma-1}c$. This is derived by solving the single-agent optimal stopping problem where the flow payoff is $\theta_t - c$ and the lump-sum stopping payoff is zero. We refer to this outcome as the "optimal coordinative outcome." To implement it,

[18]This argument is common to pre-emption models. To our knowledge, Dutta and Rustichini (1993) is the first to make such an argument in a stochastic setting. Weeds (2002) also makes the same argument and analyzes two distinct types of MPE that are akin to "coordinative equilibrium" and "pre-emptive equilibrium" in our paper.

[19]This argument also relies on the assumption that the one-dimensional stochastic state variable has a continuous path and can evolve in both directions, which hold in our Brownian setting.

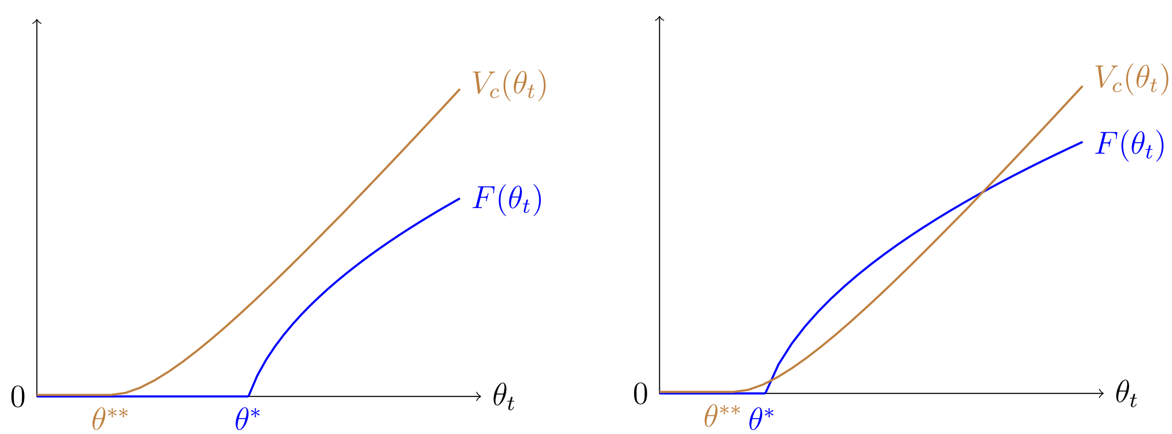
[20]Notably, in a pre-emptive equilibrium both players intend to stop at Stage 1 when $\theta_t \in (\theta^*, \tilde{\theta})$, but the one who (fails the coin flip and) proceeds to Stage 2 will still contribute in this range of θ_t.

[21]Notably, it is weakly dominant to contribute when $\theta_t = \tilde{\theta}$. A trivial coordinative equilibrium possibly exists if both players' stopping sets include $\tilde{\theta}$ and exclude $(\theta^*, \tilde{\theta}) \cup (\tilde{\theta}, \infty)$, but such an equilibrium (if existing) is always Pareto dominated. Hence, we only consider the coordinative equilibrium where players' strategies are not weakly dominated.

we can let players adopt the stopping set $\Theta^1 = \Theta^2 = (0, \theta^{**})$.[22] Such a strategy profile gives each player the following value function[23]

$$V_c(\theta_t) = \begin{cases} -\frac{c}{r}\left[1 - \left(\frac{\theta_t}{\theta^{**}}\right)^{\gamma}\right] + \frac{\theta_t}{r-\mu}\left[1 - \left(\frac{\theta_t}{\theta^{**}}\right)^{\gamma-1}\right] & \text{, when } \theta_t > \theta^{**} \\ 0 & \text{, when } \theta_t \leq \theta^{**}. \end{cases} \tag{2}$$

Since this strategy profile gives the highest possible value to joint-stopping players, the existence of coordinative equilibrium boils down to whether this strategy profile constitutes an equilibrium. To check this, we only need to tell whether $V_c(\theta_t) \geq F(\theta_t)$ holds for $\forall \theta_t \in (\theta^{**}, \infty)$ since each player can unilaterally deviate with an early stop (i.e., stopping at time t when $\theta_t > \theta^{**}$). As we will rigorously show in Proposition 1, the above condition is equivalent to $\beta \geq \beta^* := \left[\frac{1-(1-\alpha)^{\gamma}}{\alpha\gamma}\right]^{\frac{1}{1-\gamma}}$. Intuitively, a large β indicates that the second mover finds it difficult to run the entire project solo, which, in turn, may deter the players' free-riding intentions in the first place. Figure 2 illustrates this result. The deviation payoff (the blue curve) weakly decreases in β, and will be point-wise lower than $V_c(\theta_t)$ (the brown curve) when β is sufficiently large (see the left panel).



(a) $\beta \geq \beta^*$ (b) $\beta < \beta^*$

Figure 2: Existence of coordinative equilibrium

The above finding is summarized in Proposition 1. We will refer to $\Theta^1 = \Theta^2 = (0, \theta^{**})$ as the

[22]This is not the only strategy profile to implement the optimal coordination outcome. Any strategy profile satisfying $\Theta^1 \cup \Theta^2 = (0, \theta^{**})$ can implement it.

[23]Derivation of θ^{**} and $V_c(\theta_t)$ is omitted as it is almost identical to that for Claim 1. Also, we use the subscript "c" to indicate the coordinative outcome.

"optimal coordinative equilibrium" in case it constitutes an equilibrium.

Proposition 1. ***(a)*** *When $\beta \geq \beta^*$, coordinative equilibrium exists. Among all coordinative equilibria, $\Theta^1 = \Theta^2 = (0, \theta^{**})$ is uniquely Pareto-optimal (up to payoff equivalence).*[24]

(b) *When $\beta < \beta^*$, coordinative equilibrium does not exist.*

Proof. See Appendix A.4. □

3.2.2 Pre-emptive Equilibrium

We then analyze the pre-emptive equilibrium where both players stop in the entire set of $(\theta^*, \tilde{\theta})$. Since it is still dominant for players to contribute when $\theta_t \geq \tilde{\theta}$, what remains undetermined in a pre-emptive equilibrium is the players' strategies when $\theta_t \in (0, \theta^*]$.

Unlike coordinative equilibrium, a pre-emptive equilibrium always exists. For instance, $\Theta^1 = \Theta^2 = (0, \tilde{\theta})$ is always an equilibrium.[25] However, this is not the best pre-emptive equilibrium for the players. Notice that $\theta^* > c$ is possible, so players can receive positive payoff by jointly contributing to the project when θ_t is below θ^* but above c.

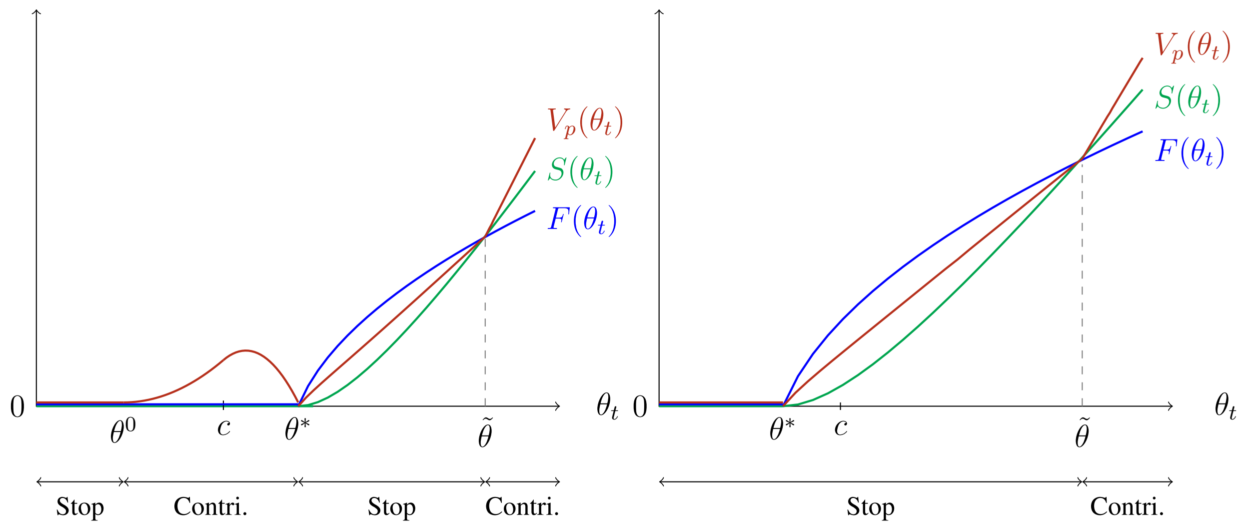
To formalize the analysis, we denote $\beta^{**} := \frac{\gamma-1}{\gamma}\frac{r}{r-\mu}$. If $\beta > \beta^{**}$, it follows that $\theta^* > c$. Similar to coordinative equilibrium, any stopping when $\theta_t \in (0, \theta^*]$ will trigger a de facto joint stop. Hence, the "optimal pre-emptive outcome" can be derived from solving the following single-agent stopping problem with $\theta_t \in (0, \theta^*)$: The flow payoff is $\theta_t - c$, the lump-sum stopping payoff is zero, and additionally, there exists an exogenous stopping point at θ^*. The solution to this single-agent problem is to stop if and only if $\theta_t < \theta^0$, where θ^0 is uniquely determined by value matching and smooth pasting conditions. To implement this outcome, we construct an "optimal pre-emptive equilibrium," namely, $\Theta^1 = \Theta^2 = (0, \theta^0) \cup (\theta^*, \tilde{\theta})$.[26] Figure 3(a) illustrates this equilibrium as well

[24]There are two sources of equilibrium multiplicity that do not impact players' payoffs. First, whether players should stop at the boundary of stopping set is indeterminate for optimality. Including θ^* in the stopping set also makes an equilibrium. Second, the same equilibrium outcome can be implemented by different strategy profiles as is shown by Footnote 22. The same argument on payoff-equivalence also applies to Proposition 2 and Theorem 1.

[25]When $\theta_t \in (0, \theta^*]$, each player finds it indifferent to stop or to contribute as long as his partner chooses to trigger a joint stop. So it is an equilibrium.

[26]It is worthwhile to point out a difference between $(0, \theta^0)$ and $(\theta^*, \tilde{\theta})$. On the equilibrium path, both players will de facto jointly stop when $\theta_t \in (0, \theta^0)$, while in contrast, only one player successfully stops when $\theta_t \in (\theta^*, \tilde{\theta})$ and the ex-post second mover will wait until θ^* is reached again. Also, this strategy profile is not the only one to implement the optimal pre-emptive outcome as in Footnote 22.

as the corresponding (expected) value function $V_p(\theta_t)$.[27] A noteworthy feature of this equilibrium is that the stopping strategy does not admit a threshold form, which rarely occurs for stopping games.



(a) $\beta > \beta^{**}$ (b) $\beta \leq \beta^{**}$

Figure 3: Optimal pre-emptive equilibrium

If $\beta \leq \beta^{**}$ (i.e., $\theta^* \leq c$), any joint contribution when $\theta_t \in (0, \theta^*]$ is not worthwhile. Hence, the unique pre-emptive equilibrium is $\Theta^1 = \Theta^2 = (0, \tilde{\theta})$.[28] Figure 3(b) depicts this equilibrium and the corresponding value function $V_p(\theta_t)$.[29] Proposition 2 summarizes these findings.

Proposition 2. ***(a)*** *If $\beta > \beta^{**}$, pre-emptive equilibrium exists. Among all pre-emptive equilibria, $\Theta^1 = \Theta^2 = (0, \theta^0) \cup (\theta^*, \tilde{\theta})$ is uniquely Pareto-optimal (up to payoff equivalence).*
(b) *If $\beta \leq \beta^{**}$, the unique (up to payoff equivalence) pre-emptive equilibrium is $\Theta^1 = \Theta^2 = (0, \tilde{\theta})$.*

[27] We use the subscript "p" to indicate pre-emption (cf. $V_c(\theta_t)$). $V_p(\theta_t)$ can be derived as follow. Due to the tie-breaking assumption, $V_p(\theta_t) = \frac{1}{2}[F(\theta_t) + S(\theta_t)]$ when $\theta_t \in [\theta^*, \tilde{\theta}]$. When $\theta_t > \tilde{\theta}$, $V_p(\theta_t)$ can be determined with the Feynman Kac Equation plus an exogenous stopping at $\tilde{\theta}$ and a boundary condition when θ_t approaches infinity. When $\theta_t < \theta^*$, $V_p(\theta_t)$ and θ^0 are jointly determined by the value matching and smooth pasting conditions at θ^0 plus a value matching condition at θ^*. Notice that smooth pasting is not required at θ^* as it is an exogenous stopping threshold.

[28] Similar to Footnote 26, $(0, \theta^*)$ is a region where players de facto jointly stop, while $[\theta^*, \tilde{\theta})$ is a region where only one player successfully stops.

[29] The closed form of $V_p(\theta_t)$ can be derived in a similar manner to Footnote 27.

3.3 Main Result

Previous analysis indicates three sources of multiplicity in the pure-strategy MPE of this stopping game. First, when $\beta \geq \beta^*$, coordinative equilibria and pre-emptive equilibria coexist. Second, there can be multiple coordinative equilibria. Third, there can be multiple pre-emptive equilibria.

In the presence of multiplicity, it is natural to adopt the Pareto criterion for equilibrium selection. The second and third sources of multiplicity can be addressed by pinning down the *optimal coordinative equilibrium* and the *optimal pre-emptive equilibrium* according to Propositions 1 and 2, respectively. As for the first source, it can be resolved by the following proposition.

Proposition 3. *If the optimal coordinative equilibrium exists, it Pareto dominates the optimal pre-emptive equilibrium.*

Proof. See Appendix A.5. □

Theorem 1. *The Pareto-optimal MPE is unique (up to payoff equivalence). If $\beta^{**} < \beta^*$, the equilibrium stopping set for both players at Stage 1 is*

$$\Theta^1 = \Theta^2 = \begin{cases} (0, \theta^{**}) & \text{, when } \beta \in [\beta^*, \infty) \\ (0, \theta^0) \cup (\theta^*, \tilde{\theta}) & \text{, when } \beta \in (\beta^{**}, \beta^*) \\ (0, \tilde{\theta}) & \text{, when } \beta \in (1, \beta^{**}]. \end{cases}$$

If $\beta^{**} \geq \beta^*$, the equilibrium stopping set for both players at Stage 1 is

$$\Theta^1 = \Theta^2 = \begin{cases} (0, \theta^{**}) & \text{, when } \beta \in [\beta^*, \infty) \\ (0, \tilde{\theta}) & \text{, when } \beta \in (1, \beta^*). \end{cases}$$

Proof. When $\beta \geq \beta^*$, there exists an optimal coordinative equilibrium (see Proposition 1), which also Pareto-dominates the set of pre-emptive equilibria (see Proposition 3). When $\beta < \beta^*$, coordinative equilibrium does not exist, so the optimal pre-emptive equilibrium will be Pareto-optimal. Its particular form depends on whether $\beta \leq \beta^{**}$ or not (see Proposition 2). □

Theorem 1 provides a reasonable prediction for the baseline model. Figure 4 illustrates this result when $\beta^{**} < \beta^*$.[30] Denote $V(\theta_t)$ as each player's equilibrium value function in this MPE. It

[30] We omit the case of $\beta^{**} \geq \beta^*$ since it is only a degenerate version where the intermediate scenario vanishes.

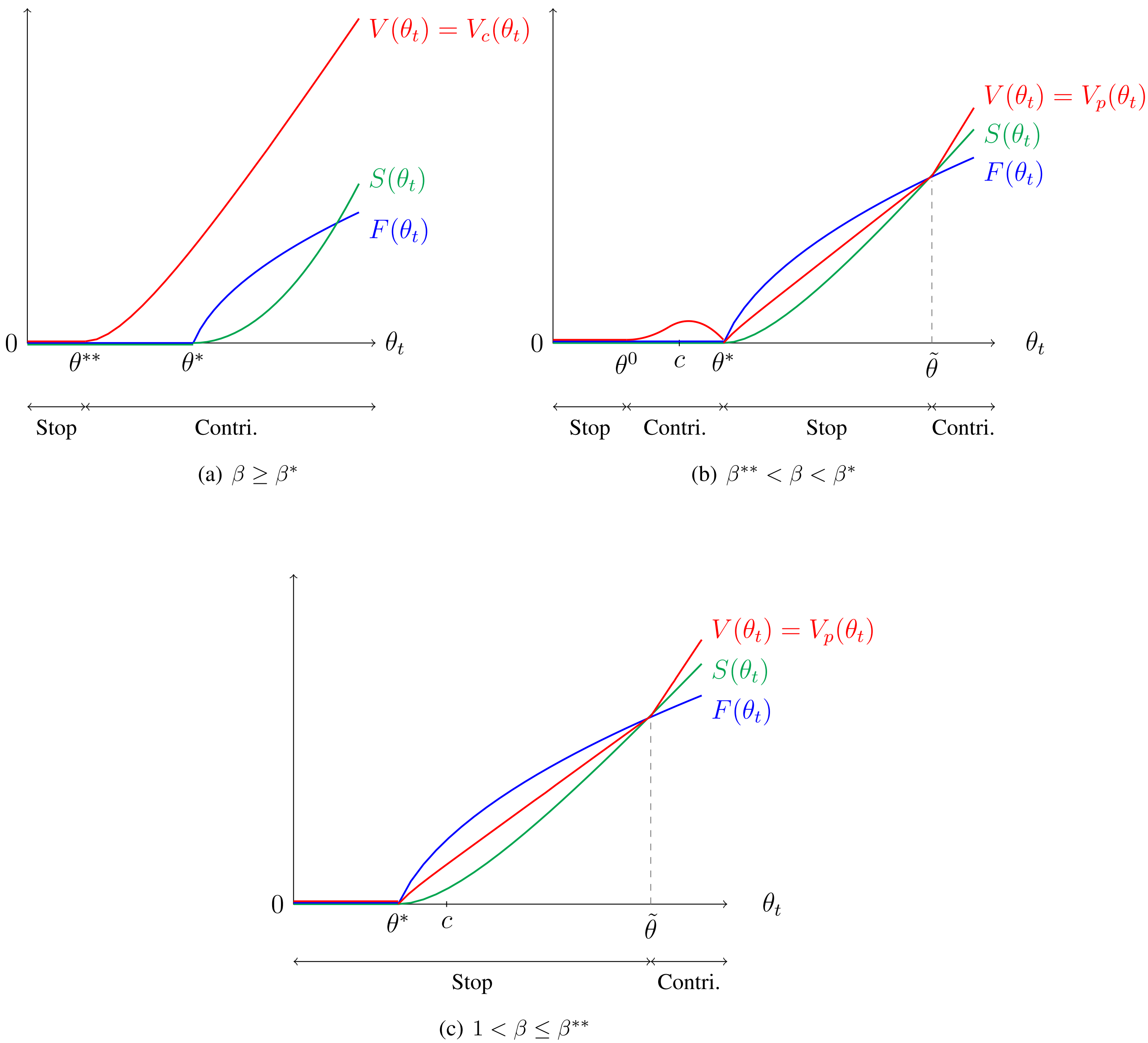


Figure 4: Pareto-optimal MPE (when $\beta^{**} < \beta^*$)

equals to $V_c(\theta_t)$ if the optimal coordinative equilibrium exists and $V_p(\theta_t)$ if not.

We highlight two features of this equilibrium. First, the non-monotonicity of $V(\theta_t)$ when $\beta^{**} < \beta < \beta^*$ indicates a "curse of productivity" (see Figure 4(b)).[31] This result can be understood from two angles. On the surface, it is a direct consequence of the players' non-monotonic stopping strategies as we specify in Theorem 1. At its core, it captures the idea that a larger θ_t can be a double-edged sword: While it generates higher revenue from the joint project, it also makes the second mover more likely to run the project solo and thus stimulates free-riding behavior.

Second, there is a "blessing of joint liability," namely, coordination is easier to sustain when the second mover is imposed with larger joint liability from the first mover's fault. This argument is backed up by the finding that a coordinative equilibrium exists only when β is sufficiently large.[32] Intuitively, although a smaller joint liability seems to alleviate the second mover's burden, it may backfire by encouraging players to free ride in the first place.

Corollary 1. *β^* strictly increases with both μ and α.*

Proof. See Appendix A.6. □

In this section, we regard β as the key parameter. However, this is somewhat arbitrary. To see how other parameters affect the equilibrium, one way is to indirectly examine how they relate to β^*, the minimal requirement of joint liability to maintain coordination. Corollary 1 performs comparative statics analysis regarding β^*. The first result indicates that the coordinative outcome can be more difficult to maintain if the joint project is more promising (i.e., larger μ). This result stems from the insight that a promising project will undermine the second mover's ability to deter the first mover's free-riding in the first place. The second result shows a straightforward finding that maintaining coordination is more difficult if the first mover has a larger incentive to free ride (i.e., larger α).

[31] This result also applies to a broad set of pre-emptive equilibria (not necessarily Pareto-optimal) if $\beta > \beta^{**}$. See Figure 3(a).

[32] We care about implementing the optimal coordinative outcome because it is the ex-post Pareto-efficient outcome, namely, we cannot have another outcome (after the realization of coin flip) that Pareto-improves this outcome. Also, it is worth noticing that this outcome does not necessarily maximize social welfare (defined as the sum of two players' values). For instance, when β is close to 1, having one player stopping early may be socially desirable. That being said, in the most natural setting where $\beta = 2$, this outcome also maximizes social welfare.

4 The Effect of Group Size

4.1 Setup

This section generalizes the model to $N \geq 2$ players and analyzes how the group size affects coordination sustainability. The players are all initially engaged in a joint project but can choose when to stop contributing (and still enjoying some partial benefits). Denote the number of contributors at time t by $n_t \leq N$. It is a non-increasing process as we assume players' defections are irreversible.

A contributor's flow payoff at time t is $\theta_t - \beta_{n_t} c$ where we assume $\beta_1 \geq \beta_2 \geq ... \geq \beta_N > 0$. She still receives the full flow revenue, while the flow cost is smaller when there are more people sharing the responsibility.[33] Similar to the baseline, a free-rider's (i.e., one who already stopped contributing) flow payoff is $\alpha\theta_t$ if $n_t \geq 1$ and 0 if $n_t = 0$. The free-riding parameter is constant as long as the project is operating, which is reasonable in contexts where the discount to a free-rider's revenue is based only on the free-riding action itself.[34] Also, we maintain the following tie-breaking assumption: In case there are multiple players attempting to stop at the same time, only one of them (selected at random by a fair randomization device) will succeed in doing so while others can withdraw their stopping decisions.

4.2 Coordination-Sustainable Group Size

Compared with the baseline model, the novel feature of Section 4 is the presence of a "Domino effect." A player's decision to stop contribution may trigger a second player to stop, which further triggers a third player to stop, and so on. We can delineate the extent of this effect and characterize the group sizes with which a coordinative equilibrium exists. To avoid unnecessary discussions on non-monotonicity, we also assume that $\beta_1 \leq \beta^{**}$.[35]

We start by defining the n-player optimal coordinative outcome.[36] Similar to Section 3.2.1, we

[33] A more natural setting would be $\beta_n = \frac{N}{n}$, but here we allow for more generality as in the baseline.

[34] The major insight of Section 4 remains effective if the free-riding parameter depends on the total quantity of contribution $n_t\beta_{n_t}$, which will bring additional discussion since $n_t\beta_{n_t}$ is generically non-monotonic in n_t. We discuss a more general model in Appendix B.2.

[35] Notice that β^{**} can be set arbitrarily large when μ is set close to r. Hence, this assumption will not contradict other setting of the paper as long as we let μ be sufficiently close to r. Appendix B.2 includes more general result without this assumption.

[36] We care about the coordinative outcome for the same reason as in the baseline model (see Footnote 32). It is ex-post Pareto efficient. It also maximizes social welfare in the most natural setting where $\beta_n = \frac{N}{n}$.

solve the single-agent optimal joint-stopping problem for n players. The optimal joint-stopping threshold is $\theta_n^* = \frac{r-\mu}{r}\frac{\gamma}{\gamma-1}\beta_n c$, and the corresponding value function for each player is[37]

$$V_n(\theta_t) = \begin{cases} -\frac{\beta_n c}{r}\left[1-\left(\frac{\theta_t}{\theta_n^*}\right)^{\gamma}\right] + \frac{\theta_t}{r-\mu}\left[1-\left(\frac{\theta_t}{\theta_n^*}\right)^{\gamma-1}\right] & \text{, when } \theta_t \geq \theta_n^* \\ 0 & \text{, when } \theta_t < \theta_n^*. \end{cases} \tag{3}$$

Instead, one who decides to free ride will keep receiving $\alpha\theta_t$ until no one contributes (i.e., $n_t = 0$). Denote $F_n(\theta_t)$ as a free-rider's value function when the project will shut down at the threshold θ_n^*, which, for instance, happens when n remaining contributors agree to implement the n-player optimal coordinative outcome.[38] It takes the following closed form.

$$F_n(\theta_t) = \begin{cases} \frac{\alpha\theta_t}{r-\mu}\left[1-\left(\frac{\theta_t}{\theta_n^*}\right)^{\gamma-1}\right] & \text{, when } \theta_t \geq \theta_n^* \\ 0 & \text{, when } \theta_t < \theta_n^*. \end{cases}$$

Before stating the characterization result, we analyze an example where the optimal coordinative equilibrium exists with three players, but not with two players.

Example 1. *Suppose* $\frac{\beta_1}{\beta_2} < \beta^*$ *and* $\frac{\beta_1}{\beta_3} \geq \beta^*$.

Figure 5 illustrates the relevant value functions for Example 1. As Lemma 2 will show (see Appendix A.7), $\frac{\beta_1}{\beta_2} < \beta^*$ implies that $V_2(\theta_t) < F_1(\theta_t)$ for some $\theta_t > \theta_1^*$, which further indicates that a two-player coordinative outcome is not an equilibrium. Since we assume $\beta_1 \leq \beta^{**}$, the unique equilibrium for $N = 2$ is that both adopt the stopping set $(0, \tilde{\theta})$ when no one has stopped yet.

We then consider $N = 3$ for Example 1. If one player deviates from the three-player coordinative outcome, he should foresee the following Domino effect: A second stop will immediately emerge if $\theta_t \leq \tilde{\theta}$, or take some time (until $\tilde{\theta}$ is reached again) to occur if $\theta_t > \tilde{\theta}$. In both situations, the third stop (and hence a complete shutdown of the project) will happen the next time when $\theta_t \leq \theta_1^*$, before which the deviating player keeps receiving a flow payoff $\alpha\theta_t$. Hence, his lump-sum stopping payoff is also $F_1(\theta_t)$. By Lemma 2, $\frac{\beta_1}{\beta_3} \geq \beta^*$ indicates that $V_3(\theta_t) \geq F_1(\theta_t)$ for

[37] To save on notation, in Section 4 we omit the subscript "c" from the coordinative value functions.
[38] The baseline model corresponds to the following special case of Section 4: Let $N = 2$, $\beta_2 = 1$, $\beta_1 = \beta$, $V_2(\theta_t) = V_c(\theta_t)$, $V_1(\theta_t) = S(\theta_t)$, and $F_1(\theta_t) = F(\theta_t)$.

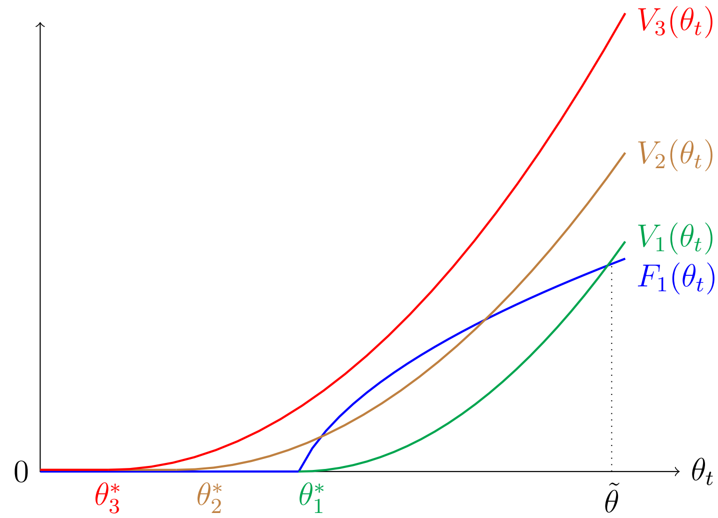


Figure 5: Value functions for Example 1

$\forall\theta_t \in (\theta_3^*, \infty)$, and thus a three-player coordinative equilibrium exists.

In this example, we can continue to analyze $N = 4$. Since three players can sustain a coordinative equilibrium, one who deviates from a four-player coordinative outcome will get a lump-sum stopping payoff $F_3(\theta_t)$.[39] Therefore, whether a four-player coordinative equilibrium exists boils down to whether $\frac{\beta_3}{\beta_4} \geq \beta^*$. Such exercise can be continued in an inductive way to determine all the group sizes where a coordinative equilibrium exists.

Jumping out of Example 1, Theorem 2 characterizes the "coordination-sustainable" group sizes. Despite the various modeling elements, the sequence $\{\beta_n\}_n$ is all we need to employ.

Theorem 2. *Let* $n^{(0)} = 1$. *Denote* $n^{(k)} = \min\left\{n : \frac{\beta_{n(k-1)}}{\beta_n} \geq \beta^*\right\}$. *An* N*-player coordinative equilibrium exists if and only if* $N = n^{(k)}$ *for some* k.

Proof. See Appendix A.7. □

The exact numbers characterized in Theorem 2 is unimportant. The more noteworthy implication of this theorem is that, the set of coordination-sustainable group sizes is (generically) not monotonic nor connected.[40] Whether a group size can sustain coordination is not a matter of

[39]This argument is premised on the conjecture that the three remaining players will implement the optimal coordinative equilibrium, which is uniquely Pareto-optimal (a direct extension from Proposition 3). Such a conjecture is reasonable: When the Pareto-optimal equilibrium is unique, Safronov and Strulovici (2018) show that players can always renegotiate to this equilibrium. In particular, any player can propose a switch to the unique Pareto-optimal equilibrium and other players will approve the proposal.

[40]Exceptions are not difficult to construct. For instance, if $\beta_n = 1$ for $n \geq 2$ while $\beta_1 = \beta > \beta^*$, the set of coordination-sustainable group sizes is $\{1, 2\}$, which is monotonic and connected.

magnitude per se (i.e., being sufficiently large or small). Instead, it depends on whether players' free-riding intentions can be properly deterred in light of the Domino effect. Based on this theorem, we also have the following corollary regarding the largest possible coordination-sustainable group size.

Corollary 2. *The set of group sizes where a coordinative equilibrium exists is bounded if and only if* $\lim_{n\to\infty} \beta_n > 0$.[41]

Proof. See Appendix A.8. □

4.3 Absence of Renegotiation

Theorem 2 assumes that the Pareto-optimal equilibrium is selected at any stage of the dynamic game. This equilibrium selection criterion is backed up by allowing renegotiation among players (see Footnote 39). A natural question to ask is, would it help coordination if we disallow renegotiation so that a Pareto-dominated equilibrium by some remaining players can work as a credible threat against a free rider?

The answer is positive. As an extension of Proposition 2(b), the following strategy profile is a (subgame perfect) equilibrium after one player stops: When $n_t \in [2, N-1]$, all remaining players adopt the stopping set $(0, \tilde{\theta})$ where $\tilde{\theta}$ is the intersection of $V_1(\theta_t)$ and $F_1(\theta_t)$. Under such a "punishment," one who deviates from the N-player coordinative outcome will trigger a Domino effect so that the project will be shut down the next time $\theta_t \le (0, \theta_1^*)$. Hence, a free-rider's stopping payoff is $F_1(\theta_t)$ regardless of N. According to Lemma 2, the N-player coordinative outcome can be sustained if and only if $\frac{\beta_1}{\beta_N} \ge \beta^*$, i.e., $N \ge n^{(1)}$.

This finding delivers two messages. First, with Pareto-dominated (subgame perfect) equilibrium utilized as a punishment against free-riding, a large group can outperform a small one in sustaining coordination. Second, the set of coordination-sustainable group sizes is significantly enlarged compared with Theorem 2. This is because players' ability to renegotiate may undermine the remaining players' commitment to punish a free-rider and thus make coordination more difficult.

[41] $\lim_{n\to\infty} \beta_n$ is well defined since β_n is weakly decreasing and bounded by zero.

5 No-First-Defect Commitment

This section considers the situation where one player is able to commit not to stop contribution first. To formalize this idea, the committed player is designated as the second mover while her partner is a designated first mover. The game proceeds in a Stackelberg manner: At Stage 1, the first mover chooses a $\tilde{\mathcal{H}}_{1t}$-adapted stopping time $\tilde{\tau}^f$, where $\tilde{\mathcal{H}}_{1t}$ is the public history up to time t that only contains information about the path of the payoff-relevant state. After the first mover stops, Stage 2 immediately starts and the second mover chooses a $\tilde{\mathcal{H}}_{2t}$-adapted stopping time $\tilde{\tau}^s \geq \tilde{\tau}^f$, where $\tilde{\mathcal{H}}_{2t}$ tells the public history of the payoff-relevant state and the first mover's action.

The second mover's optimal strategy at Stage 2 is identical to the baseline: She finds it optimal to adopt the Markov strategy that can be represented by a stopping set $\Theta^s = (0, \theta^*)$. We then induce backwards to the first mover's optimal stopping problem at Stage 1: He receives a flow payoff $\theta_t - c$ until stopping and collecting a lump sum payoff $F(\theta_t)$. Since this problem is time-homogeneous, it is without loss of generality to only consider Markov strategy. Hence, the first mover's strategy can also be represented by a stopping set $\Theta^f \subseteq \Theta$. We denote the value functions for the designated first and second movers at Stage 1 as $U_f(\theta_t)$ and $U_s(\theta_t)$, respectively.

Proposition 4. *The first mover's optimal stopping strategy is unique (up to payoff equivalence).*[42]
(a) *When $\beta \geq \beta^*$, the optimal stopping set is $\Theta^f = (0, \theta^{**})$.*
(b) *When $\beta < \beta^*$, there exist three thresholds, $0 < \theta^1 < \theta^2 < \theta^3$ such that the optimal stopping set is $\Theta^f = (0, \theta^1) \cup (\theta^2, \theta^3)$.*

Proof. See Appendix A.9. □

As is shown by Proposition 4, the first mover's optimal strategy at Stage 1 depends on β. When $\beta \geq \beta^*$, the first mover finds it optimal to implement the optimal coordinative outcome since $V_c(\theta_t) \geq F(\theta_t)$ for $\forall \theta_t \in [\theta^{**}, \infty)$. He will choose to trigger a de facto joint stop when θ_t is below θ^{**}, as illustrated by Figure 6(a).

When $\beta < \beta^*$, implementing the coordinative outcome is not optimal for the first mover. Figure 6(b) depicts the first mover's value function $U_f(\theta_t)$ under his optimal strategy. It coincides with $F(\theta_t)$ when $\theta_t \in \Theta^f$ and smoothly pastes with $F(\theta_t)$ at the three thresholds (θ^1, θ^2, and θ^3),

[42] The only source of payoff-equivalent multiplicity in this single-agent optimal stopping problem is the indeterminacy of whether to stop at the stopping set boundary. This differs from the stopping game considered in the baseline.

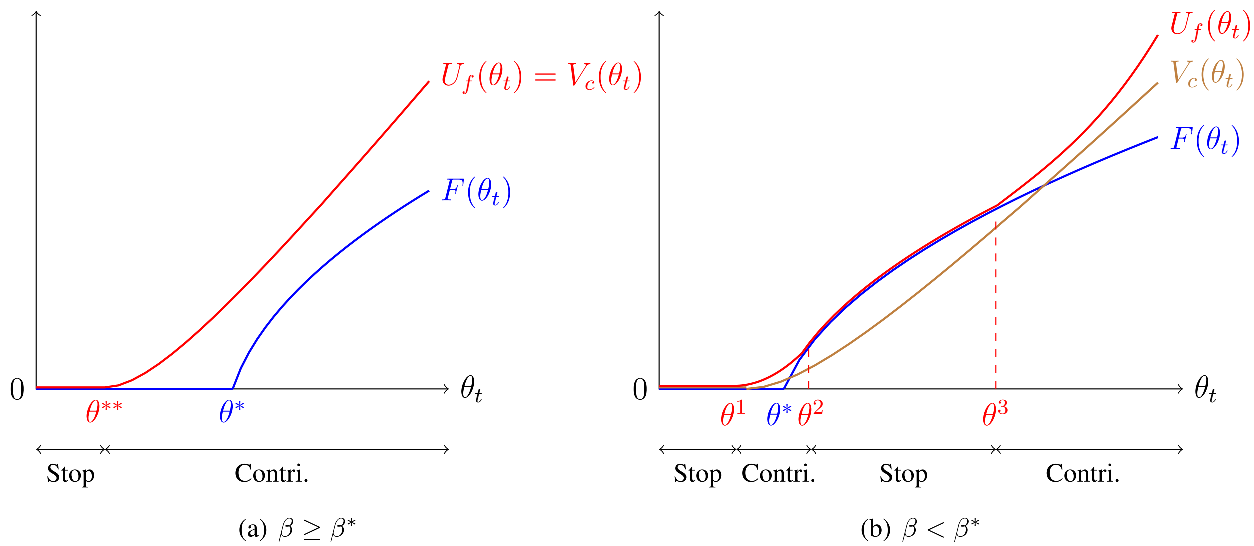


Figure 6: (Designated) first mover's optimal strategy

which are pinned down by value matching and smooth pasting conditions (see Appendix A.9). The noteworthy feature of this optimal strategy is its non-monotonicity, i.e., not in the form of a threshold strategy. In particular, the first mover chooses to contribute in two disconnected intervals: $[\theta^3, \infty)$ where the first mover's contribution is well-rewarded and free-riding is unnecessary, and $[\theta^1, \theta^2]$ where the first mover's free-riding intention is deterred so he would rather grit his teeth and run the project together with the second mover. Free-riding occurs only when $\theta_t \in (\theta^2, \theta^3)$, as it is tempting to do so while not accompanied by a severe concern of triggering a stopping spiral. Such a non-monotonic strategy, to our knowledge, is novel to the literature that regards optimal stopping. It is mainly driven by the non-standard lump sum stopping payoff $F(\theta_t)$, especially its kink at θ^* (see Claim 3).

Compared with the baseline, pre-emption is avoided when one player commits not to stop first. Roughly speaking, pre-emption is an amplifier of the harm made by free-riding, and that explains why the equilibrium outcome is identical in Theorem 1 and Proposition 4 when $\beta \geq \beta^*$ as the free-riding intention is deterred in the first place. It is natural that the first mover is better off in this section compared with the baseline. However, somewhat surprisingly, the second mover may also be better off. In other words, having one player commit not to stop first can be a Pareto improvement. We formally show this finding in the following theorem.

Theorem 3. *If $\beta < \beta^*$, there exists $\theta^f < \tilde{\theta}$ such that $U_s(\theta_t) > V(\theta_t)$ when $\theta_t > \theta^f$.*

Proof. See Appendix A.10. □

The intuition behind Theorem 3 is as follow. By committing not to stop first, the benefit from avoiding pre-emption may outweigh the cost of being "inferior" in the partnership. In particular, the cost is less prominent when θ_t is large since the second mover might even be better off compared with the first mover. That explains why the commitment is more likely to be a Pareto improvement with a large θ_t.

6 Extensions

6.1 Reversible Defections

In this section, we will discuss an alternative setting where players' defections are reversible. The flow payoff is still determined by Table 1, but now players can freely switch between *Contribute* and *Defect*. By comparing this setting with the baseline, our goal is to understand how the irreversibility of players' defections plays a role in the previous results. Despite the well-known difficulties in defining and analyzing continuous-time stochastic games,[43] we will construct a simple grim trigger strategy profile and show that it already attains the first-best outcome. This enables us to avoid discussing the complicated strategy space in continuous-time games.

We start by defining the social welfare as the sum of players' utility. Due to reversibility, the stage game at each moment t does not depend on players' past actions. Therefore, the first-best outcome can be determined by maximizing the players' total flow payoff at each moment t. Specifically, this boils down to whether to have two, one, or zero contributors for each θ_t.

We first look at the case where $\beta \geq \frac{1}{1-\alpha}$. Under this parametric assumption, the first-best outcome is to let both players defect when $\theta_t < c$ and contribute when $\theta_t \geq c$.[44] In this outcome, two players always jointly defect or contribute.[45]

Now we will show that this first-best outcome is implementable with a grim trigger strategy.

[43]For details on these difficulties and potential solutions, see Simon and Stinchcombe (1989), Bergin (1992), and Bergin and MacLeod (1993) for examples.

[44]Notice that $\beta \geq \frac{1}{1-\alpha}$ indicates $\beta > 1 + \alpha$, which is the actual condition for the first-best outcome to take this form. Details are provided in Appendix A.11.

[45]This is similar to the optimal coordinative outcome in the baseline, but is different as we do not impose the irreversibility constraint here.

Under the assumption $\beta \geq \frac{1}{1-\alpha}$, it is not difficult to derive the stage-game Nash equilibrium (NE henceforth) as follow:

$$\begin{cases} (C,C) & \text{, when } \theta_t \in (\beta c, \infty) \\ (C,C) \,\&\, (D,D) & \text{, when } \theta_t \in \left[\frac{c}{1-\alpha}, \beta c\right] \\ (D,D) & \text{, when } \theta_t \in \left(0, \frac{c}{1-\alpha}\right). \end{cases} \tag{4}$$

Apparently, the strategy profile where players contribute if and only if $\theta_t \geq \frac{c}{1-\alpha}$ is an equilibrium since the stage-game NE is played at each moment. We will refer to it as the non-cooperative equilibrium and use it as the grim trigger to implement the first-best outcome: Let both players contribute if and only if $\theta_t \geq c$, and if anyone deviates, both players switch to the non-cooperative equilibrium. Notice that the first-best outcome does not coincide with the non-cooperative equilibrium only when $\theta_t \in \left[c, \frac{c}{1-\alpha}\right)$, which is the only range of θ_t that we need to check whether players will deviate from the first-best outcome. Fortunately, since (C,C) Pareto-dominates (D,D) in this range, and there is not a one-period deviation benefit for the deviating player in a continuous-time setting, we conclude that the first-best outcome is implementable.

A similar argument also applies when $\beta < \frac{1}{1-\alpha}$. The central idea is that, the one-period deviation benefit vanishes in a continuous-time setting, so a strategy profile can be supported by an equilibrium if its action profile at any moment is a Pareto improvement over a corresponding stage-game NE. We summarize our observation in the following proposition.

Proposition 5. *When defections are reversible, the first-best outcome is implementable with a grim trigger strategy.*

Proof. See Appendix A.11. □

Compared with the baseline model, Proposition 5 indicates that free-riding is an issue only when defections are irreversible. Intuitively, a deviating player can be punished in the future when his defection is reversible, while irreversibility makes that punishment physically impossible. Correspondingly, the curse of productivity also disappears with reversible defections.

6.2 Endogenous Cost Allocation

Our analysis up to this point considers exogenous cost structure for players. In this subsection, we study an alternative setting where a designer can choose how to allocate C, the total flow cost of running the project, to N players. As in the baseline, players' decisions to opt out of contribution are irreversible. We denote $a_{it} = 1$ if Player i is still contributing at time t, and $a_{it} = 0$ if Player i has already opted out. At each moment t, the designer assigns a flow cost c_{it} to Player i such that: (1) $\sum_{i=1}^{N} c_{it} = C$ if at least one player has not opted out; (2) $c_{it} = 0$ if $a_{it} = 0$. A complete plan of $\{c_{it}\}_{t\geq 0, i\in[1,N]}$ based on any information available to the designer is called a "cost allocation scheme." The setting on Player i's flow revenue remains unchanged: given that the project is operating, Player i collects a flow revenue θ_t if $a_{it} = 1$ and $\alpha\theta_t$ if $a_{it} = 0$.

Since the total amount of required cost is now fixed regardless of how many people are contributing, inefficiency occurs when the project is still operating but some players have opted out. Hence, the socially optimal outcome (subject to the irreversibility constraint) is a joint stop by all players at the threshold $\theta_{\text{opt}} = \frac{r-\mu}{r}\frac{\gamma}{\gamma-1}\frac{C}{N}$, which is solved from the optimal stopping problem with flow payoff $N\theta_t - C$ and zero stopping payoff. This outcome gives each player a value function $\tilde{V}(\theta_t) = \max\left\{-\frac{C}{Nr}\left[1-\left(\frac{\theta_t}{\theta_{\text{opt}}}\right)^{\gamma}\right] + \frac{\theta_t}{r-\mu}\left[1-\left(\frac{\theta_t}{\theta_{\text{opt}}}\right)^{\gamma-1}\right], 0\right\}$. To implement this outcome, one way is to perfectly align the interests of each individual and the society, i.e., to evenly split the flow cost among N players before anyone opts out.

To sustain this arrangement, we need to properly design the cost allocation plan in case one or more players opt out. We construct an extreme plan to minimize players' free-riding incentives: After one or more players opt out, we place the entire cost C on one of the remaining players until nobody stays. As we will show in Proposition 6, such a scheme provides the worst possible punishment for a free-rider by inducing remaining players to gradually opt out and terminate the project when $\theta_{\text{punish}} = \frac{r-\mu}{r}\frac{\gamma}{\gamma-1}C$ is reached. Hence, a free-rider will collect a stopping payoff $\tilde{F}(\theta_t) = \max\left\{\frac{\alpha\theta_t}{r-\mu}\left[1-\left(\frac{\theta_t}{\theta_{\text{punish}}}\right)^{\gamma-1}\right], 0\right\}$. To ensure that players will not deviate from the social optimum when $\theta_t > \theta_{\text{opt}}$, we need $\tilde{V}(\theta_t) \geq \tilde{F}(\theta_t)$ which is equivalent to $N \geq \beta^*$. Proposition 6 formalizes this finding. Roughly speaking, it indicates that a larger group size is better at deterring free-riding behavior as long as we design the Domino effect in a proper way.

Proposition 6. *There exists a cost allocation scheme to implement the socially optimal outcome if* *and only if* $N \geq \beta^*$.

Proof. See Appendix A.12. □

6.3 Separate Processes

In the baseline model, we assume that players' flow revenues follow an identical stochastic process. We can relax this assumption and allow separate processes for player's revenues. To be more specific, we let Player i's flow revenue be θ_{it} if he is contributing, and $\alpha\theta_{it}$ if he defects while Player j still contributes. One interpretation of this setting is that each player is running his own project but still liable to the defection made by the other player.

The problem becomes a stopping game with two-dimensional underlying states, which usually does not admit a tractable solution.[46] Despite the difficulty of formal analysis, we conjecture that some insights from the baseline model still hold here. For example, Player i's value function may be non-monotonic to the productivity of his own project: A large output can backfire by making him more committed to cover Player j's defection, which stimulates Player j to free ride. In that sense, hiding θ_{it} from Player j's observation might be beneficial to both players.

7 Concluding Remarks

This paper analyzes the free-riding problem in jointly liable partnerships. We characterize the unique MPE of a stochastic stopping game with payoff externalities, and find that players may suffer from a more lucrative project and benefit from a larger joint liability. Both findings are built on understanding a second mover's willingness to undertake the entire project.

We then generalize the model to more than two players. Under this generalization, a player's free-riding decision may lead to a Domino effect, which guides us to develop an inductive method to determine whether a coordinative equilibrium exists for a particular group size. Our result indicates that a group size is coordination-sustainable not because it is too large or too small, but instead because it can successfully deter players' intentions to free ride other group members.

[46] One exception is Ke and Villas-Boas (2019) who analyze a two-dimensional optimal stopping problem. Their model admits a tractable solution under particular parameters.

Our framework is tractable and can be further extended to address more questions regarding the design of jointly liable partnerships. For instance, how does players' complementarity in the joint project affect the equilibrium outcome? If players run individual projects, would it be helpful to hide a player's output from his partners? When players hold private information about their own payoff, how can they better coordinate in the project? We will leave these questions for future exploration.

A Proofs

A.1 Proof for Claim 1

The corresponding Hamilton-Jacobian-Bellman equation for the optimal stopping problem is

$$0 = max\left\{0 - S(\theta), -rS(\theta) + \theta - \beta c + S'(\theta)\mu\theta + \frac{\sigma^2}{2}S''(\theta)\theta^2\right\}. \quad (5)$$

The general solution for the homogenous ODE, $-rS(\theta) + \theta - \beta c + S'(\theta)\mu\theta + \frac{\sigma^2}{2}S''(\theta)\theta^2 = 0$ is

$$S(\theta) = -\frac{\beta c}{r} + \frac{\theta}{r-\mu} + k_1\theta^{\gamma^-} + k_2\theta^{\gamma^+},$$

where γ^+ and γ^- are the positive and negative root of $\Gamma(x) = \mu x + \frac{\sigma^2}{2}x(x-1) - r = 0$.[47] The solution to Equation (5) must also admit this form when $S(\theta) > 0$. Utilizing the boundary condition, $\lim_{\theta\to\infty}\left[S(\theta) - \frac{\theta}{r-\mu} + \frac{\beta c}{r}\right] = 0$, we infer that $k_2 = 0$.[48] To save notation, we replace γ^- by γ henceforth.

The unique optimal solution is to adopt a stopping threshold θ^* such that $S(\theta^*) = 0$ (value matching) and $S'(\theta^*) = 0$ (smooth pasting). Solving these two conditions yields $\theta^* = \frac{r-\mu}{r}\frac{\gamma}{\gamma-1}\beta c$. We can also pin down the value of k_1, which gives us the closed form in Claim 1.

A.2 Proof for Claim 2

Notice that the first mover is not doing any optimization at Stage 2. When $\theta_t < \theta^*$, the first mover's default will immediately trigger the second mover's subsequent default, and hence, both will gain a zero lump sum from that moment onwards, i.e., $F(\theta_t) = 0$. When $\theta_t \geq \theta^*$, the Feynman-Kac formula is

$$0 = -rF(\theta) + \alpha\theta + F'(\theta)\mu\theta + \frac{\sigma^2}{2}F''(\theta)\theta^2,$$

[47] $\Gamma(x)$ is a convex parabola with $\Gamma(0) = -r < 0$, so it must admit two roots of different signs. Specifically, $\gamma^- = \frac{\sigma^2-2\mu-\sqrt{(\sigma^2-2\mu)^2+8r\sigma^2}}{2\sigma^2}$ and $\gamma^+ = \frac{\sigma^2-2\mu+\sqrt{(\sigma^2-2\mu)^2+8r\sigma^2}}{2\sigma^2}$.

[48] This boundary condition comes from the fact that the probability of the process θ_t being absorbed by the boundary θ^* in finite time, when $\theta^* \to \infty$, approaches zero. In other words, the option value of stopping is zero in the limit, so $S(\theta)$ should be arbitrarily close to $\frac{\beta\theta}{r-\mu} - \frac{c}{r}$ then.

whose general solution is in the form of

$$F(\theta) = \frac{\alpha\theta}{r-\mu} + k_3\theta^{\gamma^-} + k_4\theta^{\gamma^+}.$$

Then we use two boundary conditions to pin down the coefficients: (1) the exogenous stopping condition $F(\theta^*) = 0$; (2) the boundary condition $\lim_{\theta\to\infty}\left[F(\theta) - \frac{\alpha\theta}{r-\mu}\right] = 0$.[49] Applying these two conditions to the general solution yields the closed form in Claim 2, where again $k_4 = 0$ and γ^- will be replaced by γ for clarity.

A.3 Proof for Lemma 1

Claim 3. *$F(\theta_t)$ is kinked at $\theta_t = \theta^*$ and strictly concave when $\theta_t \in [\theta^*, \infty)$, while $S(\theta_t)$ is differentiable at $\theta_t = \theta^*$ and strictly convex when $\theta_t \in [\theta^*, \infty)$.*

Proof. The left derivative of $F(\theta_t)$ at θ^* is 0, while the right derivative equals to $F'_+(\theta^*) = \frac{\alpha}{r-\mu} - \frac{\alpha\gamma}{(r-\mu)(\theta^*)^{\gamma-1}}(\theta^*)^{\gamma-1} = \frac{\alpha(1-\gamma)}{r-\mu} > 0$, so $F(\theta_t)$ has a kink at θ^*. Also, when $\theta_t > \theta^*$, we have $F''(\theta_t) = -\frac{\alpha\gamma(\gamma-1)}{(r-\mu)(\theta^*)^{\gamma-1}}\theta_t^{\gamma-2} < 0$, justifying the strict concavity argument. Differentiability of $S(\theta_t)$ at θ^* comes directly from the smooth pasting condition when we derive $S(\theta_t)$, i.e., $S'(\theta^*) = 0$. Strict convexity of $S(\theta_t)$ when $\theta_t > \theta^*$ comes from $S''(\theta_t) = -\frac{\beta c\gamma}{r(\theta^*)^\gamma}\theta_t^{\gamma-2} > 0$. □

In Claim 3, the property of $S(\theta_t)$ is standard for an optimal stopping problem. For $F(\theta_t)$, the kink occurs since the stopping threshold is not chosen by the first mover. The concavity argument, however, deserves some explanation. We decompose $S(\theta_t)$ as $S(\theta_t) = \frac{\theta_t}{r-\mu} - \frac{\beta c}{r} + \left(\frac{\theta_t}{\theta^*}\right)^\gamma \frac{\beta c}{r(1-\gamma)}$ when $\theta_t \geq \theta^*$. The first two terms are linear in θ_t and represents the value if she never exercises the stopping option at Stage 2, while the last term is non-linear in θ_t and reflects the option value. Similarly, $F(\theta_t)$ can be decomposed as $F(\theta_t) = \frac{\alpha\theta_t}{r-\mu} - \frac{\alpha}{(r-\mu)(\theta^*)^{\gamma-1}}\theta_t^\gamma$, where the first term is linear in θ_t and equals to the lifetime payoff he will receive if the project is never terminated by the second mover, while the second term is non-linear in θ_t and accounts for the termination loss. In Figure 7, we depict both value functions' asymptotic lines, namely, the value that only includes the linear terms.[50] As is shown by the gap between $S(\theta_t)$ and its asymptotic line, the option value

[49] The rationale of the second boundary condition is the same as Footnote 48.

[50] We call them asymptotic lines because the non-linear terms in both value functions diminish as θ_t approaches infinity, due to the fact that the probability of θ_t being absorbed by θ^* in finite time becomes negligible.

(i.e., the non-linear term) increases super-linearly as θ_t decreases towards the stopping threshold θ^*, which explains the convexity of $S(\theta_t)$ when $\theta_t \geq \theta^*$. Similarly, the termination loss in $F(\theta_t)$ also increases super-linearly as θ_t decreases towards the stopping threshold, which accounts for the concavity of $F(\theta_t)$ when $\theta_t \geq \theta^*$.

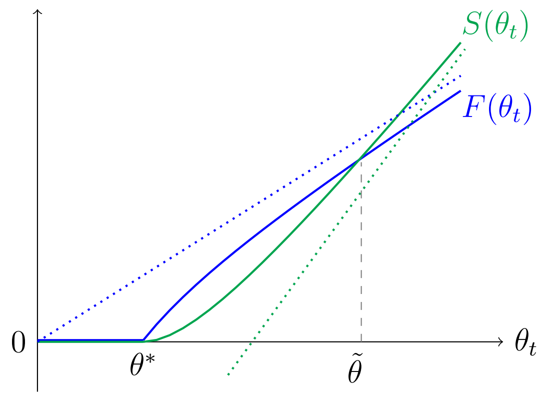


Figure 7: Value functions (solid) and their asymptotic lines (dotted)

With Claim 3 in hand, we continue to prove Lemma 1. Denote $\Delta(\theta_t) = F(\theta_t) - S(\theta_t)$. We first point out that $\Delta(\theta_t)$ is bounded by the two asymptotic lines:

$$\Delta(\theta_t) < \frac{\alpha}{r-\mu}\theta_t - \frac{1}{r-\mu}\theta_t + \frac{c}{r} = \frac{\alpha - 1}{r-\mu}\theta_t + \frac{c}{r}. \tag{6}$$

Therefore, when θ_t is sufficiently large (i.e., $\theta_t > \frac{c(r-\mu)}{r(1-\alpha)}$), the RHS of (6) is negative, and hence $\Delta(\theta_t) < 0$. Meanwhile, Claim 3 indicates that the right derivative of $\Delta(\theta_t)$ is positive at $\theta_t = \theta^*$, which implies that $\Delta(\theta^* + \epsilon) > 0$ with $\epsilon > 0$ arbitrarily small. By the continuity of $\Delta(\theta_t)$, there must be at least one zero of the function $\Delta(\theta_t)$ in the range of (θ^*, ∞).

Such a zero point is also unique. Due to the convexity of $S(\theta_t)$ and the concavity of $F(\theta_t)$ (Claim 3), we infer that $\Delta''(\theta_t) < 0$. A strictly concave function can at most admit two zero points, which are θ^* and $\tilde{\theta}$ in our case. Strict concavity of $\Delta(\theta_t)$ also indicates that $\Delta(\theta_t) > 0$ for $\theta_t \in (\theta^*, \tilde{\theta})$ and $\Delta(\theta_t) < 0$ for $\theta_t \in (\tilde{\theta}, \infty)$.

A.4 Proof for Proposition 1

Equation (1) indicates that $F(\theta_t)$ is non-increasing in β, while $F(\theta_t)$ is strictly concave and $V_c(\theta_t)$ is strictly convex when $\theta_t \in (\theta^*, \infty)$ (Claim 3). These properties imply the existence of a critical value β^* under which $V_c(\theta_t)$ tangentially intersects with $F(\theta_t)$ in the range of (θ^*, ∞). We denote such a point of tangency as $\bar{\theta}$, so $F(\bar{\theta}) = V_c(\bar{\theta})$ and $F'(\bar{\theta}) = V_c'(\bar{\theta})$ must hold, i.e.,

$$-\frac{c}{r} + \frac{(1-\alpha)\bar{\theta}}{r-\mu} + \left(\frac{\bar{\theta}}{\theta^{**}}\right)^{\gamma} K = 0 \tag{7}$$

$$\frac{(1-\alpha)\bar{\theta}}{r-\mu} + \gamma\left(\frac{\bar{\theta}}{\theta^{**}}\right)^{\gamma} K = 0, \tag{8}$$

where $K = \frac{c}{r} - \frac{\theta^{**}}{r-\mu} + \frac{\alpha\theta^*}{r-\mu}\left(\frac{\theta^{**}}{\theta^*}\right)^{\gamma} = \frac{(1-\alpha\gamma\beta^{1-\gamma})c}{r(1-\gamma)}$. We then subtract (8) from (7)$*\gamma$ and get

$$\bar{\theta} = \frac{1}{1-\alpha}\frac{r-\mu}{r}\frac{\gamma}{\gamma-1}c = \frac{1}{1-\alpha}\theta^{**}. \tag{9}$$

Then we plug Equation (9) into Equation (8) and get $1 - \beta^{1-\gamma}\alpha\gamma = (1-\alpha)^{\gamma}$, which gives $\beta^* = \left[\frac{1-(1-\alpha)^{\gamma}}{\alpha\gamma}\right]^{\frac{1}{1-\gamma}}$. To complete the analysis, we also need to prove that $\beta^* > 1$. It suffices to show that $\frac{1-(1-\alpha)^{\gamma}}{\alpha\gamma} > 1$, i.e., $\alpha\gamma + (1-\alpha)^{\gamma} > 1$. Let $\Lambda(\alpha) = \alpha\gamma + (1-\alpha)^{\gamma}$. Since $\Lambda(0) = 1$ and $\Lambda'(\alpha) = \gamma[1-(1-\alpha)^{\gamma-1}] > 0$, it is verified that $\Lambda(\alpha) > 1$.

If $\beta \geq \beta^*$, we know from the construction of β^* that $V_c(\theta_t) \geq F(\theta_t)$ for $\forall\theta_t$. Thus the coordinative equilibrium exists. If $\beta < \beta^*$, we need to show that players want to deviate in $(\theta^*, \tilde{\theta})$, the region with first mover advantage.

Claim 4. *When $\beta < \beta^*$, there exists $\theta_t \in (\theta^*, \tilde{\theta})$ such that $F(\theta_t) > V_c(\theta_t)$.*

Proof. According to the construction of β^*, $F(\theta_t)$ and $V_c(\theta_t)$ must intersect (non-tangentially) in the range of (θ^*, ∞) when $\beta < \beta^*$. Moreover, the intersections must be in the range of $(\theta^*, \tilde{\theta})$. This is because $V_c(\theta_t) > S(\theta_t) \geq F(\theta_t)$ when $\theta_t \geq \tilde{\theta}$, which indicates that $V_c(\theta_t)$ and $F(\theta_t)$ should not intersect in the range of $[\tilde{\theta}, \infty)$. □

A.5 Proof for Proposition 3

It suffices to show that $V_c(\theta_t) \geq V_p(\theta_t)$ for $\forall\theta_t$ when $\beta \geq \beta^*$. This can be proved by the following three arguments. First, $V_c(\theta_t) > V_p(\theta_t)$ for $\theta_t \in [\theta^*, \tilde{\theta}]$. This is because the existence of the optimal coordinative equilibrium implies that $V_c(\theta_t) \geq F(\theta_t)$ in this range, while $V_p(\theta_t) = \frac{1}{2}[S(\theta_t) + F(\theta_t)] < F(\theta_t)$.

Second, $V_c(\theta_t) > V_p(\theta_t)$ for $\theta_t > \tilde{\theta}$. We regard $V_c(\theta_t)$ (resp. $V_p(\theta_t)$) as the value to one who receives $\theta_t - c$ until stopping with a lump-sum payoff $V_c(\tilde{\theta})$ (resp. $V_p(\tilde{\theta})$) when $\tilde{\theta}$ is reached. Hence, this argument holds because $V_c(\tilde{\theta}) > V_p(\tilde{\theta})$.

Third, $V_c(\theta_t) \geq V_p(\theta_t)$ for $\theta_t < \theta^*$. The reason is similar to the second argument while now we exploit the fact that $V_c(\theta^*) > V_p(\theta^*)$.[51]

A.6 Proof for Corollary 1

The monotonicity w.r.t. α is easy to prove: $\frac{\partial\left[\frac{1-(1-\alpha)^\gamma}{\alpha\gamma}\right]}{\partial\alpha} = \frac{(1-\alpha)^{\gamma-1}[\alpha(1+\gamma)-1]-1}{\gamma\alpha^2} > 0$. Since γ strictly decreases in μ,[52] for the argument about μ it suffices to show that β^* strictly decreases in γ. We first replace $\frac{1}{1-\alpha}$ by z and express β^* as $\beta^* = f(\gamma) := \left[\frac{z^{1-\gamma}-z}{-(z-1)\gamma}\right]^{\frac{1}{1-\gamma}}$. We would like to show that $f'(\gamma) < 0$. Let $f(\gamma) = [g(\gamma)]^{\frac{1}{1-\gamma}}$ and $h(\gamma) = ln(g(\gamma))$. Since

$$
\begin{aligned}
f'(\gamma) &= \frac{1}{1-\gamma} g(\gamma)^{\frac{1}{1-\gamma}-1} g'(\gamma) + g(\gamma)^{\frac{1}{1-\gamma}} ln(g(\gamma)) \frac{1}{(1-\gamma)^2} \\
&= \frac{g(\gamma)^{\frac{1}{1-\gamma}}}{1-\gamma} \left[\frac{g'(\gamma)}{g(\gamma)} + \frac{ln(g(\gamma))}{1-\gamma}\right] \\
&= \frac{f(\gamma)}{1-\gamma} \left[h'(\gamma) - \frac{h(1) - h(\gamma)}{1-\gamma}\right],
\end{aligned}
$$

it suffices to show that

$$
h'(\gamma) - \frac{h(1) - h(\gamma)}{1-\gamma} < 0. \tag{10}
$$

Notice that $\frac{h(1)-h(\gamma)}{1-\gamma}$ is the slope of the secant line between γ and 1 on the curve of $h(\cdot)$, one sufficient condition for Condition (10) to hold is that $h(\gamma)$ is convex, i.e., $g(\gamma)$ is log-convex.

To prove the log-convexity of $g(\gamma)$, we only need to show that $z^{\gamma x} g(\gamma)$ is convex for $\forall x \in \mathbb{R}$.[53] We adopt Taylor Expansion w.r.t. γ on $z^{\gamma x} g(\gamma)$ to show that

[51] We can also infer that $\theta^0 < c$, due to the usual logic of strategic delay in optimal stopping problem. We also expect $\theta^0 > \theta^{**}$, since the exogenous stopping at θ^* decreases the option value of waiting.

[52] This result is standard in the literature and can be derived from the closed-form of γ shown in Section 3.1.

[53] See Page 70 of Niculescu and Persson (2006) for reference.

$$
\begin{aligned}
z^{\gamma x} g(\gamma) &= \frac{z}{(z-1)\gamma}\left(z^{\gamma x} - z^{\gamma x - \gamma}\right) \\
&= \frac{z}{(z-1)\gamma}\left[\sum_{n=0}^{\infty}(x ln(z))^n \gamma^n - \sum_{n=0}^{\infty}((x-1) ln(z))^n \gamma^n\right] \\
&= \frac{z}{(z-1)}\sum_{n=1}^{\infty}(ln(z))^n \left[x^n - (x-1)^n\right]\gamma^{n-1}.
\end{aligned}
$$

Since $z > 1$, and $x^n - (x-1)^n > 0$ for $\forall n \geq 1, \forall x \in \mathbb{R}$, we conclude that $z^{\gamma x} g(\gamma)$ is convex.

A.7 Proof for Theorem 2

Step 1. We prove a lemma that generalizes Proposition 1. It aims to derive the critical condition such that $V_n(\theta_t)$ and $F_{n'}(\theta_t)$ tangentially intersect.

Lemma 2. *For given $n > n'$,*
***(a)** if $\frac{\beta_{n'}}{\beta_n} \geq \beta^*$, $V_n(\theta_t) \geq F_{n'}(\theta_t)$ for $\forall \theta_t$;*
***(b)** if $\frac{\beta_{n'}}{\beta_n} < \beta^*$, $V_n(\theta_t) < F_{n'}(\theta_t)$ for some θ_t.* [54]

Proof. The proof is a revision of Appendix A.4. Equations (8) and (9) can be revised as

$$
\frac{(1-\alpha)\bar{\theta}}{r-\mu} + \gamma\left(\frac{\bar{\theta}}{\theta_n^*}\right)^{\gamma} K' = 0 \tag{11}
$$

$$
\bar{\theta} = \frac{1}{1-\alpha}\frac{r-\mu}{r}\frac{\gamma}{\gamma-1}\beta_n c = \frac{1}{1-\alpha}\theta_n^*, \tag{12}
$$

where $K' = \frac{\beta_n c}{r} - \frac{\theta_n^*}{r-\mu} + \frac{\alpha\theta_{n'}^*}{r-\mu}\left(\frac{\theta_n^*}{\theta_{n'}^*}\right)^{\gamma} = \frac{\left(1-\alpha\gamma\left(\frac{\beta_n}{\beta_{n'}}\right)^{-\gamma}\right)\beta_n c}{r(1-\gamma)}$. Substitute Equation (12) into Equation (11), we get

$$
1 - \left(\frac{\beta_{n'}}{\beta_n}\right)^{1-\gamma}\alpha\gamma = (1-\alpha)^{\gamma}.
$$

Hence, $\frac{\beta_{n'}}{\beta_n} = \beta^*$ is the critical condition where $V_n(\theta_t)$ tangentially intersects with $F_{n'}(\theta_t)$. □

[54] Proposition 1 corresponds to the special case with $n' = 1$, $n = 2$, $\beta_1 = \beta$, and $\beta_2 = 1$ here.

Step 2. We then prove the theorem by induction. As we already show in Example 1, the coordinative outcome is not an equilibrium when the group size N is larger than 1 and smaller than $n^{(1)}$, since players will be tempted to deviate and get $F_1(\theta_t)$ when it outweighs $V_N(\theta_t)$. Meanwhile, a $n^{(1)}$-player coordinative equilibrium exists according to the definition of $n^{(1)}$.

By induction, if n is larger than $n^{(i-1)}$ and smaller than $n^{(i)}+1$, a deviation from the coordinative outcome will give a stopping payoff of $F_{n^{(i-1)}}(\theta_t)$ since the Domino effect will work until $n^{(i-1)}$ players are remaining, i.e., the project will run until $\theta^*_{n^{(i-1)}}$ is reached. By Lemma 2, we conclude that a coordinative outcome is not an equilibrium when $n \in [n^{(i-1)}, n^{(i)}-1]$, and it is an equilibrium when $n = n^{(i)}$.

A.8 Proof for Corollary 2

We first prove "only if". Suppose M is the largest group size to maintain a coordinative equilibrium. By Lemma 2, $\beta_n \geq \frac{\beta_M}{\beta^*}$ for $\forall n > M$. Since $\frac{\beta_M}{\beta^*}$ is bounded away from 0, we conclude that β_n does not converge to zero.

We then prove "if". By Lemma 2, $\beta_{n^{(i)}} \leq \frac{\beta_1}{(\beta^*)^{i-1}}$. Hence, the maximal number of coordination-sustainable group size $I = \left\lceil log_{\beta^*}\left(\frac{\beta_1}{\lim\limits_{n\to\infty}\beta_n}\right)\right\rceil$ is finite since $\beta^* > 1$ and $\lim\limits_{n\to\infty}\beta_n > 0$. This indicates that the corresponding set must be bounded.

A.9 Proof for Proposition 4

The proof is mostly contained in the analysis following Proposition 4. Here we complement some analysis regarding the three thresholds. Before further analysis, the associated Hamilton-Jacobian-Bellman equation for this optimal stopping problem is

$$0 = max\{F(\theta_t) - U_f(\theta_t), -rU_f(\theta_t) + \theta_t - c + U_f'(\theta_t)\mu\theta_t + \frac{\sigma^2}{2}U_f''(\theta_t)\theta_t^2\}.$$

Step 1. First, we show that the thresholds in Proposition 4 exist by construction. Scenario (a) is straightforward as we already have the closed-form. For Scenario (b), we start from θ^3. Notice that the conditions to pin down θ^3 are exactly Equations (7) and (8), and thus we conclude that $\theta^3 = \frac{1}{1-\alpha}\theta^{**}$ as in Equation (9). For θ^1 and θ^2, the general solution of $U_f(\theta_t)$ when $\theta_t \in [\theta^1, \theta^2]$ is

$U_f(\theta_t) = -\frac{c}{r} + \frac{\theta_t}{r-\mu} + k_6\theta_t^{\gamma^-} + k_7\theta_t^{\gamma^+}$.[55] The value matching and smooth pasting conditions for θ^1 and θ^2 are thus (we denote $k_5 = -\frac{1}{(r-\mu)\gamma(\theta^{**})^{\gamma-1}}$)

$$
\begin{aligned}
-\frac{c}{r} + \frac{\theta^1}{r-\mu} + k_6(\theta^1)^{\gamma^-} + k_7(\theta^1)^{\gamma^+} &= 0 \quad (13)\\
\frac{\theta^1}{r-\mu} + \gamma^- k_6(\theta^1)^{\gamma^-} + \gamma^+ k_7(\theta^1)^{\gamma^+} &= 0 \quad (14)\\
-\frac{c}{r} + \frac{(1-\alpha)\theta^2}{r-\mu} + (k_6-k_5)(\theta^2)^{\gamma^-} + k_7(\theta^2)^{\gamma^+} &= 0 \quad (15)\\
\frac{(1-\alpha)\theta^2}{r-\mu} + \gamma^-(k_6-k_5)(\theta^2)^{\gamma^-} + \gamma^+ k_7(\theta^2)^{\gamma^+} &= 0. \quad (16)
\end{aligned}
$$

Claim 5. $k_7 > 0$ *and* $k_6 > k_5$.

Proof. Let (16) − (15)$*\gamma^-$ and (16) − (15)$*\gamma^+$, we have

$$\frac{c\gamma^-}{r} + (1-\gamma^-)\frac{(1-\alpha)\theta^2}{r-\mu} + k_7(\gamma^+ - \gamma^-)(\theta^2)^{\gamma^+} = 0 \quad (17)$$

$$\frac{c\gamma^+}{r} + (1-\gamma^+)\frac{(1-\alpha)\theta^2}{r-\mu} + (k_6-k_5)(\gamma^- - \gamma^+)(\theta^2)^{\gamma^-} = 0 \quad (18)$$

By construction, we already require that $\theta^2 < \theta^3 = \frac{\theta^{**}}{1-\alpha}$. Plug it into Equations (17) and (18) we conclude that $k_7 > 0$ and $k_6 > k_5$. □

From (14) − (13)$*\gamma^-$ and (14) − (13)$*\gamma^+$, we have

$$
\begin{aligned}
\frac{c\gamma^-}{r} + (1-\gamma^-)\frac{\theta^1}{r-\mu} + k_7(\gamma^+ - \gamma^-)(\theta^1)^{\gamma^+} &= 0\\
\frac{c\gamma^+}{r} + (1-\gamma^+)\frac{\theta^1}{r-\mu} + k_6(\gamma^- - \gamma^+)(\theta^1)^{\gamma^-} &= 0.
\end{aligned}
$$

We can thus express k_6 and k_7 as functions of θ^1, i.e.,

$$
\begin{aligned}
k_7(\theta^1) &= \frac{c\gamma^-}{r(\gamma^- - \gamma^+)}(\theta^1)^{-\gamma^+} + \frac{1-\gamma^-}{(r-\mu)(\gamma^- - \gamma^+)}(\theta^1)^{1-\gamma^+} \quad (19)\\
k_6(\theta^1) &= \frac{c\gamma^+}{r(\gamma^+ - \gamma^-)}(\theta^1)^{-\gamma^-} + \frac{1-\gamma^+}{(r-\mu)(\gamma^+ - \gamma^-)}(\theta^1)^{1-\gamma^-}. \quad (20)
\end{aligned}
$$

[55] Notice that the boundary condition when $\theta_t \to \infty$ no longer holds here.

We then construct the following function that takes θ^1 as a parameter,

$$\tilde{U}(\theta_t; \theta^1) = -\frac{c}{r} + \frac{\theta_t}{r-\mu} + k_6(\theta^1)\theta_t^{\gamma^-} + k_7(\theta^1)\theta_t^{\gamma^+}. \tag{21}$$

Finding θ^1, θ^2, k_6, and k_7 that satisfy Equations (13) to (16) is thus equivalent to finding a value for θ^1 such that $\tilde{U}(\theta_t; \theta^1)$ tangentially intersects with $F(\theta_t)$ for $\theta_t > \theta^1$. If we let the point of tangency be θ^2, Equations (13) to (16) will be satisfied.

We further let $\tilde{\Delta}(\theta_t; \theta^1) = \tilde{U}(\theta_t; \theta^1) - F(\theta_t)$. It suffices to find a value of θ^1 so that $\tilde{\Delta}(\theta_t; \theta^1)$ tangentially intersects with the x-axis. This is possible as we have the following claim that shows the strict convexity of $\tilde{\Delta}(\theta_t; \theta^1)$.

Claim 6. *$\tilde{\Delta}(\theta_t; \theta^1)$ is strictly convex in θ_t for $\forall\theta^1$.*

Proof. $\tilde{\Delta}''(\theta_t; \theta^1) = \gamma^-(\gamma^- - 1)(k_6 - k_5)\theta_t^{\gamma^- - 2} + \gamma^+(\gamma^+ - 1)k_7\theta_t^{\gamma^+ - 2} > 0$ as we already know from Claim 5 that $k_7 > 0$ and $k_6 > k_5$, and also $\gamma^- < 0$ and $\gamma^+ > 1$. □

On one hand, let $\theta^1 = \theta^{**}$, we have $\tilde{\Delta}(\theta_t; \theta^{**}) = S(\theta_t)$. According to the assumption of $\beta < \beta^*$, we should have $\inf_{\theta_t \in (\theta^*, \infty)} \tilde{\Delta}[\theta_t; \theta^{**}) < 0$ as illustrated by the right panel of Figure 6. On the other hand, if let $\theta^1 = \epsilon$ with $\epsilon > 0$ sufficiently small, it is not difficult to see that $\inf_{\theta_t \in [\theta^*, \infty)} \tilde{\Delta}(\theta_t; \epsilon) > 0$. By continuity w.r.t. θ^1, there must exist $\hat{\theta}^1 \in (0, \theta^{**})$ such that $\inf_{\theta_t \in [\theta^*, \infty)} \tilde{\Delta}(\theta_t; \hat{\theta}^1) = 0$. According to Claim 6 and the fact that $\tilde{\Delta}(\infty; \theta^1) = \infty$, we conclude that the infimum of $\inf_{\theta_t \in [\theta^*, \infty)} \tilde{\Delta}(\theta_t; \hat{\theta}^1) = 0$ is attainable. Let $\hat{\theta}^2$ be the point of infimum, we can verify that $\tilde{\Delta}(\hat{\theta}^2; \hat{\theta}^1) = 0$, $\tilde{\Delta}'(\hat{\theta}^2; \hat{\theta}^1) = 0$, and $\tilde{\Delta}(\theta_t; \hat{\theta}^1) > 0$ for $\forall\theta_t \in [\theta^*, \infty)/\{\hat{\theta}^2\}$.

We also need to verify that $\hat{\theta}^2$ is consistent with the presumption that $\theta^2 < \theta^3$. This can be proved as follow. Combining Equations (17) and (19), we get

$$\frac{c\gamma^-}{r}\left[(\theta^2)^{\gamma^+} - (\theta^1)^{\gamma^+}\right] + \frac{(1-\gamma^-)}{r-\mu}\left[\theta^1(\theta^2)^{\gamma^+} - (1-\alpha)\theta^2(\theta^1)^{\gamma^+}\right] = 0, \tag{22}$$

which gives us $\hat{\theta}^2 < \frac{1}{1-\alpha}\hat{\theta}^1 < \frac{1}{1-\alpha}\theta^{**} = \theta^3$. Hence, we conclude that $\tilde{U}(\theta_t; \hat{\theta}^1) = -\frac{c}{r} + \frac{\theta_t}{r-\mu} + k_6(\hat{\theta}^1)\theta_t^{\gamma^-} + k_7(\hat{\theta}^1)\theta_t^{\gamma^+}$ will smoothly paste with $F(\theta_t)$ at the two thresholds, $\hat{\theta}^1$ and $\hat{\theta}^2$, while satisfying $\tilde{U}(\theta_t; \hat{\theta}^1) > F(\theta_t)$ for $\forall\theta_t \in (\hat{\theta}^1, \hat{\theta}^2)$.

Step 2. We next show the uniqueness of these thresholds specified in Proposition 4. Scenario (a) is straightforward as we already have the closed-form. For Scenario (b), θ^3 also has a closed-form, so what remains to be proved is the uniqueness of θ^1 and θ^2. Following the analysis in Step 1, it suffices to show that there exists only one $\hat{\theta}^1$ satisfying $\inf_{\theta_t \in [\theta^*, \infty)} \tilde{\Delta}(\theta_t; \hat{\theta}^1) = 0$.

One sufficient condition for the above argument to hold is that $\inf_{\theta_t \in [\theta^*, \infty)} \tilde{\Delta}(\theta_t; \theta^1)$ is single-crossing w.r.t. θ^1 (i.e., crosses the x-axis only once). By Envelope Theorem, we only need to show that $\frac{\partial \tilde{\Delta}}{\partial \theta^1}(\hat{\theta}^2, \hat{\theta}^1)$ is either positive or negative. We have

$$\begin{aligned}
\frac{\partial \tilde{\Delta}}{\partial \theta^1}(\hat{\theta}^2, \hat{\theta}^1) \cdot \hat{\theta}^1 &= k_6'(\hat{\theta}^1)\hat{\theta}^1 \left(\hat{\theta}^2\right)^{\gamma^-} + k_7'(\hat{\theta}^1)\hat{\theta}^1 \left(\hat{\theta}^2\right)^{\gamma^+} \\
&= (1-\gamma^-)k_6 \left(\hat{\theta}^2\right)^{\gamma^-} + (1-\gamma^+)k_7 \left(\hat{\theta}^2\right)^{\gamma^+} + \frac{c}{r(\gamma^+ - \gamma^-)} \left[\gamma^- \left(\frac{\hat{\theta}^2}{\hat{\theta}^1}\right)^{\gamma^+} - \gamma^+ \left(\frac{\hat{\theta}^2}{\hat{\theta}^1}\right)^{\gamma^-}\right] \\
&= \frac{c}{r} + k_5(1-\gamma^-) \left(\hat{\theta}^2\right)^{\gamma^-} + \frac{c}{r(\gamma^+ - \gamma^-)} \left[\gamma^- \left(\frac{\hat{\theta}^2}{\hat{\theta}^1}\right)^{\gamma^+} - \gamma^+ \left(\frac{\hat{\theta}^2}{\hat{\theta}^1}\right)^{\gamma^-}\right] \\
&= k_5(1-\gamma^-) \left(\hat{\theta}^2\right)^{\gamma^-} + \frac{c}{r(\gamma^+ - \gamma^-)} \left[\gamma^+ - \gamma^- + \gamma^- \left(\frac{\hat{\theta}^2}{\hat{\theta}^1}\right)^{\gamma^+} - \gamma^+ \left(\frac{\hat{\theta}^2}{\hat{\theta}^1}\right)^{\gamma^-}\right] \\
&< k_5(1-\gamma^-) \left(\hat{\theta}^2\right)^{\gamma^-} + \frac{c}{r(\gamma^+ - \gamma^-)} \left[\gamma^+ - \gamma^- + \gamma^- - \gamma^+\right] \\
&= k_5(1-\gamma^-) \left(\hat{\theta}^2\right)^{\gamma^-} \\
&< 0.
\end{aligned}$$

The first equality comes directly from Equation (21). The second equality is obtained by plugging in the expressions of $k_6(\theta_t)$ and $k_7(\theta_t)$ in Equations (19) and (20). The third equality makes use of Equations (15) and (16). The fourth equality combines like terms. The first inequality comes from the following fact: the function $\Phi(x) = \gamma^- x^{\gamma^+} - \gamma^+ x^{\gamma^-}$ is decreasing in x when $x \geq 1$, while $\frac{\hat{\theta}^2}{\hat{\theta}^1} > 1$. The last inequality comes from $k_5 < 0$. We can eventually conclude that the single-crossing condition holds, so there exists a unique pair of $(\hat{\theta}^1, \hat{\theta}^2)$ satisfying Equations (13) to (16).

Step 3. Finally, we argue that the strategy suggested in Proposition 4 is optimal for the first mover, i.e., the constructed $U_f(\theta_t)$ satisfies the HJB equation. Strulovici and Szydlowski (2015) point out that, to verify this argument, it suffices to prove the following three conditions. First, $U_f(\theta_t)$ is nowhere below $F(\theta_t)$, the lump sum stopping payoff. Second, $U_f(\theta_t)$ is smooth, i.e., everywhere continuous and first-order differentiable. Third, $-rU_f(\theta_t) + \theta_t - c + U_f'(\theta_t)\mu\theta_t + \frac{\sigma^2}{2}U_f''(\theta_t)\theta_t^2 \leq 0$ whenever $U_f(\theta_t) = F(\theta_t)$. From our construction, it is not difficult to check that these three conditions hold for both Scenarios (a) and (b), as is also illustrated by Figure 6.

A.10 Proof for Theorem 3

First, we show that $\theta^3 < \tilde{\theta}$. This is because θ^3 is smaller than the largest intersection of $V_c(\theta_t)$ and $F(\theta_t)$ (according to Appendix A.9), while $\tilde{\theta}$ must be larger than that intersection since $V_c(\theta_t) > S(\theta_t)$. Roughly speaking, the endogenous first mover in the baseline stops earlier than the designated first mover in Section 5.

We then prove Theorem 3 for $\theta_t = \tilde{\theta}$. Notice that $S(\tilde{\theta})$ is equivalent to the value of a player (when $\theta_t = \tilde{\theta}$) who keeps receiving a flow payoff $\theta_t - \beta c$ until exogenously stopping at θ^3 and collecting $S(\theta^3)$. Meanwhile, $U_s(\tilde{\theta})$ is equivalent to the value of a player (when $\theta_t = \tilde{\theta}$) who keeps receiving a flow payoff $\theta_t - c$ until exogenously stopping at θ^3 and collecting $S(\theta^3)$. Comparing these two scenarios, we conclude that $U_s(\tilde{\theta}) > S(\tilde{\theta})$. Meanwhile, from Lemma 1 we know that $V(\tilde{\theta}) = S(\tilde{\theta})$. This gives us $U_s(\tilde{\theta}) > V(\tilde{\theta})$.

For $\forall\theta_t > \tilde{\theta}$, $V(\theta_t)$ is equivalent to the value of a player who keeps receiving a flow payoff $\theta_t - c$ until exogenously stopping at $\tilde{\theta}$ and collecting $V(\tilde{\theta})$. Meanwhile, $U_s(\theta_t)$ is equivalent to the value of a player who keeps receiving a flow payoff $\theta_t - c$ until exogenously stopping at $\tilde{\theta}$ and collecting $U_s(\tilde{\theta})$. These two arguments, together with $U_s(\tilde{\theta}) > V(\tilde{\theta})$, prove $U_s(\theta_t) > V(\theta_t)$ when $\theta_t > \tilde{\theta}$.

The two paragraphs above show that $U_s(\theta_t)$ is strictly greater than $V(\theta_t)$ when $\theta_t \geq \tilde{\theta}$. Finally, according to the continuity of $U_s(\theta_t)$ and $V(\theta_t)$, there must exist $\theta^f < \tilde{\theta}$ such that $U_s(\theta_t) - V(\theta_t) \geq 0$ when $\theta_t \geq \theta^f$.

A.11 Proof for Proposition 5

We first derive the first-best outcome. As we already mentioned in Section 6.1, it suffices to find the optimal number of contributors for each θ_t, i.e., comparing $2\theta_t - 2c$ (two contributors), $(1+\alpha)\theta_t - \beta c$ (one contributor) and zero (no contributor). It is not difficult to verify that: When $\beta \geq 1 + \alpha$, the optimum is to have two contributors when $\theta_t \geq c$ and no contributor otherwise;

when $\beta < 1+\alpha$, it is optimal to have two contributors when $\theta_t \geq \frac{2-\beta}{1-\alpha}c$, one contributor when $\theta_t \in \left[\frac{\beta}{1+\alpha}c, \frac{2-\beta}{1-\alpha}c\right)$, and zero contributor when $\theta_t < \frac{\beta}{1+\alpha}c$.

We next derive the stage-game NE as we need it to define the grim trigger strategy. When $\beta \geq \frac{1}{1-\alpha}$, the NE is shown by Equation (4). When $\beta < \frac{1}{1-\alpha}$, the NE is

$$\begin{cases} (C,C) & \text{, when } \theta_t \in \left(\frac{1}{1-\alpha}c, \infty\right) \\ (C,D) \;\&\; (D,C) & \text{, when } \theta_t \in \left[\beta c, \frac{1}{1-\alpha}c\right] \\ (D,D) & \text{, when } \theta_t \in (0, \beta c)\,. \end{cases} \tag{23}$$

We can construct non-cooperative equilibria based on stage-game NE. It is worth noticing that, one non-cooperative equilibrium is particularly undesirable to Player 1 as we implement (D,C) when $\theta_t \in \left[\beta c, \frac{1}{1-\alpha}c\right]$. Vice versa, there also exists one that is undesirable to Player 2 when (C,D) is implemented instead.

Finally, we show that the first-best outcome is implementable with a grim trigger strategy. If $\beta \geq \frac{1}{1-\alpha}$, the analysis is already included in Section 6.1. If $\beta \in \left[1+\alpha, \frac{1}{1-\alpha}\right)$, we can adopt the following trigger strategy: In case one player deviates from the first-best outcome, we let the players switch to the non-cooperative equilibrium that is most undesirable to this deviating player, as we specified by Equation (23). With this harsh punishment in place, the first-best outcome is implementable.

If $\beta < 1+\alpha$, the first-best outcome involves indeterminacy since when $\theta_t \in \left[\frac{\beta}{1+\alpha}c, \frac{2-\beta}{1-\alpha}c\right)$, we should let one player contribute and the other defect. To ensure fairness, we let the players take turns to contribute. As the frequency of alternation gets sufficiently large, we can use $\frac{1}{2}\left[(1+\alpha)\theta_t - \beta c\right]$ to represent each player's flow payoff. In order to implement this fair first-best outcome, we can adopt the following grim trigger strategy: If one player deviates from this outcome, we let the players switch to the non-cooperative equilibrium that is most undesirable to the one who deviates. Since the fair first-best outcome generates a flow payoff that always Pareto-dominates the stage-game NE most undesirable to the deviator, we conclude that the first-best outcome can be achieved.

A.12 Proof for Proposition 6

Step 1. We first show that, under the punishment scheme specified in Section 6.2, the project will be terminated when θ_{punish} is reached the next time. Without loss of generality, we allocate the entire cost to Player i at Stage i (i.e., $i-1$ players already opted out) for $i \in [2, N]$.

At Stage N, Player N will adopt the stopping threshold at θ_{punish}. At Stage $(N-1)$, Player $(N-1)$ is facing a stopping problem with flow payoff $\theta_t - C$ and stopping payoff $\tilde{F}(\theta_t)$. We can show that the optimal strategy is to adopt a stopping threshold at $\frac{\theta_{\text{punish}}}{1-\alpha}$. Then we induce backward to Stage $(N-2)$: After Player $(N-2)$ stops, the project will keep operating until the next time θ_{punish} is reached. Hence, the lump-sum stopping payoff for Player $(N-2)$ is also $\tilde{F}(\theta_t)$ and his stopping threshold is also $\frac{\theta_{\text{punish}}}{1-\alpha}$. By induction, we infer that the stopping payoff for Player 1 is also $\tilde{F}(\theta_t)$.

Step 2. Then we show that the social optimum is implementable if $N \geq \beta^*$, i.e., $\tilde{V}(\theta_t) \geq \tilde{F}(\theta_t)$ for $\forall \theta_t > \theta_{\text{opt}}$. This can be easily seen as we substitute $\beta_{n'} = N$ and $\beta_n = 1$ into Lemma 2(a).

We also need to show that the social optimum is not achievable if $N < \beta^*$. According to Lemma 2(b), there exists θ_t such that $\tilde{V}(\theta_t) < \tilde{F}(\theta_t)$, while it is impossible for all players to achieve higher value than $\tilde{V}(\theta_t)$. Therefore, free-riding must occur.

B Miscellaneous Analysis

B.1 Tie-Breaking Assumption

We provide three justifications for this tie-breaking assumption that commonly appears in the literature of stochastic timing games. First, a player's stopping decision may need to be validated through an authorization procedure, while the authority in charge can only approve of one application at a time. Usually, the approval depends on who is first in line or other rationing methods (Grenadier, 1996). Second, there might be a random delay between a player's stopping decision (or the expression of intention) and the actual exercise of that decision. If the length of delay follows an atomless continuous distribution that is identical and independent for both players, each player will stop first with 50% probability. The current setting can be regarded as a limit case when the random delay converges in probability to zero.[56] Third, players may establish a communication scheme (or other coordination devices) to avoid miscoordination, namely, an undesirable joint stop. One will prefer to stop only when his partner does not, so players will benefit from a coordination device that allows one of them to withdraw the stopping decision in case both attempt to stop.

Denote Player i's continuation value upon a simultaneous stop by $M_i(\theta_t)$. The tie-breaking assumption is equivalent to $M_1(\theta_t) = M_2(\theta_t) = \frac{1}{2}[S(\theta_t) + F(\theta_t)]$. One possible way to relax this assumption is to assume $\min\{F(\theta_t), S(\theta_t)\} \leq M_i(\theta_t) \leq \max\{F(\theta_t), S(\theta_t)\}$ for $i = 1, 2$ instead. This alternative assumption retains the main results but does not provide new insights, so in the baseline model we still stick to the original assumption for simplicity.

B.2 A General Model on Group Size

The characterization on coordination-sustainable group sizes in Section 4 depends on two assumptions: First, the free-riding parameter is constant for $n_t \geq 1$; second, $\beta_1 \leq \beta^{**}$. Here we provide a more general model without these assumptions. In particular, we denote a free-rider's flow revenue as $\alpha_{n_t}\theta_t$ with no additional assumptions. The idea of Domino effect remains in place, but we no longer have a neat characterization result. Instead, we develop an algorithm to determine which group sizes can sustain coordination. This algorithm also enables us to determine the exit waves of players.

The algorithm adopts an inductive structure. We start from examining the subgame with one remaining player. As we already show, this last remaining player will adopt the stopping set $\tilde{\Theta}^1 = (0, \theta_1^*)$. Inductively, suppose we know the players' equilibrium stopping sets $\tilde{\Theta}^k$ and equilibrium value function $\tilde{V}_k(\theta_t)$ for $k = 1, 2, ..., n-1$,[57] we can examine whether a n-player coordinative equilibrium exists in following steps.

Step 1. *Derive the optimal coordinative value function $\tilde{V}_{c,n}(\theta_t)$. This is identical to Equation (3). Here we rephrase it as $\tilde{V}_{c,n}(\theta_t)$ for consistency with future steps.*

Step 2. *Derive a player's stopping payoff $\tilde{F}_n(\theta_t)$. The challenge of this step is that, for $\forall\theta_t$ we*

[56] See Weeds (2002) for an explicit formulation of this justification with a micro foundation and Dutta and Rustichini (1993) for a reduced-form formulation.

[57] We restrict attention to symmetric equilibrium.

need to first determine how the remaining $(n-1)$ *players gradually opt out in the future, based on previous derivations of* $\tilde{\Theta}^k$ *for* $k = 1, 2, ..., n-1$*. Then we can tell how* α_{n_t} *evolves stochastically and thus calculate the expected stopping payoff for a free-rider,* $\tilde{F}_n(\theta_t)$*.*

Step 3. *If* $\tilde{V}_{c,n}(\theta_t) \geq \tilde{F}_n(\theta_t)$ *for* $\forall \theta_t \geq \theta_n^*$*, then a* n*-player coordinative equilibrium exists. Therefore,* $\tilde{\Theta}^n = (0, \theta_n^*)$*, and* $\tilde{V}_n(\theta_t) = \tilde{V}_{c,n}(\theta_t)$*.*

Step 4. *If* $\tilde{V}_{c,n}(\theta_t) < \tilde{F}_n(\theta_t)$ *for some* $\theta_t \geq \theta_n^*$*, then a* n*-player coordinative equilibrium fails to exist. Similar to the baseline model, we need to first determine the "first mover advantage region(s)" by comparing* $\tilde{F}_n(\theta_t)$ *and* $\tilde{V}_{n-1}(\theta_t)$*, where each first mover advantage region is a connected set of* θ_t*. Then we can derive the optimal pre-emptive equilibrium in the following way:*[58]

Step 4.1 *Identify the first mover advantage regions that are subject to pre-emption and let players stop in these regions.*

Step 4.2 *Derive players' optimal stopping strategy outside the first mover advantage regions. Derive the corresponding value function* $\tilde{V}_{p,n}(\theta_t)$*. If it is everywhere higher than* $\tilde{F}_n(\theta_t)$*, we are done. If not, return to Step 4.1 and identify more first mover advantage regions that are subject to pre-emption.*

With the optimal pre-emptive equilibrium in hand, we are able to derive players' stopping set $\tilde{\Theta}^n$ *and equilibrium value function* $\tilde{V}_n(\theta_t) = \tilde{V}_{p,n}(\theta_t)$*.*

[58] See Dutta and Rustichini (1993) for a similar description of this algorithm.

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